Which product to advertise? Optimal price advertising with heterogeneous consumers[*]

Marie-Laure Allain [†]   Sebastien Cerles[‡]   Claire Chambolle[§]

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Abstract

When consumers do not see prices before they shop, retailers advertise the prices of some goods to attract customers into their stores. In a setting where a monopolist sells two goods but can advertise only one, we show that the optimal advertising strategy depends on the heterogeneity of consumers' tastes and shopping costs. When consumers have relatively high shopping costs, it becomes profitable to advertise the product with more dispersed valuations, because it induces stronger one-stop shopping complementarities. Such an advertising strategy can even yield a higher consumer surplus than under perfect information.

1 Introduction

While manufacturers advertise to promote a specific product against its competitors, retailers typically sell thousands of different products, and their advertising strategy primarily aims to attract more customers into their stores. In a setting where consumers have imperfect information

[†]CREST, CNRS, Institut Polytechnique de Paris, 5 avenue Henry Le Chatelier, 91120 Palaiseau, France. e-mail: allain@ensae.fr.

[‡]Université Paris-Saclay, INRAE, AgroParisTech, Paris-Saclay Applied Economics, 91120, Palaiseau, France, and CREST. e-mail: sebastien.cerles@inrae.fr.

[§]Université Paris-Saclay, INRAE, AgroParisTech, Paris-Saclay Applied Economics, 91120, Palaiseau, France. e-mail: claire.chambolle@inrae.fr.

about prices before visiting a store and must incur shopping costs to learn those prices, retailers can advertise the prices of a small subset of the products they sell to attract customers.

Such marketing practices, of which weekly price leaflets are a prime example, raise several questions. First, from a retailer's standpoint, it is important to define which products to advertise: should a supplier advertise products that are equally valued by most consumers, that is, staples such as bread or toilet paper, or is it a better strategy to advertise more polarizing goods, such as soy milk or coconuts, for which consumers' valuations are more heterogeneous. Then, how are consumers affected by these different advertising strategies? Ultimately, from a regulator's perspective, it is essential to characterize under which circumstances it may be optimal to regulate price advertising or price disclosure strategies.

To answer these questions, we investigate the advertising and pricing behavior of a multi-product monopolist. In our model, a monopolist retailer sells two goods, denoted by A and B. It can inform consumers about the price of one of these goods by sending an advertisement, thereby signaling a commitment to that price. Consumers exhibit homogeneous valuations for good A, while their valuations for good B are heterogeneous. They are interested in purchasing at most one unit of each good, are privately informed about their valuations for good B, and are unaware of unadvertised prices before they reach the store. They form rational expectations regarding the price of the other good. Consumers must incur a shopping cost to visit the store and learn all prices. A key assumption of our model is that consumers differ in their shopping costs. We document how this model relates to important product markets such as food retail.

We first investigate a benchmark with no information. We show that there cannot be a market without price information, because consumers incur shopping costs before visiting the store, which hold them up to the point where they would rather stay home than buy anything. This result is consistent with the Diamond paradox.

We then study a benchmark model with full information. This allows us to describe the optimal pricing strategy of the monopolist when there are no price commitment issues.

We then solve our advertising model to analyze the impact of incomplete information, namely, when the monopolist can advertise only one of the two goods (henceforth, partial information). We characterize subgame equilibria depending on whether the retailer advertises good A or good B.

We show that advertising good A, for which consumers have homogeneous valuations, generates a larger demand-expansion effect than advertising good B. The reason is that the discount

on good A is relevant for all consumers regardless of their valuation for good B, whereas an additional discount on good B attracts only consumers with sufficiently high valuations for good B.

Moreover, we exhibit that good B, for which consumers have heterogeneous valuations, begets more complementarities than good A, because all consumers who visit the store after having received an advert for good B end up purchasing good A as well, whereas only some of the consumers who visited the store after having received an advert for good A end up purchasing good B as well. Because of this complementarity effect, the optimal advertised price of good B is always lower than the advertised price of good A.

Advertising product B is therefore relatively more costly for the monopolist and becomes profitable only when the complementarity effect outweighs the demand expansion effect, which occurs when shopping costs are high. As a consequence, when shopping costs are relatively low, the monopolist chooses to advertise good A, and when shopping costs are relatively high, the monopolist chooses to advertise good B.

Eventually, we show that consumers always prefer the situation in which good B is advertised over the situation in which good A is advertised. We also demonstrate that this complementarity effect is so potent that consumers may end up better off when the monopolist advertises good B than they would be under perfect information.

The importance of this complementarity effect can be traced back to Ramsey (1927), who analyzes the optimal pricing of a multiproduct monopoly. The model shows that complementarities among products affect the optimal price-cost margins, such that the more complementary a product is to the others, the lower its margin should be. This is because a lower price for a complementary product increases the demand and surplus for the other products. The pro-competitive impact of the complementarity generated by one-stop shoppers has been empirically assessed by Thomassen et al. (2017).[1]

We contribute to the literature on price advertisement and incomplete information markets by modeling two types of goods: one with constant valuations, following Lal and Matutes (1994), and one with heterogeneous valuations, following Rhodes (2014). We also introduce

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[1]Thomassen et al. (2017) find that supermarkets have more market power when they lower their prices than when they raise them, because they can attract more consumers who also buy complementary products. The authors empirically quantify the pro-competitive nature of this complementarity effect, showing that one-stop shoppers drive more competition between retailers than multi-stop shoppers, even though the latter tend to cherry-pick the best price for each good.

heterogeneous shopping costs among consumers, which affect their decisions to visit and buy from the retailer.

Our model draws from Lal and Matutes (1994), who assume that consumers have homogeneous valuations for all products sold by the firms. In their model, a firm's advertised price for one product reveals no information about its price for the other product. By contrast, we allow consumer valuations for one of the two goods to be heterogeneous and drawn from a continuous distribution. As a result, changes in the advertised price affect the composition of consumers who choose to visit the store, which in turn influences the firm's optimal pricing strategy for the other product.

In this regard, our model is closer to Rhodes (2014) than to Lal and Matutes (1994), because we also consider multiproduct retailing with heterogeneous valuations. Rhodes (2014) shows that a lower advertised price attracts more price-sensitive consumers, and thus lowers the firm's incentive to charge high prices for the other products. In our model, this Rhodes effect plays an important part, and it is precisely what makes possible the existence of an equilibrium under incomplete information. However, because we assume heterogeneous shopping costs rather than homogeneous ones, discounts on the heterogeneously valued good attract consumers who have both high shopping costs and a large valuation for this good. This creates incentives to charge high prices for the heterogeneously valued good, and constitutes a major brake to the Rhodes effect.

As an extension, we investigate a model where consumers have homogeneous shopping costs, as in Rhodes (2014). We show that advertising a single product allows the monopolist to perfectly mimic the full information situation, making accounting for heterogeneous shopping costs a crucial component in our novel mechanism.

We also contribute to the literature on search costs and shopping costs. As Choi, Dai, and Kim (2018) shows, higher search costs may benefit consumers. However, they assume perfect price information but imperfect information about the quality of the match between a consumer and a seller. Thus, their results are driven by the fact that lower search costs enable more searches, which in turn improve the average quality of matches and consumers' willingness to pay. In our model, higher search costs may benefit consumers by changing the discount policy of a monopolist retailer.

Our model and results also relate to and differ from Ellison (2005), who shows that partial information usually benefits firms rather than consumers. This is because Ellison (2005) focuses

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on a model with competing firms that can restrict consumers' knowledge of prices through the use of add-on pricing. Their results rely on firms being able to use partial information to discriminate between two kinds of consumers. Our results, however, are in line with some papers showing how partial information may benefit consumers more than full information (See Zapechelnyuk, 2020; Luco, 2019; Lizzeri, 1999).[2]

Our article thus put forward a novel mechanism to explain why partial information may benefit consumers more than full information. This has important policy implications. Indeed, we show that, provided consumers have sufficiently high shopping costs, and these costs are heterogeneous enough, it is detrimental to consumers to promote full information. In other words, a regulator should not facilitate full price information, as partial information incentivizes retailers to offer very large discounts, which benefit consumers. Whereas the existing literature, mostly empirical (Luco, 2019; Ater and Rigbi, 2023), finds that the anti-competitive effects of full information may explain why partial information benefits consumers, we highlight complementarity as a new channel through which partial information may benefit consumers.

The paper is organized as follows. Section 2 presents our mainline model; Section 3 analyzes a benchmark without information; Section 4 characterizes a benchmark with full information; Section 5 considers advertising and determines on which product advertising is most profitable for the retailer. Eventually, Section 6 discusses the effect of the advertising strategies on consumer surplus and derives some policy implications. Section 7 provides an extension in which consumers have homogeneous search costs, and Section 8 concludes.

2 Model

A monopolist retailer sells two goods, A and B, at a marginal cost normalized to zero. It is located at 0 on a segment of length 1. A mass 1 of consumers is uniformly distributed along

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[2]Zapechelnyuk (2020) studies a problem of quality certification in a moral hazard setting, where a producer chooses a quality and a price of a product, and a consumer observes the price and a rating assigned by a certifier. The paper demonstrates that partial information may be preferable to full information for two reasons. First, partial information may induce more efficient price-quality trade-offs, as the producer faces less price competition from lower-quality products. Second, partial information may create more incentives for quality improvement. Conducing an empirical evaluation of a price disclosure obligation for Chilean petrol stations, Luco (2019) show that, in some areas, perfect information was detrimental to consumers. The authors resort to tacit collusion as the most likely theoretical rationale behind their results.

the segment, and a consumer located at a distance x from the retailer incurs shopping costs tx. We assume $t > 0$.[3] Shopping costs can either be interpreted as transportation costs, as in a Hotelling model à la Lal and Matutes (1994), or as search costs, as in Rhodes (2014). We assume all consumers have an equal valuation $v_A = a$ for good A, while they differ in their valuations v_B for good B. More precisely, v_B is uniformly distributed between $a - z$ and $a + z$ (with $z \leq a$), so that the average valuation for each good is the same. We assume $z > \frac{a}{3}$.[4] Note that both v_B and x are uniformly distributed, and we assume the two distributions are independent: the location x of a consumer is not correlated to her valuation v_B for good B.

The utility of each consumer is additively separable. The utility of a consumer located in x is the sum of her valuations for each good she buys, minus the prices she pays and the shopping costs. We normalize the outside option to 0. As a result, the purchasing behavior of a consumer located in x stems from the following maximization program:

$$U(x, v) = \max(v_A + v_B - p_A - p_B - tx, v_A - p_A - tx, v_B - p_B - tx, 0)$$

Goods A and B are neither substitutes nor complements, but shopping costs induce some complementarities, because a consumer who visits the store incurs tx regardless of the number of products she buys.

Before they reach the store, consumers are uninformed about prices, unless advertised. The price of an advertised good is known by all, and the price of each good is observed by consumers once they are in the store. We assume the retailer can advertise at most one good, and we normalize advertising costs to 0. This corresponds to the situation in which retailers only advertise a subset of the products they sell.

The timing of the game is as follows.

Stage 1: The monopolist chooses which product to advertise and at what price.

Stage 2: The monopolist sets the price of the unadvertised good.

Stage 3: Consumers learn the price of the advertised good and form expectations regarding the price of the unadvertised one. They decide whether to visit the store and which goods to buy.

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[3] Solving the model is possible for $t = 0$, but this special case without shopping costs tantamount to a perfect information setting with $t = 0$.

[4] This assumption guarantees that consumer heterogeneity is sufficiently large for the optimal price to be interior when all consumers are present in the store. We provide further justification for this assumption in Section A.2.

Formally, the first two stages are simultaneous. However, the informational structure is not the same in these two stages. The sequential timing reflects that the monopolist must commit to the advertised price but can freely modify the other price as long as the consumers are not yet in the store.

We look for subgame perfect Bayesian-Nash equilibria.

We believe this model corresponds to several important product markets, such as food retail or consumer staples. Indeed, these are markets in which the number of available goods is so large consumers usually display limited to no price recall (see Monroe and Lee, 1999). These are also markets in which consumers' taste heterogeneity itself varies across products: there is less consumer heterogeneity for peanut butter or salt than for Jerusalem artichokes or luxury olive oil. Moreover, as shown by Bagwell (2007), retailers' advertising mostly contains information about prices, rather than information about products' characteristics. We thus focus on a setting where valuations are known in advance and where search is a sunk cost incurred to learn prices.[5]

Our modeling of shopping cost is also consistent with empirical evidence. Using consumer panel data, Smith and Thomassen (2012) show that consumers visit on average 1.71 shops per week, but that the standard deviation is 1.31. This is coherent with two features from our model: on the one hand, that consumers do not visit more shops imply that they incur some sort of shopping costs. These shopping costs can be interpreted as transportation costs, that is, the physical cost of going to the shop in terms of gasoline, or as search costs, that is, the time and energy taken for visiting a store. On the other hand, that consumers do not all visit the same number of shops imply they incur heterogeneous shopping costs. This can be due to consumers living closer or further away from the store, or to individuals having more or less available time.

3 Benchmark: imperfect information and no advertising

We first show that there is no market when firms cannot advertise prices, that is, when consumers observe all prices once in the store. Without advertising, we denote $p_A^\emptyset$ and $p_B^\emptyset$ the prices of goods A and B, where $\emptyset$ stresses the absence of information.

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[5]Other articles instead assume that prices are known in advance but the value of the match between a consumer and a product is learned after search has occurred (see for instance Chen, Li, and Zhang (2022))

Lemma 1. *Without advertising, $p_A^\emptyset = a$ and $p_B^\emptyset = a + z$, so that no consumer ever visits the store.*

Proof. See Appendix A.1. □

The economic force behind Lemma 1 is a standard hold-up problem with shopping costs. When consumers do not observe prices before entering, all consumers who decide to visit reveal that their expected net surpluses from paying the shopping cost are non-negative. Anticipating that, the retailer has an incentive to raise prices, thereby holding-up consumers who have already paid the shopping cost. Consumers correctly infer this, so that paying the search cost is not optimal and the market unravels.

The logic behind this result can be traced back to the Diamond paradox (Diamond, 1971). While the classic statement of the paradox concerns monopoly pricing despite many sellers, our setting is closer to Lal and Matutes (1994), who show that, without advertising and when consumers hold homogeneous valuations for all goods, the Diamond paradox extends to multiproduct retailers. However, in a very general setting, Rhodes (2014) highlights that taste heterogeneity across several products may sustain market existence in the absence of advertising, because consumers who incur the shopping cost to buy one or several goods for which they have very high valuations may still be interested in buying other goods for which their valuations are closer to the average, and the retailer thus has an incentive to keep prices low enough to maximize its profit on in-store consumers. In our more restricted framework, we deliberately combine a product with homogeneous valuations (good A, similar to Lal and Matutes, 1994) and a product with heterogeneous valuations (good B, similar to Rhodes, 2014). Lemma 1 shows that the presence of a single anchor good with homogeneous valuation is enough to reintroduce the hold-up logic.

This benchmark provides a baseline for the rest of the paper: by ruling out trade when no prices are advertised, it highlights why price advertising can be essential and motivates our main question: how the retailer's incentives to advertise differ across two goods with distinct demand structures.

4   Benchmark model with perfect information

In order to describe the effect of advertising, we first solve another benchmark model with perfect information. This setting corresponds to the case in which consumers are informed

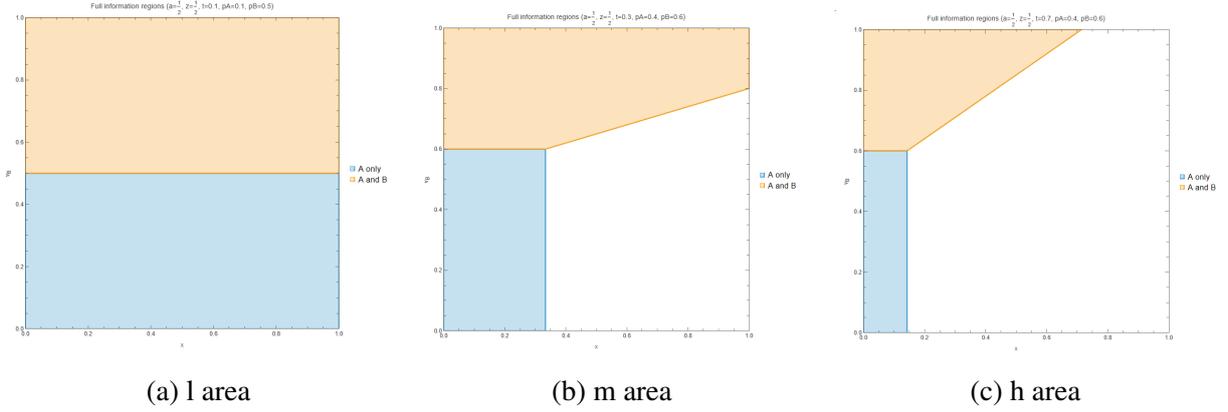| (a) l area | (b) m area | (c) h area |

Figure 1: The different possible shapes of the demand depending on prices, tastes, and shopping costs (Full Information)

about all prices before visiting the store. Within our framework, it can also be interpreted as a situation in which the monopolist can freely advertise all prices.

A consumer located at a distance x from the retailer incurs shopping costs tx to visit the store. Once in the store, the consumer can either buy good A alone, or good B alone, or both goods. Of course, the consumer can also choose not to visit the store. We normalize this outside option to 0. As a result, the purchasing behavior of a consumer located in x whose valuation for good B is equal to v_B stems from the following maximization program:

$$U(x, v_B) = \max(v_A + v_B - p_A - p_B - tx, v_A - p_A - tx, v_B - p_B - tx, 0)$$

In the subsequent discussion, we denote the equilibrium prices in the full information case as p_A^{fs} and p_B^{fs}, where the superscript f stands for full information, and where the superscript $s = \{l, m, h\}$ represents different levels of shopping costs: low, medium, or high[6]. Similarly, we denote the demand for each product and profit under full information respectively D_A^{fs}, D_B^{fs} and π^{fs}. The profit is defined as $\pi^{fs} \equiv p_A^{fs} D_A^{fs} + p_B^{fs} D_B^{fs}$.

Under full information, Figure 1 shows the different configurations of consumer purchasing behaviors, according to the level of shopping costs. D_A^{fs} corresponds to the sum of the blue and yellow areas, and D_B^{fs} corresponds to the yellow area. We rule out dominated strategies where $p_A > a$ (zero demand for product A). The first graph corresponds to the demand when

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[6]"Low", "medium", or "high" is more thoroughly defined in each section, because it depends on the information structure

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shopping costs are low enough for all consumers to visit the store. In this situation, since all consumers have the same valuation $v_A = a$ for good A, all consumers buy good A. Moreover, all consumers whose valuation v_B for good B is above p_B buy good B. In this low shopping costs area ($s = l$), demands are thus:

$$
\begin{aligned}
D_A^{fl}(p_A, p_B) &= 1 & (1)\\
D_B^{fl}(p_A, p_B) &= \mathbb{P}(v_B \geq p_B) = \int_{p_B}^{a+z} \frac{1}{2z} dv_B & (2)
\end{aligned}
$$

The second graph in Figure 1 corresponds to the situation in which shopping costs are medium ($s = m$). In this case, consumers' decisions to visit the store depend on their location x and their valuation v_B for good B. Intuitively, some consumers reside sufficiently close to the store so their surplus from purchasing good A alone outweighs the associated shopping cost. Yet, other consumers may choose to visit the store because their valuation of good B is high enough so that the combined surplus of purchasing goods A and B exceeds the shopping cost. We define the m area as the set of situations in which some consumers do not visit the store, but all consumers with maximal valuation for good B do. More precisely, consumers whose x is such that $v_A - p_A - tx \geq 0$ visit the store regardless of their valuations for good B. These consumers always buy good A, and buy good B whenever $v_B \geq p_B^{fm}$. Consumers whose location $x > \frac{a - p_A^{fm}}{t}$ is too far away to buy good A alone can still visit the store to purchase both goods provided $v_B \geq p_A^{fm} + p_B^{fm} + tx - a$. In the m area, $\forall x \in [0,1] : p_A^{fm} + p_B^{fm} + tx - a < a + z$, and demands are:

$$
\begin{aligned}
D_A^{fm}(p_A, p_B) &= \int_0^{\frac{a-p_A}{t}} \int_{a-z}^{a+z} \frac{1}{2z} dv_B dx + \int_{\frac{a-p_A}{t}}^{1} \int_{p_A+p_B+tx-a}^{a+z} \frac{1}{2z} dv_B dx & (3)\\
D_B^{fm}(p_A, p_B) &= \int_0^{\frac{a-p_A}{t}} \int_{p_B}^{a+z} \frac{1}{2z} dv_B dx + \int_{\frac{a-p_A}{t}}^{1} \int_{p_A+p_B+tx-a}^{a+z} \frac{1}{2z} dv_B dx & (4)
\end{aligned}
$$

The third graph in Figure 1 corresponds to the situation in which shopping costs are high ($s = h$). As in the m area, consumers' decisions to visit the store depend on their location x as well as their valuation v_B for good B. The difference with the m area is that shopping costs are now so high that consumers at the far end of the segment never visit the store, regardless of their valuation for good B. Formally, $\exists \tilde{x}, \forall v_B \in [a-z, a+z]$ defined by $v_A + v_B - p_A - p_B - t\tilde{x} = 0$

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such that no consumer whose $x \geq \tilde{x} = \frac{2a+z-p_A^{fh}-p_B^{fh}}{t}$ ever visits the store. Demands in the h area are:

$$D_A^{fh} \ (p_A, p_B) = \int_0^{\frac{a-p_A}{t}} \int_{a-z}^{a+z} \frac{1}{2z} dv_B dx + \int_{\frac{a-p_A}{t}}^{\frac{2a+z-p_A-p_B}{t}} \int_{p_A+p_B+tx-a}^{a+z} \frac{1}{2z} dv_B dx \quad (5)$$

$$D_B^{fh} \ (p_A, p_B) = \int_0^{\frac{a-p_A}{t}} \int_{p_B}^{a+z} \frac{1}{2z} dv_B dx + \int_{\frac{a-p_A}{t}}^{\frac{2a+z-p_A-p_B}{t}} \int_{p_A+p_B+tx-a}^{a+z} \frac{1}{2z} dv_B dx \quad (6)$$

In each area, prices are set to maximize the profit. As the frontiers of the three areas depend on p_A and p_B, the monopoly's profit is defined piecewise. The global profit function is continuous, and we characterize local maxima on each area to define the global maximum. Formally, for $s \in \{l, m, h\}$, we compute the derivatives of π^{fs} with respect to p_A^{fs} and p_B^{fs}. Locally, the profit is concave except in the case $s = l$ where we have a corner solution, and thus maximized by the following two FOCs:

$$\frac{\partial \pi^{fs}}{\partial p_A} = p_A \frac{\partial D_A^{fs}}{\partial p_A} + D_A^{fs} + p_B \frac{\partial D_B^{fs}}{\partial p_A} = 0 \quad (7)$$

$$\frac{\partial \pi^{fs}}{\partial p_B} = p_A \frac{\partial D_A^{fs}}{\partial p_B} + D_B^{fs} + p_B \frac{\partial D_B^{fs}}{\partial p_B} = 0 \quad (8)$$

Eventually, depending on the shopping cost parameter t, we solve the optimal pricing decisions of the retailer. For any given shopping cost t, Lemma 2 describes the equilibrium prices:

Lemma 2. *When shopping costs are heterogeneous, under perfect information, there exist two thresholds $\underline{t}^f \equiv \frac{a(3z-a)}{7z-a} > 0$ and $\bar{t}^f \equiv 2a + z - p_A^{fm} - p_B^{fm} = 2a + z - p_A^{fh} - p_B^{fh}$ such that:*

- *when $t \leq \underline{t}^f$: in equilibrium, all consumers visit the store. The price of good A is $p_A^{fl} = a - t$ and the price of good B is $p_B^{fl} = \frac{a+z}{2}$.*

- *when $\underline{t}^f \leq t \leq \bar{t}^f$: in equilibrium, all consumers whose location x is close to 0 visit the store. Among consumers whose location x is close to 1, only those whose valuation v_B for good B is high enough visit the store. p_A^{fm} and p_B^{fm} are the solution of the system of FOCs (7) and (8) for $s = m$.*

- *when $t \geq \bar{t}^f$: in equilibrium, all consumers whose location x is close to 0 visit the store. No consumer whose location x is close to 1 visits the store. p_A^{fh} and p_B^{fh} are the solutions of the system of FOCs (7) and (8) for $s = h$.*
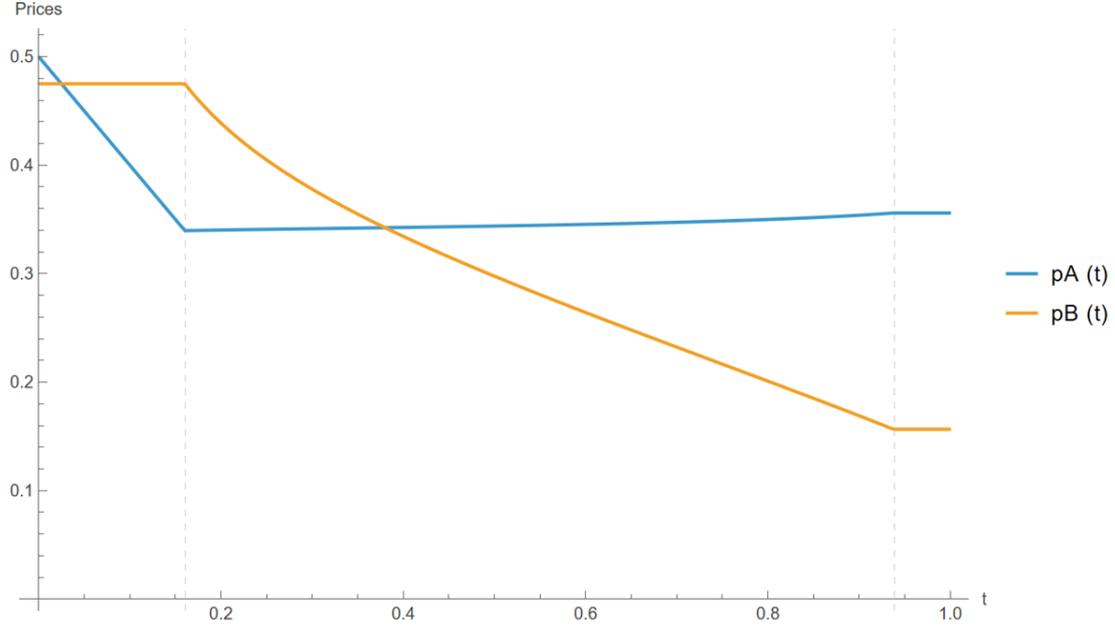
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Figure 2: Equilibrium prices as a function of t and s in the full information benchmark with heterogeneous shopping costs ($a = 0.5$, $z = 0.45$)

Proof. See proof in the appendix A.2. $\qquad\square$

With $a = 0.5$ and $z = 0.45$[7], Figure 1 displays equilibrium prices in each different area, while Figure 2 shows the prices of goods A and B as a function of the shopping cost parameter t. It is important to note that, as long as $t \leq \underline{t}^f$, the monopolist decreases p_A^{fl} to attract all consumers, and thus keeps p_B^{fl} constant. However, in the m and h areas, it is optimal to discount p_B^{fs} rather than p_A^{fs} to attract consumers. In Lemma 3, we show that p_A^{fs} is actually weakly increasing in t in the m and h areas. Note that in the particular case in which $a = z = \frac{1}{2}$, as shown in Figure 2, we have p_A^{fm} constant in t.

Lemma 3. *Under full information and with heterogeneous shopping costs, when the solution is interior:* $\frac{\partial p_A^{fm}}{\partial t} \geq 0$ *and* $\frac{\partial p_A^{fh}}{\partial t} = 0$.

Proof. See proof in the appendix A.3. $\qquad\square$

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[7]With $a = z = 0.5$, p_A^{fm} is flat in t. We show the graph with $z = 0.45$ to convince the reader that it can indeed be increasing in t.

Intuitively, this result relates to some important features of our model: when the shopping cost is so high it becomes unprofitable to attract all consumers to the store, lowering the price of good B allows to attract consumers willing to buy both goods, whereas lowering the price of good A also attracts consumers who are not interested in buying good B. In other words, discounting good B begets more complementarities between the two goods, and it is profitable to slightly increase the price of good A to benefit from this complementarity effect. Graphically, on Figure 1, this corresponds to the expansion of the yellow area and the reduction of the blue area. In the h area, however, it is now optimal to keep p_A constant. The intuition behind this result is more subtle, and requires to understand what distinguishes area m from area h. In the m area, some consumers located in $x = 1$ whose valuation for good B is below $a + z$ still visit the store. This is no longer the case in the h area. These corner consumers imply that, for any given price p_A, the distribution of v_B is more heavily upward-distorted in the m area than in the h area. As a consequence, for any given price p_A, the optimal price p_B is higher in the m area than in the h area. To countervail this effect, the retailer uses p_A to attract more consumers with average valuations for good B. Thus, the more corner consumers visit the store, the more the retailer discounts p_A. Since an increase in t diminishes the number of corner consumers, the countervailing value of p_A becomes weaker, and the retailer has an incentive to raise p_A. In the h area, corner consumers no longer visit the store, so that any increase in t no longer diminishes the countervailing value of p_A, and the incentive to raise p_A as t increases disappear.

5   Advertising

We now focus on partial information, which corresponds to our main model. We respectively describe the subgame equilibria that prevail when the monopolist advertises good A, and when the monopolist advertises good B. Eventually, we endogenize the retailer's advertising strategy.

5.1   The monopolist advertises good A

In the subsequent discussion, we denote the equilibrium prices when the monopolist advertises good A as p_A^{As} and p_B^{As}, where the superscript A stands for "good A is advertised". Similarly, we denote the profit when the monopolist advertises good A as π^{As}. The superscript $s = \{l, m, h\}$ here again refers to different levels of shopping costs.

As in the previous section, a consumer located at a distance x from the retailer incurs shopping costs tx to visit the store. The main difference with the previous section is that consumers must now decide whether to visit the store based on their anticipations regarding the unobserved price of good B, $\widehat{p}_B$. As a result, they first decide whether or not to visit the store based on their anticipations, and if they decide to pay the shopping cost, they eventually choose the goods they buy once in the store, based on the actual observed price of good B, p_B. Formally, a consumer located in x whose valuation for good B is equal to v_B visits the store if and only if:

$$\max(v_A + v_B - p_A - \widehat{p}_B - tx, v_A - p_A - tx, v_B - \widehat{p}_B - tx) \geq 0$$

Then, once in the store, her purchasing behavior stems from the following maximization program:

$$U(x, v_B) = \max(v_A + v_B - p_A - p_B, v_A - p_A, v_B - p_B, 0)$$

As in the full information benchmark, this consumer program allows for three different shapes of demands. Under partial information, Figure 3b displays these different possible shapes.

Although the demands are very similar to the full information benchmark, they now depend on the anticipated price $\widehat{p}_B$ of good B as well as the actual in-store price p_B. The l area is defined as the area in which all consumers visit the store.

$$
\begin{aligned}
D_A^{Al}(p_A, \widehat{p}_B, p_B) &= 1 & (9) \\
D_B^{Al}(p_A, \widehat{p}_B, p_B) &= \mathbb{P}(v_B \geq p_B) = \int_{p_B}^{a+z} \frac{1}{2z} dv_B & (10)
\end{aligned}
$$

In the l area, when the shopping costs are very low, the best strategy of the monopolist is to sell to all consumers. p_A^{Al} is set to its maximum level such that all consumers still visit the store. In other words, the consumer who is the least likely to visit the store, that is, the one who lives in $x = 1$ and does not value good B at all ($v_B = 0$), must be indifferent between visiting the store or not: $p_A^{Al} = a - t$. Since all consumers visit the store to buy good A, p_B^{Al} is set to its monopoly level: $p_B^{Al} = \frac{a+z}{2}$. Up to this point, this is similar to the perfect information benchmark.
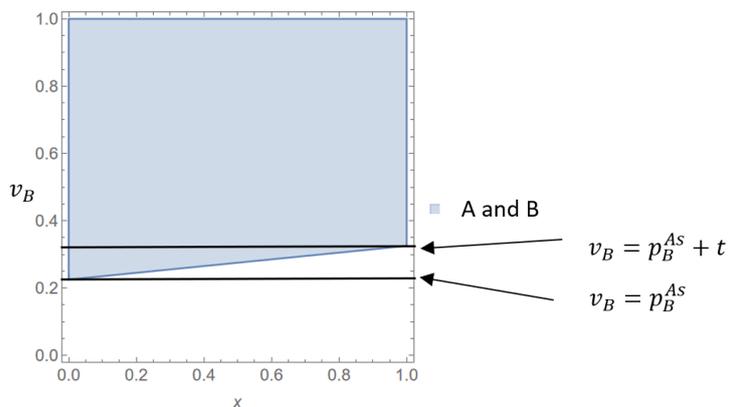
Let us now solve the cases in which some consumers cease to visit the store, that is, the m and h areas.

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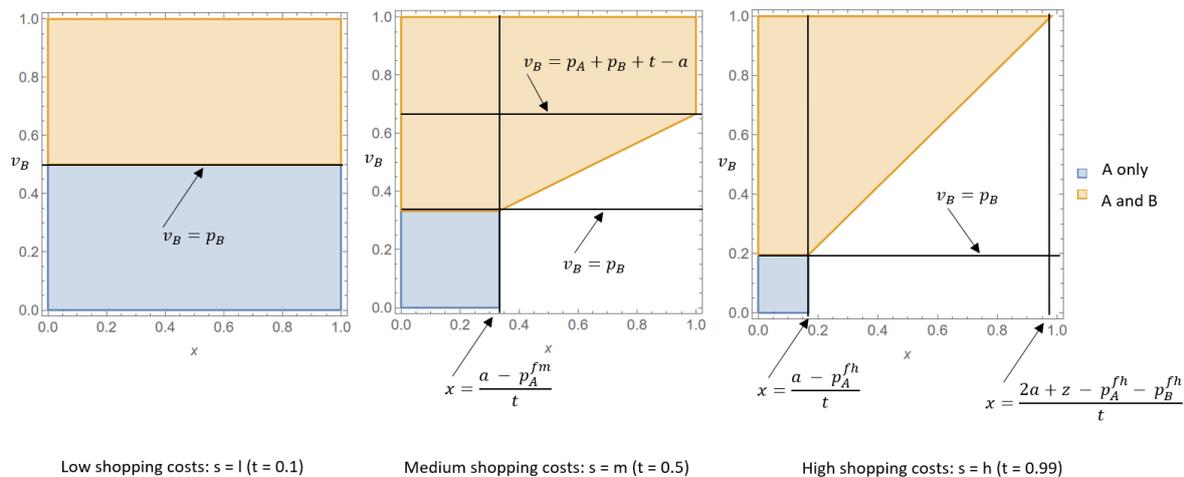
(a) Full Information
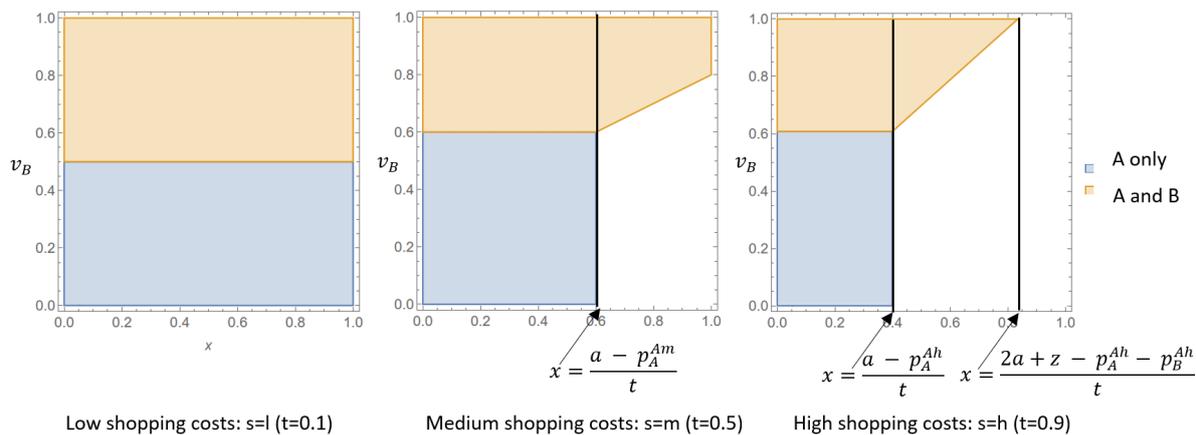
(b) Advertising good A

(c) Advertising good B

Figure 3: Equilibrium purchasing decisions of consumers depending on x, v, s, and the information structure ($a = z = \frac{1}{2}$)

When $t \leq 2a + z - p_A - \widehat{p}_B$, that is, when all consumers whose valuation for good B equals $a + z$ visit the store, we have:

$$D_A^{Am}(p_A, p_B, \widehat{p}_B) = \int_0^{\frac{a-p_A}{t}} \int_{a-z}^{a+z} \frac{1}{2z} dv_B dx + \int_{\frac{a-p_A}{t}}^1 \int_{p_A+\widehat{p}_B+tx-a}^{a+z} \frac{1}{2z} dv_B dx$$

$$D_B^{Am}(p_A, p_B, \widehat{p}_B) = \int_0^{\frac{a-p_A}{t}} \int_{p_B}^{a+z} \frac{1}{2z} dv_B dx + \int_{\frac{a-p_A}{t}}^1 \int_{\max(p_A+\widehat{p}_B+tx-a,p_B)}^{a+z} \frac{1}{2z} dv_B dx$$

When $t \geq 2a + z - p_A - \widehat{p}_B$, that is, when some consumers whose valuation for good B equals $a + z$ do not visit the store, we have:

$$D_A^{Ah}(p_A, p_B, \widehat{p}_B) = \int_0^{\frac{a-p_A}{t}} \int_{a-z}^{a+z} \frac{1}{2z} dv_B dx + \int_{\frac{a-p_A}{t}}^{\frac{2a+z-p_A-\widehat{p}_B}{t}} \int_{p_A+\widehat{p}_B+tx-a}^{a+z} \frac{1}{2z} dv_B dx$$

$$D_B^{Ah}(p_A, p_B, \widehat{p}_B) = \int_0^{\frac{a-p_A}{t}} \int_{p_B}^{a+z} \frac{1}{2z} dv_B dx + \int_{\frac{a-p_A}{t}}^{\frac{2a+z-p_A-\widehat{p}_B}{t}} \int_{\max(p_A+\widehat{p}_B+tx-a,p_B)}^{a+z} \frac{1}{2z} dv_B dx$$

These expressions differ from the full information benchmark. More precisely, while the demands for good A only depend on the anticipated price $\widehat{p}_B$, the demands for good B depend on both the anticipated price $\widehat{p}_B$ and the real price p_B.

First, it is important to note that $p_B \geq \widehat{p}_B$, because consumers incur the shopping cost tx before visiting the store, so that the monopolist has an incentive to hold them up by setting a price above their expectations, rather than below. Thus, we rule out $p_B < \widehat{p}_B$ as such prices reflect dominated strategies.

In the second stage, taking p_A as given, the monopolist chooses p_B to maximize her profit, $p_A D_A(p_A, p_B, \widehat{p}_B) + p_B D_B(p_A, p_B, \widehat{p}_B)$. Assuming rational expectations, consumers must correctly anticipate the price of B, implying that in any Bayesian equilibrium, $p_B = \widehat{p}_B$. This defines the Bayesian equilibrium of the stage-2 subgame as a function of p_A in each area of the parameter space.

Lemma 4. *In stage-2 Bayesian equilibrium, given p_A, we have*

$$p_B^{Am}(p_A) = \frac{a^2 + (p_A + t)^2 - 2a(p_A + 2t) - 2tz}{2(p_A - a - t)}$$

and

$$p_B^{Ah}(p_A) = 3a - 2p_A + z - \sqrt{2}\sqrt{(a - p_A)(3a - 2p_A + z)}$$

Both $p_B^{Am}(p_A)$ and $p_B^{Ah}(p_A)$ are increasing in p_A.

Proof. See proof in the appendix A.4. □

The equilibrium price of B increases in p_A, because when consumers receive an ad for good A, they know a higher advertised price implies fewer consumers pay the shopping cost to buy good A only. In other words, a high percentage of in-store consumers plan to buy good B, and these consumers value good B above the population average. Since the monopolist cannot commit to the price of good B, consumers anticipate that a high average valuation among in-store consumers of good B implies a high price for good B.

In the first stage, the monopolist sets p_A to maximize her profit $\pi^{As} = p_A D_A^{As}(p_A, p_B^{As}(p_A)) + p_B^{As}(p_A) D_B(p_A, p_B^{As}(p_A))$. Solving for equilibrium prices is possible for numerical values of a and z, but there is no tractable closed-form in the general setting.

Proposition 1. *When shopping costs are heterogeneous, and the monopolist advertises only good A, there exist two thresholds $\underline{t}^A = \underline{t}^f$ and $\bar{t}^A \in [\underline{t}^f, \bar{t}^f]$*

- *when $t \leq \underline{t}^A$: all consumers visit the store. Prices are equal to the equivalent full information benchmark: $p_A^{Al} = p_A^{fl} = a - t$ and $p_B^{Al} = p_B^{fl} = \frac{a+z}{2}$.*

- *when $\underline{t}^A \leq t \leq \bar{t}^A$: all consumers whose location x is close to 0 visit the store. Among consumers whose location x is close to 1, only those whose valuation v_B for good B is close to $a + z$ visit the store. For any t in the interval: $p_A^{Am} < p_A^{fm}$ and $p_B^{Am} > p_B^{fm}$.*

- *when $t \geq \bar{t}^A$: all consumers whose location x is close to 0 visit the store. No consumer whose location x is close to 1 visits the store. For any t in the interval: $p_A^{Ah} < p_A^{fs}$ and $p_B^{Ah} > p_B^{fs}$ for $s \in \{m, h\}$.*

Proof. See proof in the appendix A.5. □

Proposition 1 stems from the fact that advertising A distorts the valuations for good B of in-store consumers with respect to the general population: as consumers do not observe the price of the unadvertised good before visiting the store, the monopolist sets p_B to maximize $p_B q_B(p_B)$, the demand of in-store consumers. When deciding whether or not to visit the store, consumers whose valuation for good B is below the anticipated price $\widehat{p}_B$ of good B visit the store when $a - p_A - tx \geq 0$, and consumers whose valuation for good B is above $\widehat{p}_B$ visit the store when $a + v - p_A - \widehat{p}_B - tx \geq 0$. In the m area, in-store consumers' valuations for B are uniformly distributed between $a - z$ and $a + z$, so that the equilibrium price p_B is equal to $\frac{a+z}{2}$.

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In the h area, consumers with a high valuation for good B are over-represented in the store, and the average valuation among in-store consumers of good B is distorted upward. Hence, the optimal price p_B is above $\frac{a+z}{2}$.

For $a = z = 0.5$, Figure 3b shows the equilibrium purchasing decisions of consumers when good A is advertised, depending on x and v_B: whenever there are consumers who do not visit the store, there is an over-representation of consumers with high valuations for good B. Contrasting Figure 3b with Figure 1 shows the extent of the distortion: whereas the full information monopolist sets a low p_B when shopping costs are medium (around 0.35 when $t = 0.5$), the advertising monopolist deviates towards an inefficiently high p_B (around 0.6 when $t = 0.5$).

This result stems from the fact that, with heterogeneous shopping costs, some consumers visit the store to purchase good A, regardless of whether or not they are interested in good B. Thus, the valuation for good B of in-store consumers is still distributed over $[a - z, a + z]$. Note that it is no longer uniformly distributed over the interval, though, which explains why the optimal price p_B^{As} is above $\frac{a+z}{2}$. There are two strengths at play, which define the equilibrium. On the one hand, the monopolist has an incentive to raise p_B^{As} to take advantage of the distorted-up distribution of in-store consumers' valuations for good B. This is the hold-up effect described in our benchmark without advertising. On the other hand, however, advertising good A attracts consumers who are willing to pay the shopping cost to buy good A only, and these consumers have average valuations for good B. Thus, the monopolist has an incentive to maintain p_B^{As} low enough to keep selling them good B. This is what we call the Rhodes-effect. The optimal p_B^{As} is the Bayesian equilibrium that balances the hold-up effect and the Rhodes-effect.

5.2 The monopolist advertises good B

In the same setting, we now characterize the subgame-perfect equilibria in which the monopolist advertises good B. We denote the equilibrium prices when the monopolist advertises good B as p_A^{Bs} and p_B^{Bs}, where the superscript B stands for "good B is advertised", and where the superscript $s = \{l, m\}$ represents the levels of shopping costs: low or medium. Similarly, we denote the profit and demands as π^{Bs}, D_A^{Bs}, and D_B^{Bs}. It is important to note that, as opposed to the case in which the monopolist advertises good A, there are only two different shapes of demand. This is because, even when shopping costs are very low, the monopolist never sets

p_B low enough to appeal to consumers with minimal valuations for good B. In other words, consumers such that $v_B = a - z$ never visit the store.

Since all consumers share the same valuation a for good A, and since the monopolist can no longer commit on the price of good A before consumers incur the shopping costs, the price of good A is always set to its maximal level such that consumers who are in the store are indifferent between purchasing good A or purchasing good B only:

$$\forall s \in \{m, l\}, p_A^{Bs} = a$$

As a consequence, no consumer ever buys a single good and $D_A^{Bs} = D_B^{Bs}$. As this is anticipated by consumers, each consumer decides whether to visit the store based on the advertised price of product B.

In this setting, in the l area, at least one consumer with $x = 1$ has a valuation v_B for good B high enough for her to visit the store. The demands are thus given by:

$$D_A^{Bl} = D_B^{Bl} = \int_0^1 \int_{p_B+tx}^{a+z} \frac{1}{2z} dv_B dx$$

By contrast, in the m area, no consumer with $x = 1$ has a valuation v_B for good B high enough for her to visit the store.[8] The demands are thus given by:

$$D_A^{Bm} = D_B^{Bm} = \int_0^{\frac{a+z-p_B}{t}} \int_{p_B+tx}^{a+z} \frac{1}{2z} dv_B dx$$

Proposition 2. *When the retailer advertises good B, it always sets $p_A^{Bs} = a$. Moreover, $p_B^{Bl} = \frac{1}{4}(2z - t)$ and $p_B^{Bm} = \frac{1}{3}(z - a)$. The switch from area l to area m occurs at $\bar{t}^B = \frac{2}{3}(2a + z)$.*

Proof. See proof in the appendix A.6. □

With $a = z = \frac{1}{2}$, Figure 3c provides an illustration of the equilibrium purchasing decisions of consumers.[9]

Note that, in the m area, p_B^{Bm} is negative when $z < a$. We have not *a priori* ruled out below cost prices. However, it is worth noticing that, in a world where below cost prices are prohibited, the corresponding equilibrium is a corner solution with $p_B^{Bm} = 0$.

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[8]Note that, in theory, there could exist two additional areas, defined when $p_B^B \le a - z$. However, it is never optimal for the monopolist to set p_B so low.

[9]The case $s = m$ cannot be achieved with $a = z = \frac{1}{2}$. We have decided to keep these parameters for the sake of homogeneity throughout the paper, and because we believe it is more important to give a clear picture of the possible demands when the monopolist advertises good A, rather than when she advertises good B.

5.3 Optimal advertising strategy

In this section, we endogenize the retailer's advertising strategy. We show that with heterogeneous shopping costs, it may be profitable for the monopolist to advertise good B.

Proposition 3. *For any given shopping cost t, the price of each good is lower when it is advertised than when the other good is advertised: $p_A^{As} < p_A^{Bs}$, and $p_B^{Bs} < p_B^{As}$.*

Proof. See proof in the appendix A.7. □

This result stresses two important facets of our model. First, the advertised good allows the monopolist to attract consumers to the store, which implies committing on a low price. Second, since the monopolist cannot commit on the price of the unadvertised good, it tends to set its price high to maximize profit over in-store consumers.

For $a = z = 0.5$, Figure 4 illustrates Proposition 3. It shows the equilibrium advertised prices as a function of the shopping costs t. The red line (p_B^{Bs}) is well below the green line (p_A^{Bs}), and the blue line (p_A^{As}) is well below the yellow line (p_B^{As}). Further, Figure 4 also shows that the monopolist advertises a lower price when it advertises good B than when it advertises good A (on the graph, $p_B^{Bs} < p_A^{As}$). This is not a general result, because the precise location of the price curves depends on parameters a and z, but it sheds some light on two key features of our model. First, advertising A allows for attracting more consumers in store than advertising B because all customers are potentially interested in purchasing good A for any price p_A^{As} below a, while only customers whose v_B is above p_B^{Bs} are sensitive to an advertisement for good B. Thus, to reach any amount of consumers, there is no need to discount A as much as B. Second, advertising B begets more complementarities. Indeed, when the retailer advertises B, $p_A^{Bs} = a$, hence all consumers who decide to visit the store because of an advertisement for B end up purchasing A, whereas when the retailer advertises A, consumers who decide to visit the store only purchase B when $p_B^{As} \leq v_B$. Thus, while an additional discount on the advertised price is always compensated for by higher sales of the other good, this effect is stronger when the monopolist advertises B. Since the monopolist internalizes the cross-product externality, the optimal advertised price for good B is lower than for good A.

Anticipating stages 2 and 3, the monopolist chooses which good to advertise. It advertises good A whenever his expected profit is larger when advertising good A rather than good B.

To summarize, the monopolist faces the following trade-offs: on the one hand, advertising A is more efficient to attract consumers in store, because all consumers are potentially interested
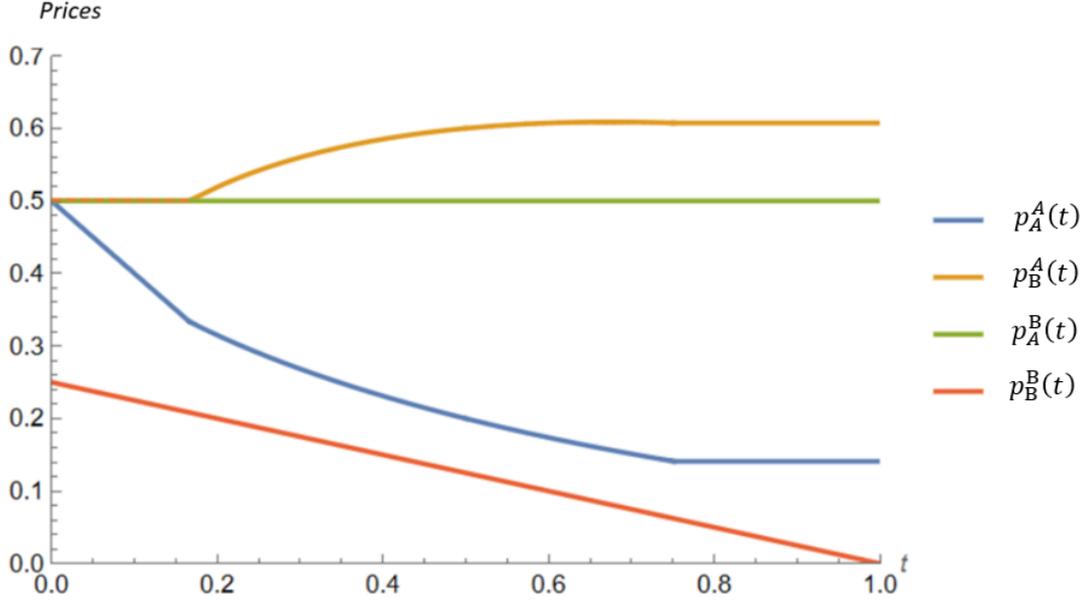
Figure 4: Equilibrium advertised prices as a function of the shopping costs t ($a = z = \frac{1}{2}$)

by the discount, while only those whose valuation is above p_B are potentially interested by an advertisement for good B. Thus, for a given discount, advertising A brings more customers to the store. On the other hand, advertising B begets more complementarity, so that all in-store customers buy the two goods.

Proposition 4. *With $a = z = 1/2$, we find $\tilde{t} = 0.35$ such that:*

- *When $t \leq \tilde{t}$ the monopolist advertises good A.*

- *When $t \geq \tilde{t}$ the monopolist advertises good B.*

For any $a > 0$ and $z \in [a/3, a]$, we show that $\Delta(t) \equiv \pi^A(t) - \pi^B(t)$ can either be negative for all $t \in [0, 1]$, positive for all $t \in [0, 1]$, or admit a single $\tilde{t} \in [0, 1]$ such that $\Delta(t) > 0$ when $t < \tilde{t}$, $\Delta(t) < 0$ when $t > \tilde{t}$, and $\Delta(\tilde{t}) = 0$. Though we cannot prove it in the general setting, numerical simulations suggest the uniqueness of the threshold.

Proof. See proof in the appendix A.8. □

Intuitively, this result stems from the fact that advertising good B begets more complementarity than advertising good A: while shopping costs are below $\tilde{t}$, advertising A allows

the monopolist to set higher prices while attracting enough demand. However, when shopping costs increase above $\tilde{t}$, the complementarity effect of advertising good B enables the monopolist to keep the aggregated demand for goods A and B higher than when it advertises good A. Thus, despite lower average prices, selling more is profitable. With $a = z = 0.5$, Figure 4 shows that, as shopping costs increase, the advertised price of good A p_A^{As} decreases more slowly than p_B^{Bs}.[10] This stems from the fact that when t increases, the marginal amount of consumers any additional discount on product A attracts decreases as well. Indeed, not only does a higher t mean higher shopping costs, but it also reflects a stronger heterogeneity in shopping costs: a consumer located in x spends tx to visit the store, and the further away from the store a consumer is, the more she is affected by an increase in t. This heterogeneity stems from the fact that, for a given valuation of the goods, the relative weight of the shopping cost borne by consumers far away from the store becomes larger. As a consequence, when t increases, a monopolist who maintains a constant advertised price for good A loses a disproportionate amount of far-away customers. Since consumers' valuation for product A is homogeneous, a monopolist who advertises good B lowers its price as t increases to attract more consumers, but must keep the price of good A independent of the shopping costs. By contrast, when the monopolist advertises good A, he finds it optimal to reduce the price of product A by less: the firm forgoes attracting additional demand and instead raises the price of product B to extract higher margins. As t increases, the proportion of far-away consumers decreases so that any further increase in t requires a smaller additional discount on the advertised price of good A.

With $a = z = 0.5$, Figure 5 shows aggregated sales as a function of shopping costs, depending on whether the monopolist advertises good A or good B. When the ad strategy switch occurs, aggregated sales are already higher when the monopolist advertises good B.

With $a = z = 0.5$, Figure 6 shows the profit of the monopolist as a function of the shopping costs, depending on whether it advertises good A or good B. With those parameters, $\tilde{t} = 0.35$.

For a large array of parameters $a > 0$ and $z \in [a/3, a]$, Figure 7 shows which kind of equilibrium prevails. The graph delivers three intuitions. First, when the heterogeneity in valuations for good B is small (z small, red area in Figure 7), the complementarity effect always dominates, so that the monopolist advertises good B for all t. Second, when valuations for good B are more heterogeneous (green area in Figure 7), the monopolist prefers advertising A

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[10]Each change in the slope corresponds to the switch from one kind of equilibrium to the next, i.e. $s = \{l, m, h\}$, as displayed on Figure 3.
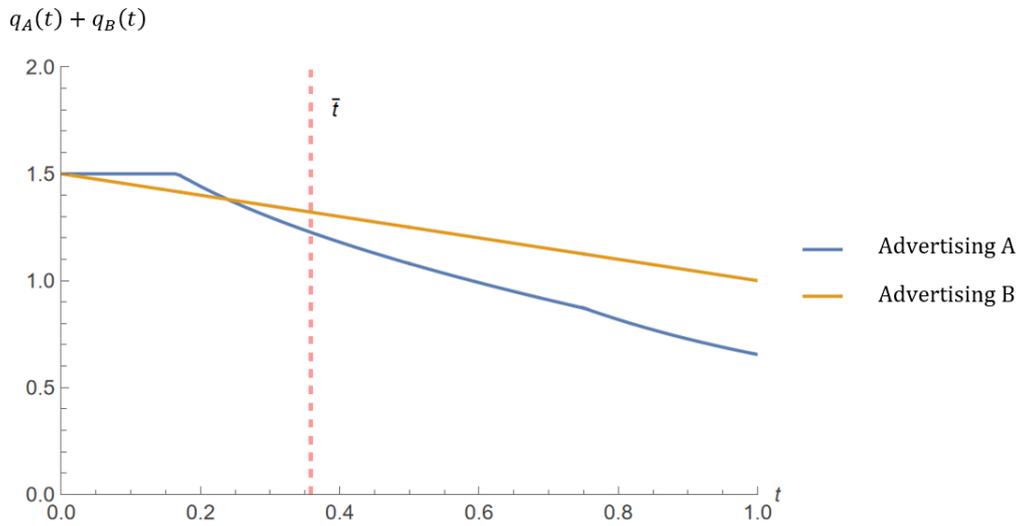
Figure 5: Aggregated sales as a function of shopping costs t, depending on which good is advertised ($a = z = \frac{1}{2}$)
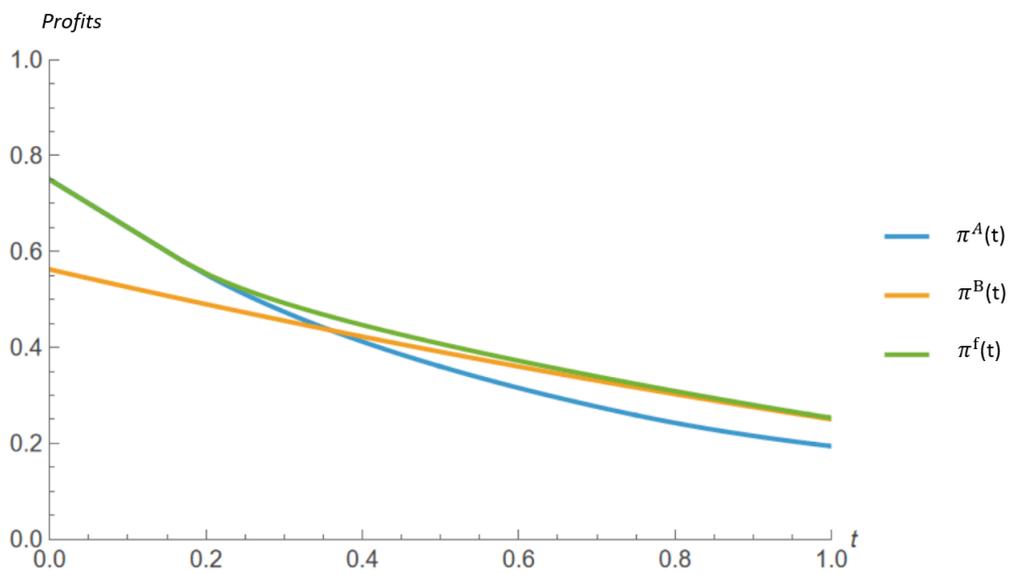

Figure 6: Profit as a function of shopping costs t, depending on the information structure ($a = z = \frac{1}{2}$)
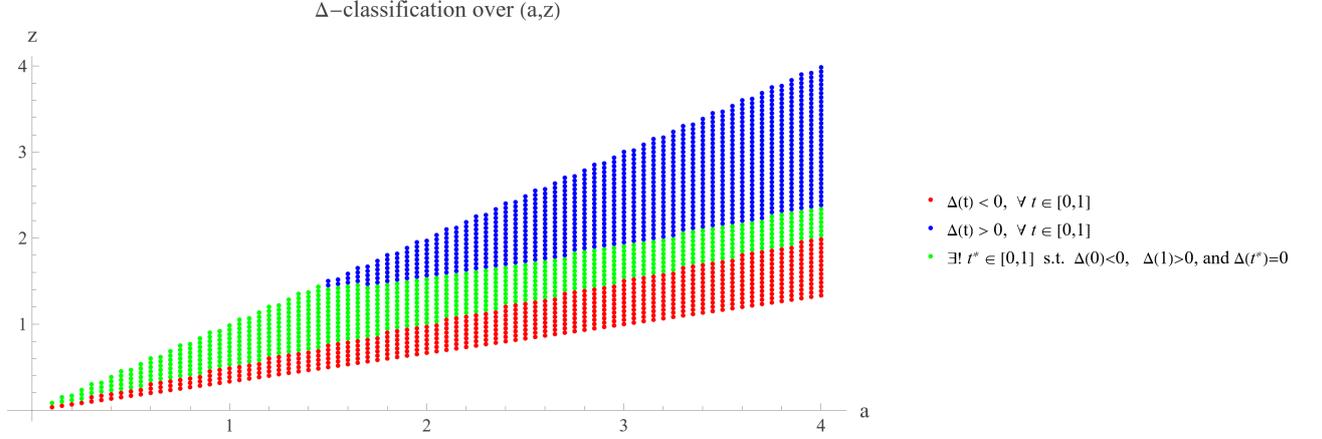
Figure 7: Optimal advertising strategies depending on t, a, and z

when shopping costs t are not too high. This is because a high z implies consumers are not densely packed along the v_B axis, which requires the monopolist to offer a large discount on p_B to bring consumers to the store and exploit the complementarity effect. Third, the blue area in Figure 7 corresponds to the case when both a and z are high. In this situation, the shopping costs are small compared to the average valuations v_A and v_B, implying that the monopolist can still attract sufficient demand when advertising good A, thereby benefiting from higher prices. In the Online Appendix, we also check that the optimal advertising strategies follow the same patterns when $t \in [0, a]$. Intuitively, this corresponds to normalizing the shopping costs with respect to a. The consequence is that the blue area disappears, all else equal. Intuitively, the blue area appears only when shopping costs are very low relative to consumers' valuations a. Thus, when shopping costs are allowed to increase up to a, the case in which the monopolist always advertises good A is no longer optimal.

6   Policy Implications

This section analyzes the effect of advertisement on consumer surplus and derives some policy implications. More precisely, we compare consumer surpluses when the monopolist advertises A, when the monopolist advertises B, and under the standard perfect information setting, in

which the monopolist commits on both prices before consumers decide whether or not to visit the store.

Proposition 5. *The aggregate consumer surplus is higher when the monopolist advertises good B than under perfect information, and it is higher under perfect information than when the monopolist advertises good A.*

Proof. See proof in the appendix A.9. □

For $a = z = 0.5$, Figure 8 shows consumer surplus as a function of shopping costs when good A is advertised, when good B is advertised, and under perfect information. When the monopolist advertises B, prices are lower than when the monopolist advertises A. This is because there is no profitable upward deviation on the price of the unadvertised good and because complementarities foster a low advertised price p_B^B. As a consequence, consumers benefit a lot from the monopolist advertising good B rather than good A.

For $a = z = \frac{1}{2}$, contrasting Figure 3c with Figure 1 shows that, whereas the full information monopolist sets $p_B^{fh} = 0.2$ when shopping costs are high ($t = 0.99$), the advertising monopolist advertises $p_B^{Bh} = 0.02$ when shopping costs are high ($t = 0.9$). Consumers benefit from these complementarities: there are very few consumers who would have visited the store in a perfect information world but who abstain from doing so in a partial information world, and those who visit the store end up paying less. With $a = z = \frac{1}{2}$, Figure 8 shows that the additional surplus consumers can expect from partial information is particularly large when shopping costs are small. That is because, under perfect information, the monopolist uses p_A to attract consumers while keeping prices high, whereas under partial information, it must advertise a very low p_B to reach enough consumers and exploit the complementarity effect with an advertisement for good B.

Our most intriguing result is that consumers benefit from a lack of information as long as the retailer advertises the price of the good for which valuations are heterogeneous. The two key conditions for this effect to arise are: there must be heterogeneity in shopping costs, and there must be at least two goods on sale, with consumers' tastes being more heterogeneous for one of them than for the others. We focus on a monopolist retailer for tractability, but there are no *a priori* reasons for our results to change qualitatively when we allow for imperfect competition.

We think these conditions are likely to be met in several important product markets, such as supermarkets. In their rulings on competition cases, Competition Authorities usually define the
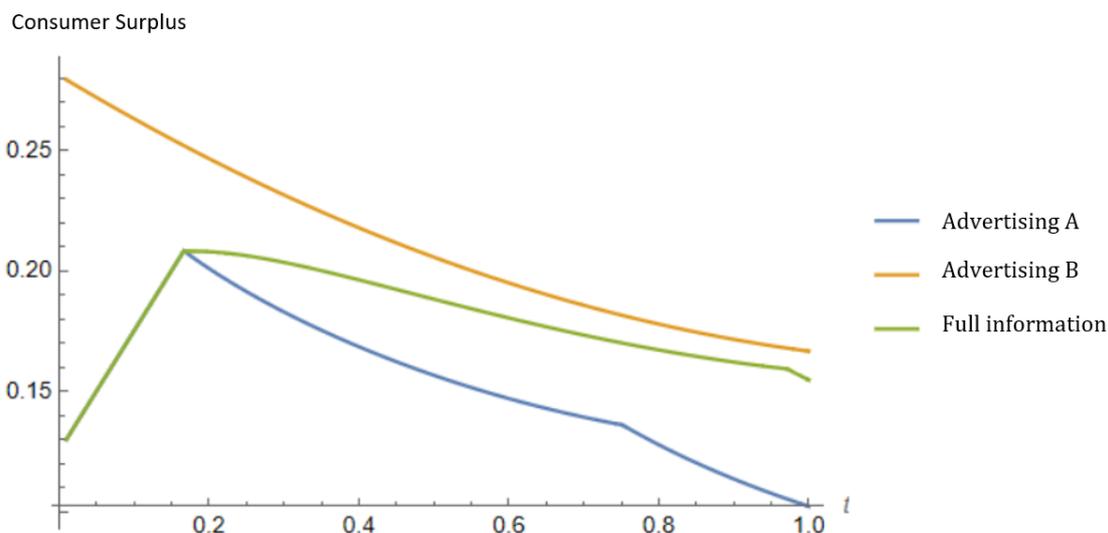
Figure 8: Consumer surplus as a function of shopping costs, when good A is advertised, when good B is advertised, and under perfect information

relevant food retail market at the local level, so that a non-negligible number of consumers face actual monopolies. Due to the number of products they sell, these supermarkets cannot easily commit to prices for all products, and consumers differ in their shopping costs because of their distance to the store, as well as their shopping preferences and availability.

By focusing on these markets, our model and findings have important policy implications: the regulator should not necessarily push for full price disclosure. Mandates for full price disclosure usually take the form of an online platform where retailers are to report prices. Such policies have already been implemented with mixed results: regarding Israeli supermarkets, Ater and Rigbi (2023) find that the mandate led to lower prices. Regarding Chilean petrol stations, Luco (2019) find that price disclosure increases their margins. However, in these papers, price disclosure affects price through its pro-competitive or anti-competitive impact. We highlight a new channel through which mandatory price disclosure may harm consumers even in the absence of competition. More precisely, we show that partial information may compel a monopolist retailer to adopt a more aggressive discount policy, thereby benefiting consumers.

Other policy implications are left for further research. In particular, it may be fruitful to model upstream producers. For instance, the retailer's advertising strategy affects producers' profits, enabling regulators to gain valuable insights into potentially anti-competitive practices.

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Indeed, through the bargaining process, the retailer may use price advertising as a threat to force producers to compete away their margins.

7 Extension: homogeneous search costs

In this extension, we solve the model under homogeneous shopping costs. This corresponds to the situation in which all consumers incur the same shopping cost, as in Rhodes (2014). More precisely, we assume that all consumers are located at $x = \frac{1}{2}$ on the Hotelling line. Alternatively, this is equivalent to assuming that all consumers must pay $\frac{t}{2}$ to visit the store, regardless of their location x.

7.1 Perfect information

Consumers are assumed to be perfectly informed about prices. The timing of the game is thus as follows: in stage 1, the monopolist sets both prices. In stage 2, consumers learn their valuations for good B, learn both prices, and decide whether or not to incur shopping costs. In stage 3, consumption occurs.

Formally, the purchasing behavior of a consumer now stems from the following maximization program:

$$U(v) = \max(v_A + v_B - p_A - p_B - \frac{t}{2}, v_A - p_A - \frac{t}{2}, v_B - p_B - \frac{t}{2}, 0)$$

Since we assume all consumers are located at $x = \frac{1}{2}$ on the Hotelling line, equilibrium prices still depend on the shopping costs t. We denote by p_A^f and p_B^f the prices of goods A and B under full information. p_A^{f0} and p_B^{f0} are the optimal prices absent transportation costs. We also distinguish p_A^{fs} and p_B^{fs}, the optimal prices under full information when shopping costs are low ($s = l$), medium ($s = m$), and high ($s = h$). Eventually, the equilibrium prices are overlined prices (e.g. $\overline{p}_A^{f0}$).

We first solve the model when shopping costs t are equal to 0.

Lemma 5. *When shopping costs are null, the monopolist charges* $\overline{p}_A^{f0} = a$ *and* $\overline{p}_B^{f0} = \frac{a+z}{2}$.

Proof. See proof in the appendix A.10. □

This first benchmark corresponds to the case in which there are no shopping-cost-induced complementarities between goods A and B. However, when shopping costs are strictly positive, the market coverage depends on t, as in the standard case with heterogeneous shopping costs.

Lemma 6. *When shopping costs are homogeneous, under perfect information, there exists a threshold $\hat{t}^f \equiv 2(a + \frac{(a+z)^2 - (2a+z-\frac{t}{2})^2}{8z})$ such that:*

- *when $t \leq \hat{t}^f$: $\overline{p}_A^{fl} = a - \frac{t}{2}$ and $\overline{p}_B^{fl} = \frac{a+z}{2}$. All consumers visit the store.*

- *when $t \geq \hat{t}^f$: $\overline{p}_A^{fh} + \overline{p}_B^{fh} = \frac{1}{2}(2a + z - \frac{t}{2})$. Only consumers whose taste for good B is above $v_0 \equiv \overline{p}_A^{fh} + \overline{p}_B^{fh} + \frac{t}{2} - a$ visit the store.*

Proof. See proof in the appendix A.11. □

The main difference with the free shopping equilibrium is that, although both goods are independent, shopping costs induce some complementarities. This is because consumers incur shopping costs only once, regardless of whether they buy one or two goods. As a result, the monopolist sells a basket of goods at a discount to subsidize transportation costs.

Figure 9 contrasts consumer surplus as a function of t when consumers have heterogeneous shopping costs and when consumers have homogeneous shopping costs. There is an important difference that underscores the need to account for consumers' heterogeneity in shopping costs. With heterogeneous shopping costs, consumer surplus increases with shopping costs, provided the monopolist does not exclude any consumers. With homogeneous goods, consumer surplus is constant with respect to shopping costs as long as the monopolist does not exclude any consumers. This is because, with heterogeneous shopping costs, an increase in the parameter t corresponds to an increase in shopping costs' heterogeneity along the x axis, so that when the monopolist subsidizes the shopping costs of the furthest consumer, any increase in t raises the economic rent of all other consumers. In other words, in a world of heterogeneous shopping costs, an increase in t makes all consumers who still visit the store better off. That is why the consumer surplus increases in t in the equilibrium where all consumers visit the store.

7.2 Advertising A

First, we consider the case in which the monopolist advertises good A and consumers have homogeneous shopping costs $\frac{t}{2}$. We use the same notations as before, and we denote by $\overline{p}_A^{As}$
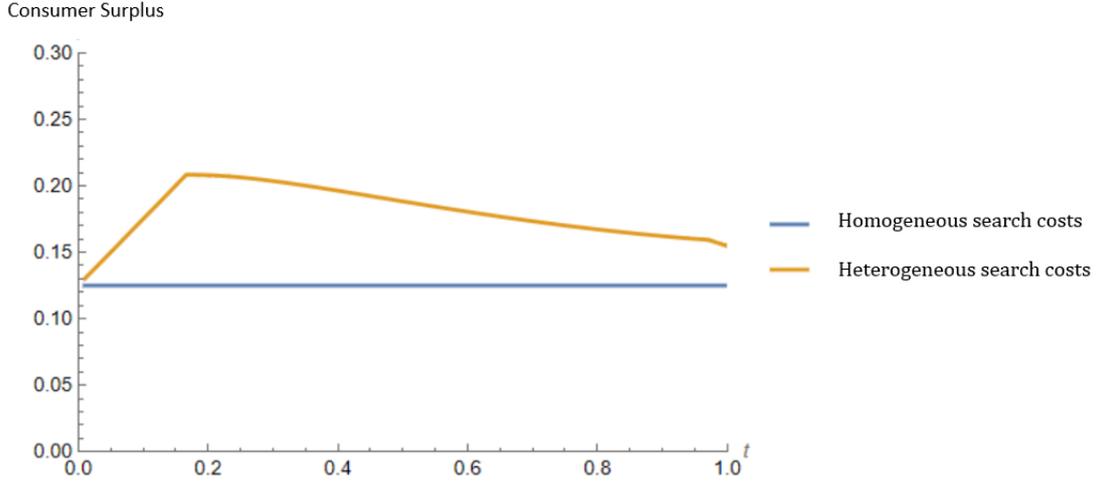
Figure 9: Consumer surplus as a function of t

Full information benchmark

and $\overline{p}_B^{As}$ (resp. $\overline{p}_A^{Bs}$ and $\overline{p}_B^{Bs}$) the prices of goods A and B when the monopolist advertises good A (resp. good B).

Lemma 7. *There exists a threshold $\hat{t}^A \equiv 2(a + \frac{(a+z)^2}{8z})$ such that:*

- *when $t \leq \hat{t}^A$: $\overline{p}_A^{Al} = \overline{p}_A^{fl} = a - \frac{t}{2}$ and $\overline{p}_B^{Al} = \overline{p}_B^{fl} = \frac{a+z}{2}$. All consumers visit the store.*

- *when $t \geq \hat{t}^A$: $\overline{p}_A^{Ah} = p_A^{\emptyset} = a$ and $\overline{p}_B^{Ah} = p_B^{\emptyset} = a + z$. No consumer ever visits the store.*

Proof. See proof in the appendix A.12. □

This result rests on the same set of intuitions we described through the no-information benchmark: when it advertises good A, the monopolist can only commit to the price of good A so that it has an incentive to hold consumers up on their shopping costs through the price of good B. As long as consumers are sure to recover their shopping costs by purchasing good A, there is no market failure. However, when they need to recover part of their shopping costs by paying a good B to visit the store, the hold-up effect is too acute, and nobody visits. Indeed, for any anticipated price $\widehat{p}_B$, the retailer knows it only faces consumers whose taste for good B is high enough to compensate for a fraction of the shopping costs. As such, it has an incentive to set p_B above $\widehat{p}_B$ to exploit the excess of taste for good B among in-store consumers. Since anticipations are rational, the only possible outcome is market collapse.

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Under both homogeneous and heterogeneous shopping costs, the monopolist has an incentive to raise the price of the unadvertised good B. The reason is selection: consumers who enter the store have, on average, a higher valuation for good B than the population as a whole, which makes it profitable to hold-up in-store consumers by setting a higher p_B. The key difference is that this pricing incentive is immediately destabilizing under homogeneous shopping costs: once the monopolist deviates by increasing p_B, the deviation undermines the candidate equilibrium, so no Bayesian equilibrium exists. By contrast, the same deviation is consistent with equilibrium when shopping costs are heterogeneous. Heterogeneity induces some consumers with low shopping costs —even those with only average valuations for good B— to visit the store. The monopolist then faces a trade-off: setting p_B too high extracts surplus from high-valuation in-store consumers, but it also reduces sales to these low-shopping-cost, average-valuation visitors. This disciplines p_B and limits the monopolist's ability to hold-up shoppers. Put differently, without shopping-cost heterogeneity, the Rhodes effect cannot offset the hold-up effect, and a Bayesian equilibrium fails to exist; with heterogeneity, the Rhodes effect provides enough discipline to sustain an equilibrium.

7.3 Advertising B

Let us now solve the partial-information model in which the monopolist advertises good B.

Lemma 8. *When the retailer advertises good B, it always sets $\overline{p}_A^B = a$ and $\overline{p}_B^B = \frac{1}{4}(2z - t)$. There exists a threshold $\hat{v} \equiv \frac{1}{4}(2z + t)$ such that only consumers whose valuations v_B for good B lie above $\hat{v}$ visit the store.*

Proof. See proof in the appendix A.13. □

Results and intuitions are similar to the case in which consumers have heterogeneous shopping costs: since all consumers have the same valuations for good A, there is no reason for the monopolist to set an unadvertised price p_A below $v_A = a$. Consumers anticipate this and thus consider only v_B and p_B when deciding whether to incur the shopping cost.

The main consequence is that advertising good B generates more complementarities than advertising good A. Indeed, when the monopolist advertises good B, all consumers who come to buy good B end up buying good A as well, whereas when it advertises good A, there are consumers who visit the store and buy good A only.

7.4 Optimal advertising strategy

We show that the monopolist's optimal advertising strategy allows it to perfectly mimic its full information price strategy. This is the same result Lal and Matutes (1994) show with heterogeneous shopping costs when both goods have constant valuations.

Under full information, Lemma 6 states that, when $t \leq \hat{t}^f$, the retailer sets $\overline{p}_A^{fl} = a - \frac{t}{2}$ to attract all consumers to the store, and $\overline{p}_B^{fl} = \frac{a+z}{2}$ to extract consumer surplus. When $t > \hat{t}^f$, it sets $\overline{p}_A^{fh} + \overline{p}_B^{fh} = \frac{1}{2}(2a + z - \frac{t}{2})$ so that only consumers who value good B enough visit the store.

Under imperfect information, Lemma 7 states that, when the monopolist advertises good A, it sets $\overline{p}_A^{Al} = \overline{p}_A^{fl}$ and $\overline{p}_B^{Al} = \overline{p}_B^{fl}$, provided it makes a positive profit, that is, provided $t \leq \hat{t}^A$. Lemma 8 states that, when the monopolist advertises good B, it always sets $\overline{p}_A^B = a$ and $\overline{p}_B^B = \frac{1}{4}(2z - t)$, so that $\overline{p}_A^B + \overline{p}_B^B = \overline{p}_A^{fh} + \overline{p}_B^{fh} = \frac{1}{2}(2a + z - \frac{t}{2})$.

Since $\hat{t}^A \geq \hat{t}^f$, the monopolist is always able to reproduce both perfect information equilibria, so that partial information affects neither profit nor consumer surplus. Thus, our main result only holds when shopping costs are heterogeneous.

8 Conclusion

Retailers typically offer a vast assortment of products to their customers. As a result, consumers are unable to recall the prices of each item perfectly. In addition, there are products that elicit diverse valuations among consumers and those that generate similar valuations across consumers. Moreover, various factors cause consumers to incur different shopping costs, whether we regard them as transportation or search costs. In a setup in which a monopolist retailer sells two products to consumers who have heterogeneous valuations for one good and homogeneous valuations for the other good, the aim of this paper is to examine the impact of price advertising on the advertising strategy of the retailer and on the welfare of consumers.

Our main finding is that consumers can benefit from having incomplete information, provided they have heterogeneous shopping costs. The key mechanism that drives this result is the differential complementarity effect, that is, the fact that advertising the good for which consumers' valuations are heterogeneous makes all consumers who visit the store buy the unadvertised good as well, whereas advertising the good for which consumers' valuations are homogeneous only makes some consumers who visit the store buy the unadvertised good as well. When the monopolist cannot commit to both prices and the shopping costs are high enough, it exploits

the complementarity effect by setting a very low advertised price. This benefits consumers, who end up paying less for the two goods than under perfect information.

This paper shows how heterogeneity in consumer valuations affects the optimal advertising and pricing strategies of a multiproduct monopolist, as well as the associated consumer surplus. More specifically, we contribute to the literature by modeling both consumers' valuation heterogeneity and their shopping cost heterogeneity.

Eventually, our model and results have two main policy implications. On the one hand, we provide a theoretical rationale for not systematically pushing for full price disclosure. On the other hand, we document how the ability to advertise important price discounts benefits consumers, so that regulating discounts or advertisements may harm consumers.

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Appendix

A Proofs

A.1 Proof Lemma 1

Let $\widehat{p}_A$ and $\widehat{p}_B$ be the prices anticipated by consumers. Without loss of generality, we assume $\widehat{p}_A \leq a$ and $a - z \leq \widehat{p}_B \leq a + z$.

All consumers whose location x and taste v_B for good B are such that $tx \leq a - \widehat{p}_A + v_B - \widehat{p}_B$ visit the store. Let D be the mass of consumers who visit the store. Everything happens as if, when the monopolist sets p_A and p_B, consumers had already paid the shopping cost, so that they make separate purchase decisions for each good: they buy good A provided $p_A \leq a$, and they buy good B provided $p_B \leq v_B$. Regarding good A, the monopolist maximizes $\Pi_A = D * p_A * \mathbb{1}_{p_A \leq a}$. The optimal price p_A is thus equal to a, independently of the anticipated price for good B. Since consumers make rational anticipations, $\widehat{p}_A = a$.

Let us now restrict our attention to the case in which $\widehat{p}_A = a$ and $a - z \leq \widehat{p}_B < a + z$. Then, all consumers whose location x and taste v_B for good B are such that $tx \leq v_B - \widehat{p}_B$ visit the store. Regarding good B, the monopolist maximizes

$$\Pi_B(p_B) = p_B \int_0^{\min(1, \frac{a+z-\widehat{p}_B}{t})} \int_{tx+\widehat{p}_B}^{a+z} \mathbb{1}_{p_B \leq v_B} f(v_B) \, dv_B \, dx$$

Note that the double integral represents demand for good B as a function of the price p_B, given that only consumers whose $v_B \geq \widehat{p}_B + tx$ are potential buyers. The double integral is needed to account for both the individual location x and the individual taste v_B for good B.

The proof shows that a profitable upward price deviation always exists. To do that, we show the derivative of the profit function is always strictly positive. On the one hand, we have:

$$\Pi_B(\widehat{p}_B) = \widehat{p}_B \int_0^{\min(1, \frac{a+z-\widehat{p}_B}{t})} \int_{tx+\widehat{p}_B}^{a+z} \frac{1}{2z} \, dv_B \, dx$$

$$= \begin{cases} \widehat{p}_B \frac{(a+z-\widehat{p}_B)^2}{4tz} & \text{if } \min(1, \frac{a+z-\widehat{p}_B}{t}) = \frac{a+z-\widehat{p}_B}{t} \\ \widehat{p}_B \frac{a+z-\widehat{p}_B - \frac{t}{2}}{2z} & \text{if } \min(1, \frac{a+z-\widehat{p}_B}{t}) = 1 \end{cases}$$

Suppose on the other hand that the retailer sets $p_B = \widehat{p}_B + \epsilon$, with $\epsilon > 0$; consumers decide to visit the store based on the price they anticipate but make their final purchase decisions based

on actual prices, hence only consumers whose v_B is above the observed price $\widehat{p}_B + \epsilon$ ultimately buy good B. The profit of the retailer would then be:

$$\Pi_B(\widehat{p}_B + \epsilon) = (\widehat{p}_B + \epsilon) * \left(\int_0^{\frac{\epsilon}{t}} \int_{\widehat{p}_B+\epsilon}^{a+z} \frac{1}{2z} \, dv_B \, dx + \int_{\frac{\epsilon}{t}}^{\min(\frac{a+z-\widehat{p}_B}{t},1)} \int_{tx+\widehat{p}_B}^{a+z} \frac{1}{2z} \, dv_B \, dx \right)$$

because in-store consumers are consumers whose $v_B \geq tx + \widehat{p}_B$ and:

when $x \leq \dfrac{\epsilon}{t}$, in-store consumers only buy good B when $v_B \geq \widehat{p}_B + \epsilon$

when $x \geq \dfrac{\epsilon}{t}$, all in-store consumers verify $v_B \geq \widehat{p}_B + \epsilon$

$$\Pi_B(\widehat{p}_B + \epsilon) = \begin{cases} \Pi_B(\widehat{p}_B) + \underbrace{\dfrac{(a + z - \widehat{p}_B)^2 - \epsilon\widehat{p}_B - \epsilon^2}{4tz}}_{>0 \text{ for } \epsilon>0 \text{ and arbitrarily small}} & \text{if } \min(1, \frac{a+z-\widehat{p}_B}{t}) = \frac{a+z-\widehat{p}_B}{t} \\[2em] \Pi_B(\widehat{p}_B) + \underbrace{\dfrac{a + z - \widehat{p}_B - \frac{t}{2} - \frac{\epsilon}{2t}(\widehat{p}_B + 2\epsilon)}{2z}}_{\text{for } \epsilon>0 \text{ and arbitrarily small}} & \text{if } \min(1, \frac{a+z-\widehat{p}_B}{t}) = 1 \end{cases}$$

Thus, once consumers are in the store, the retailer always has an incentive to charge ϵ more than $\widehat{p}_B$ for all $\widehat{p}_B < a + z$. As a consequence, the only rationally anticipated prices are $p_A^{\emptyset} = a$ and $p_B^{\emptyset} = a + z$, so that no consumer ever visits the store.

A.2 Proof Lemma 2

To solve for equilibrium prices, we begin by noting there are only two possible cases: either all consumers visit the store, or only those who live close enough to the store and/or who enjoy good B enough visit the store.

A.2.1 All consumers visit the store (l area)

When the monopolist decides to sell to all consumers, p_A is set to its maximum level such that all consumers still visit the store. In other words, we have a corner solution, as the monopolist's profit strictly increases in p_A in the l area. However, to remain in the l area, the consumer who is the least likely to visit the store, that is, the one who lives in $x = 1$ and does not value good B at all ($v_B = 0$), must be indifferent between visiting the store and staying home: $p_A^{fl} = a - t$. Since all consumers visit the store to buy good A, p_B is set to its monopoly level: $p_B^{fl} = \frac{a+z}{2}$. Note that we have assumed $z > \frac{a}{3}$ to ensure $\frac{a+z}{2} \geq a - z$, so that this solution is interior. In

other words, we decide to focus on the case where good B is sufficiently heterogeneous for the monopolist to find it profitable not to sell it to some consumers. This generates a profit:

$$\Pi(p_A^{fl}, p_B^{fl}) = p_A^{fl}.1 + p_B^{fl}.\mathbb{P}(v_B \geq p_B^{fl}) = a - t + (\frac{a+z}{2})(\frac{a+z}{4z}) = a - t + \frac{(a+z)^2}{8z}$$

A.2.2 Not all consumers visit the store (m and h areas)

m **area:** When $t \leq 2a + z - p_A - p_B$, there are consumers located in $x = 1$ who visit the store (see Figure 3a). In that case, demands for A and B are:

$$D_A^{fm}(p_A, p_B) = \frac{2a(2p_A + p_B + 2t + z) - 3a^2 - p_A^2 - 2p_A(p_B + t + z) - t(2p_B + t - 2z)}{4zt}$$

$$D_B^{fm}(p_A, p_B) = \frac{2a(p_A + 2t) - a^2 - p_A^2 - 2p_A t - t(2p_B + t - 2z)}{4zt}$$

Note that $\Pi^{fm}(p_A, p_B) = p_A D_A^{fm}(p_A, p_B) + p_B D_B^{fm}(p_A, p_B)$ is concave in its arguments. Maximizing with respect to p_A and p_B triggers the following system of FOCs:

$$FOC_A^{fm} \equiv -(3a^2 - 2a(4p_A + 2(p_B + t) + z) + 3p_A^2 + 6p_A p_B + 4p_A(t + z) + t(4p_B + t - 2z)) = 0$$

$$FOC_B^{fm} \equiv -(a^2 + 3p_A^2 + 4p_A t - 4a(p_A + t) + t(4p_B + t - 2z)) = 0$$

Solving $\text{FOC}_B^{\text{fm}} = 0$ for p_B (holding p_A fixed) yields the best response

$$p_B = -\frac{a^2 + 3p_A^2 + 4p_A t - 4a(p_A + t) + t(t - 2z)}{4t}, \tag{11}$$

Substituting (11) into $\text{FOC}_A^{\text{fm}} = 0$ delivers a cubic equation in p_A: $-2a^3 + 11a^2 p_A - 18ap_A^2 + 9p_A^3 + 4a^2 t - 12ap_A t + 12p_A^2 t - 2at^2 + 3p_A t^2 + 8atz - 14p_A tz = 0$. Then, p_A^{fm} is the only solution of this cubic equation that satisfies our constraints (positive demands, m area, non-imaginary prices). We are thus able to find closed-form solutions for p_A^{fm} and p_B^{fm}, but the expressions are too intricate to be written down. They are available in the Online Appendix. Figure 2 provides an illustration when $a = z = \frac{1}{2}$.

h **area:** When $t \geq 2a + z - p_A - p_B$, no consumer located in $x = 1$ ever visits the store, no matter her valuation for good B (see Figure 3a). In that case demands for A and B are:

$$D_A^{fh}(p_A, p_B) = \frac{a - p_A}{t} + \frac{(2a + z - p_A - p_B)^2}{4zt} - \frac{(a - p_A)(2a + z - p_A - p_B)}{2zt} + \frac{(a - p_A)^2}{4zt}$$

$$D_B^{fh}(p_A, p_B) = \frac{(a + z - p_B)(a - p_A)}{2zt} + \frac{(2a + z - p_A - p_B)^2}{4zt} - \frac{(a - p_A)(2a + z - p_A - p_B)}{2zt} + \frac{(a - p_A)^2}{4zt}$$

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Note that $\Pi^{fh}(p_A, p_B) = p_A D_A^{fh}(p_A, p_B) + p_B D_B^{fh}(p_A, p_B)$ is concave in its arguments. Maximizing with respect to p_A and p_B triggers the following system of FOCs:

$$FOC_A^{fh}(p_A, p_B) \equiv a^2 - 4ap_B + 3p_B^2 + 6az - 8p_Az - 4p_Bz + z^2 = 0$$

$$FOC_B^{fh} \equiv 3a^2 + 3p_B(2p_A + p_B) - 4a(p_A + 2p_B - z) - 4(p_A + p_B)z + z^2 = 0$$

Solving $FOC_B^{fh} = 0$ for p_B (holding p_A fixed) yields the best response

$$p_B = \frac{1}{3}\left(2a - \sqrt{-(a+z)(3a+z)}\right) \qquad (12)$$

Substituting (12) into $FOC_A^{fh} = 0$ produces a cubic equation in p_A. Then, p_A^{fh} is the only solution of this cubic equation that satisfies our constraints (positive demands, h area, non-imaginary prices). We are thus able to find closed-form solutions for p_A^{fh} and p_B^{fh}, but the expressions are too intricate to be written down. They are available in the Online Appendix. Figure 2 provides an illustration when $a = z = \frac{1}{2}$.

Frontiers between the l, m, and h areas: In equilibrium, the monopolist implements the strategy that maximizes its profit: $\underline{t}^f$ corresponds to the threshold above which it is no longer profitable to attract all consumers to the store, that is, when $\Pi(p_A^{fl}, p_B^{fl}) = \Pi(p_A^{fm}, p_B^{fm})$, and $\bar{t}^f = 2a + z - p_A^{fm} - p_B^{fm}$ corresponds to the threshold above which it is profitable to exclude all consumers located in $x = 1$, including those whose valuation for good B is $v_B = a + z$. This is the case when $\Pi(p_A^{fm}, p_B^{fm}) = \Pi(p_A^{fh}, p_B^{fh})$.

When $t = \underline{t}^f$, $\pi^{fm}(p_A^{fl}, p_B^{fl}) = \pi^{fl}(p_A^{fm}, p_B^{fm})$, $p_A^{fl} = p_A^{fm}$, and $p_B^{fl} = p_B^{fm}$. This triggers $\underline{t}^f = \frac{a(a-3z)}{a-7z}$. Similarly, when $t = \bar{t}^f$, $\pi^{fm}(p_A^{fh}, p_B^{fh}) = \pi^{fh}(p_A^{fm}, p_B^{fm})$, $p_A^{fh} = p_A^{fm}$, and $p_B^{fh} = p_B^{fm}$. Symbolic solving is too demanding to find a closed-form expression for $\bar{t}^f$, but it is possible to numerically compute $\bar{t}^f$ for any value of a and z.

A.3 Proof Lemma 3

See the online appendix for a numeric proof on Mathematica. The proof is straightforward: we check that, for any value, the derivative of the price with respect to t is positive.

A.4 Proof Lemma 4

We can first solve the m case by backward induction. Once customers are in the store, the monopolist sets p_B^{Am} to maximize $\pi^{Am} = p_A^{Am} D_A^{Am} + p_B^{Am} D_B^{Am}$.

This implies

$$p_B^{Am} = \frac{1}{3}[-2a + 2(p_A + \widehat{p}_B)$$

$$+ \sqrt{a^2 + p_A^2 + (\widehat{p}_B)^2 - 2a(p_A + \widehat{p}_B - 6t) + 2p_A(\widehat{p}_B - 3t) - 6\widehat{p}_B t - 3t^2 + 6tz]}$$

Since consumers hold rational anticipations, the anticipated price $\widehat{p_B^{Am}}$ solves $\widehat{p_B^{Am}} = p_B^{Am}$. This triggers the solution:

$$\widehat{p_B^{Am}} = \frac{a^2 + (p_A + t)^2 - 2a(p_A + 2t) - 2tz}{2(p_A - a - t)}$$

In the m area, $a > p_A^{Am} > a - t$ and $2a + z - p_A^{Am} - p_B^{Bm} - t > 0$. This is enough to show

$$\frac{\partial \widehat{p_B^{Am}}}{\partial p_A} = \frac{a^2 - 2ap_A + p_A^2 + 4at - 2p_A t - 3t^2 + 2tz}{2(a - p_A + t)^2} > 0$$

Using the same method to solve the h case, one gets

$$\widehat{p_B^{Ah}} = 3a - 2p_A + z - \sqrt{2}\sqrt{(a - p_A)(3a - 2p_A + z)}$$

In the h area, $a > p_A^{Am} > a - t$ and $2a + z - p_A^{Am} - p_B^{Bm} - t < 0$. This is enough to show

$$\frac{\partial \widehat{p_B^{Ah}}}{\partial p_A} = -2 + \frac{5a - 4p_A + z}{\sqrt{2}\sqrt{(a - p_A)(3a - 2p_A + z)}} > 0$$

A.5 Proof Proposition 1

First, let us focus on the "m" area. We need to show $p_A^{Am} < p_A^{fm}$ and $p_B^{Am} > p_B^{fm}$ when $\underline{t}^A \leq t \leq \overline{t}^A$.

Under perfect information, the best-response price function

$$p_B^{fm}(p_A) = -\frac{a^2 + 3p_A^2 + 4p_A t - 4a(p_A + t) + t(t - 2z)}{4t}$$

is obtained from the first order condition $\frac{\partial \pi^{fm}(p_A, p_B)}{\partial p_B} = 0$. We can write

$$\frac{\partial p_B^{fm}(p_A)}{\partial p_A} = \frac{2a - 2t - 3p_A}{2t} < 0$$

in the "m" area, because $p_A > a - t$ and $p_A > 0$. As a consequence, $p_B^{fm}(p_A)$ decreases in p_A.

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When the monopolist advertises good A, the best-response price function

$$p_B^{Am}(p_A) = -\frac{a^2 + (p_A + t)^2 - 2a(p_A + 2t) - 2tz}{2(a - p_A + t)}$$

is the solution of the equation $p_B(p_A, \widehat{p}_B) = \widehat{p}_B$ where $p_B(p_A, \widehat{p}_B)$ stems from the first order condition $\frac{\partial \pi^{Am}(p_A, p_B, \widehat{p}_B)}{\partial p_B} = 0$. We can write

$$\frac{\partial p_B^{Am}(p_A)}{\partial p_A} = \frac{a^2 - 2ap_A + p_A^2 + 4at - 2p_A t - 3t^2 + 2tz}{2(a - p_A + t)^2} > 0$$

in the "m" area, because $a > p_A > a - t$ and $2a + z - p_A - p_B - t > 0$. As a consequence, $p_B^{Am}(p_A)$ increases in p_A.

Since $p_B^{fm}(p_A)$ decreases in $p_A > a - t$ and $p_B^{Am}(p_A)$ increases in $p_A > a - t$, and since $p_B^{fl}(p_A) = p_B^{Al}(p_A)$ as long as $p_A \leq a - t$:

$$\forall t \in [\underline{t}^A, \bar{t}^A], p_B^{fm}(p_A) \leq p_B^{Am}(p_A)$$

To prove $p_A^{Am} < p_A^{fm}$, let us introduce some additional notations. With $\pi^m = p_A * D_A^m + p_B * D_B^m$ the profit in the m area, the first order conditions of the profit are given by:

$$FOC_A^f = \frac{\partial \pi^m}{\partial p_A} = D_A^m + p_A \frac{\partial D_A^m}{\partial p_A} + p_B \frac{\partial D_B^m}{\partial p_A}$$

$$FOC_B^f = \frac{\partial \pi^m}{\partial p_B} = D_B^m + p_B \frac{\partial D_B^m}{\partial p_B} + p_A \frac{\partial D_A^m}{\partial p_B}$$

When the retailer advertises good A, the optimal choice of p_A is given by the maximization of its profit anticipating the optimal price of product B $p_B^{Am}(p_A)$, which boils down to maximizing $\pi(p_A, p_B^{Am}(p_A))$. This leads to the following FOC of the sequential program:

$$SFOC_A^{Am}(p_A) = \frac{\partial \pi(p_A, p_B^{Am}(p_A))}{\partial p_A} = FOC_A^f(p_A, p_B^{Am}(p_A)) + \underbrace{\frac{\partial p_B^{Am}(p_A)}{\partial p_A}}_{>0} FOC_B^f(p_A, p_B^{Am}(p_A))$$

Under perfect information, solving the problem sequentially or simultaneously is equivalent. We have shown that, for all $p_A \in [a - t, a]$, $p_B^{Am}(p_A) > p_B^{fm}(p_A)$, which implies that $FOC_B^f(p_A, p_B^{Am}(p_A)) < FOC_B^f(p_A, p_B^{fm}(p_A)) = 0$ because the concavity of the program implies that $FOC_B(p_A, p_B)$ is decreasing in p_B.

Hence, we have

$$SFOC_A^{Am}(p_A^{fm}) \leq FOC_A^f(p_A^{fm}, p_B^{Am}(p_A^{fm}))$$

Moreover:

$$\frac{\partial FOC_A^f}{\partial p_B} = \frac{2a - 2t - 3p_A}{2tz} < 0$$

in the m area, because $p_A > a - t$, and $p_A > 0$. As a result, for all $p_A \in [a - t, a]$, $FOC_A^f(p_A, p_B^{Am}(p_A)) < FOC_A^f(p_A, p_B^{fm}(p_A))$.

Hence we have

$$SFOC_A^{Am}(p_A^{fm}) \leq FOC_A^f(p_A^{fm}, p_B^{Am}(p_A^{fm})) < FOC_A^f(p_A^{fm}, p_B^{fm}(p_A^{fm})) = 0.$$

Because $SFOC_A^{Am}(p_A)$ is decreasing in p_A, $SFOC_A^{Am}(p_A^{fm}) < 0$ implies that $p_A^{Am} < p_A^{fm}$. Since $p_B^{fm}(p_A)$ is decreasing in p_A, we thus have $p_B^{fm}(p_A^{fm}) < p_B^{fm}(p_A^{Am})$. Eventually:

$$p_B^{fm}(p_A) < p_B^{Am}(p_A) \implies p_B^{fm}(p_A^{Am}) < p_B^{Am}(p_A^{Am}) \implies p_B^{fm}(p_A^{fm}) < p_B^{Am}(p_A^{Am}),$$

hence the result.

The proof for the "h" area is exactly similar.

Let us now prove $\underline{t}^A = \underline{t}^f$. As shown in Lemma 2, $\underline{t}^f = \frac{a(a-3z)}{a-7z}$. Using the method already used in A.2, we know that when $t = \underline{t}^A$, p_A^{Al} and p_B^{Al} must clear the FOCs for π^{Am}. Plugging p_A^{Al} and p_B^{Al} in the FOCs for π^{Am} triggers

$$FOC_A^{fm}(p_A^{Al}, p_B^{Al}) = \frac{5a^2 - 2a(3\widehat{p}_B + 2t + z) + 4\widehat{p}_B t - 2\widehat{p}_B z + 12tz + z^2}{8tz}$$

$$FOC_B^{fm}(p_A^{Al}, p_B^{Al}) = -\frac{3a^2 - 8a\widehat{p}_B + 6az + 4(\widehat{p}_B)^2 - 8\widehat{p}_B z + 3z^2}{16tz}$$

Taking the natural solution where the anticipated price $\widehat{p}_B$ is equal to p_B:

$$FOC_A^{fm}(p_A^{Al}, p_B^{Al}) = \frac{a^2 - a(t + 3z) + 7tz}{4tz}$$

This FOC is solved for $\underline{t}^A = \frac{a(a-3z)}{a-7z} = \underline{t}^f$.

Eventually, let us prove $\overline{t}^A \leq \overline{t}^f$. To do so, we check that, when $t = \overline{t}^f$, the consumer located in $x = 1$ whose valuation for good B is equal to $a + z$ prefers not to visit the store:

$$2a + z - p_A^{Am}(\overline{t}^f) - p_B^{Am}(\overline{t}^f) - \overline{t}^f \leq 0$$

This implies that when $t = \overline{t}^f$ and the monopolist advertises product A, the conditions to be in the m area are no longer met: hence the result.

A.6 Proof Proposition 2

When the monopolist advertises B, everything happens as if the monopolist set the price of good A once consumers are already in the store. Since all consumers have the same valuation $v_A = a$ for good A, all consumers who are in the store purchase A when $p_A \leq a$, and no consumer purchases A when $p_A > a$. The optimal price for good A is thus $p_A^B = a$, and all in-store consumers purchase A.

Anticipating this price, consumers decide to visit the store when $v_B - p_B^{Bs} - tx \geq 0$.

When $s = l$, that is, when $\min(p_B^{Bs} + t, a + z) = p_B^{Bs} + t$, demands are:

$$D_B^{Bl} = D_A^{Bl} = \frac{a + z - p_B^{Bl} - \frac{t}{2}}{2z}$$

The FOC, $\frac{2z - t + 4p_B^{Bl}}{4z} = 0$, defines the optimal price $p_B^{Bl} = \frac{1}{4}(2z - t)$.

When $s = m$, that is, when $\min(p_B^{Bs} + t, a + z) = a + z$:

$$D_B^{Bm} = D_A^{Bm} = \frac{(a + z - p_B^{Bm})^2}{4tz}$$

The FOC, $\frac{(a + 3p_B^{Bm} - z)(p_B^{Bm} - a - z)}{4tz} = 0$, defines the optimal price $p_B^{Bm} = \frac{1}{3}(z - a)$.

The threshold $\bar{t}^B$ such that the equilibrium area switches from l to m is given by: $\pi^{Bl}(p_A^{Bl}, p_B^{Bl}) = \pi^{Bm}(p_A^{Bm}, p_B^{Bm})$ and $\bar{t}^B = a + z - p_B^{Bl}$. Thus, $\bar{t}^B = \frac{2}{3}(2a + z)$.

A.7 Proof Lemma 3

To prove $\forall t \in [0, 1] : p_A^{As} < p_A^{Bs}$, it is enough to note that $\forall t, p_A^{Bs} = a$, whereas $\forall t > 0, p_A^{As} < a$. Indeed, when the monopolist advertises good A, it must post a price below consumers valuation a to attract buyers in the store.

To prove $\forall t \in [0, 1] : p_B^{Bs} < p_B^{As}$, it is enough to note that $\frac{\partial p_B^{As}}{\partial t} \geq 0$, $\frac{\partial p_B^{Bs}}{\partial t} \leq 0$, and in $t = 0$ we have $\frac{z}{2} = p_B^{Bl} \leq p_B^{Al} = \frac{a+z}{2}$.

A.8 Proof Proposition 4

Let $\pi^A(t)$ and $\pi^B(t)$ denote the equilibrium profit when the monopolist advertises A and B, respectively, and define the profit difference $\Delta(t) \equiv \pi^A(t) - \pi^B(t)$. With $a = z = 1/2$, we prove: (i) $\Delta(t) > 0$ for t small; (ii) $\Delta(t) < 0$ for t large. The intermediate value theorem then guarantees $\tilde{t}$ exists.

(i) Low t: advertising A dominates. When t is small (l area), all consumers visit the store. When the monopolist advertises A, we have shown in Section 5.1 that the equilibrium prices coincide with the full-information prices: $p_A^{Al} = a - t$ and $p_B^{Al} = (a+z)/2$. The resulting profit is:

$$\pi^{Al}(t) = (a - t) \cdot 1 + \frac{a+z}{2} \cdot \frac{a + z - \frac{a+z}{2}}{2z} = a - t + \frac{(a+z)^2}{8z}$$

When the monopolist advertises B, Proposition 2 gives $p_A^{Bl} = a$ and $p_B^{Bl} = \frac{2z-t}{4}$ with every visitor buying both goods. Demand of visitors are $\frac{a+z-p_B^{Bl}-\frac{t}{2}}{2z}$, so the resulting profit is:

$$\pi^{Bl}(t) = \left(a + p_B^{Bl}\right) \cdot \frac{a + z - p_B^{Bl} - \frac{t}{2}}{2z} = \frac{(4a - t + 2z)^2}{32z}$$

Hence, in the l area:

$$\Delta(t) = \pi^{Al}(t) - \pi^{Bl}(t) = \frac{12a(2z - a) + 8at - 28zt - t^2}{32z} \tag{13}$$

In particular, $\Delta(0) = \frac{3a(2z-a)}{8z} > 0$ provided $z > \frac{a}{2}$. Thus, for $t > 0$ small enough, we have $\pi^A(t) > \pi^B(t)$. See the Online Appendix for a numerical value for $\Delta(0)$.

(ii) High t: advertising B dominates. For large t, when $a = z = 1/2$, advertising B remains in the l area, where the price of B is $p_B^{Bl} = \frac{1}{4}(2z - t)$ and $p_A^{Bl} = a$: every visitor buys both goods and the mass of visitors is $\frac{a+z-p_B^{Bl}-\frac{t}{2}}{2z}$. Therefore:

$$\pi^{Bl}(t) = \frac{(4a - t + 2z)^2}{32z}$$

Under advertising A, the relevant region for high values of t is the h area. As seen in Section 5.1, this optimal prices in this region satisfy three conditions: the monopolist maximizes her profit with respect to p_B^{Ah}, that is $\frac{\partial \pi_A^h}{\partial p_B^{Ah}} = 0$, the monopolist maximizes her profit with respect to p_A^{Ah}, that is $\frac{\partial \pi_A^h}{\partial p_A^{Ah}} = 0$, and consumers form rational anticipations, that is, $\widehat{p_B^{Ah}} = p_B^{Ah}$. We show $\pi^{Ah}(1) < \pi^{Bl}(1)$. See the Online Appendix for the numerical value of $\pi^{Ah}(1)$.

General case See the Online Appendix for numerical evidence.

A.9 Proof Proposition 5

See the Online Appendix for numerical evidence over a dense grid of parameters.

A.10 Proof Lemma 5

When $t = 0$, all consumers visit the store, and buy a good whenever their valuation for this good is above its price. As a consequence, the monopolist sets the price of each good independently.

All consumers buy good A as long as $p_A \leq a$. The optimal price is thus $\overline{p}_A^{f0} = a$.

Each consumer buys good B as long as $p_B \leq v_B$. The profit made on good B is thus equal to

$$\Pi_B(p_B) = \mathbb{P}(v_B \geq p_B) * p_B = (1 - \frac{p_B - (a - z)}{a + z - (a - z)})p_B$$

Hence $\overline{p}_B^{f0} = \frac{a+z}{2}$. Note that we have assumed $z > \frac{a}{3}$ to ensure $\frac{a+z}{2} \geq a - z$, so that the solution is interior.

A.11 Proof Lemma 6

To solve for equilibrium prices, we begin by noting there are only two possible cases: either all consumers visit the store, or only those who value good B enough.

When the monopolist decides to sell to all consumers, p_A is set to its maximum level such that all consumers still visit the store: $\overline{p}_A^{fl} = a - \frac{t}{2}$. Since all consumers visit the store to buy good A, p_B is set to its monopoly level: $\overline{p}_B^{fl} = \overline{p}_B^{f0} = \frac{a+z}{2}$. This strategy leaves the retailer with a profit $\overline{\Pi}^{fl} \equiv a - \frac{t}{2} + \frac{(a+z)^2}{8z}$.

Regarding the second strategy, the monopolist maximises $\Pi(p_A, p_B) = \mathbb{P}(v_B \geq v_0) * (p_A + p_B) = \frac{a+z-v_0}{2z}(p_A + p_B)$ where $v_0 = p_A + p_B + \frac{t}{2} - a$ is the valuation for good B of the consumer who is indifferent between staying at home and visiting the store. Optimal prices are all combinations of p_A and p_B such that $\overline{p}_A^{fh} + \overline{p}_B^{fh} = \frac{1}{2}(2a + z - \frac{t}{2})$. This strategy leaves the retailer with a profit $\overline{\Pi}^{fh} \equiv \frac{(2a+z-\frac{t}{2})^2}{8z}$.

In equilibrium, the monopolist implements the strategy that maximises its profit:

$$\overline{\Pi}^{fl} \leq \overline{\Pi}^{fh} \iff t \geq 2(a + \frac{(a + z)^2 - (2a + z - \frac{t}{2})^2}{8z})$$

For instance, when $a = z = \frac{1}{2}$, $\hat{t}^f = 1$.

When shopping costs t lie above this $\hat{t}^f$, the monopolist prefers the strategy that excludes some consumers from the market.

A.12 Proof Lemma 7

To solve for equilibrium prices, we begin by noting there are two *a priori* possible cases: either all consumers visit the store, or only those who value good B enough.

In the first case, p_A is set to its maximum level such that all consumers still visit the store: $\overline{p}_A^{Al} = \overline{p}_A^{fl} = a - \frac{t}{2}$. Since all consumers visit the store to buy good A, p_B is set to its monopoly level: $\overline{p}_B^{Al} = \overline{p}_B^{fl} = \frac{a+z}{2}$. This strategy leaves the retailer with a profit $\overline{\overline{\Pi}}^{Al} \equiv a - \frac{t}{2} + \frac{(a+z)^2}{8z}$.

Regarding the second case, we need to show that, when only consumers who value good B enough decide to incur shopping costs to visit the store, there is no market. The proof is similar to the benchmark case without information, and consists in showing that, for any anticipated price $\widehat{p}_B < a + z$, the monopolist has an incentive to unexpectedly raise p_B above $\widehat{p}_B$.

Thus, $\hat{t}^A$ is such that the monopolist retailer makes no profit when all consumers visit the store:

$$\overline{\overline{\Pi}}^{Al} = 0 \iff \hat{t}^A = 2(a + \frac{(a+z)^2}{8z})$$

A.13 Proof Lemma 8

When the monopolist advertises B, everything happens as if the monopolist set the price of good A once consumers are already in the store. Since all consumers have the same valuation $v_A = a$ for good A, all consumers who are in the store purchase A when $p_A \leq a$, and no consumer purchases A when $p_A > a$. The optimal price for good A is thus $\overline{p}_A^B = a$, and all in-store consumers purchase A.

Anticipating this price, consumers decide to visit the store when $v_B - \overline{p}_B^B - \frac{t}{2} \geq 0$. As a consequence, there exists a threshold $\hat{v} \equiv \overline{p}_B^B + \frac{t}{2}$ such that only consumers whose valuations v_B for good B are above $\hat{v}$ visit the store.

Taking that into account, the monopolist sets $\overline{p}_B^B$ to maximise its profit:

$$\Pi(\overline{p}_B^B) = (\overline{p}_A^B + \overline{p}_B^B)\mathbb{P}(v_B \geq \hat{v}) = (a + \overline{p}_B^B)\frac{a + z - \overline{p}_B^B - \frac{t}{2}}{2z}$$

The solution is $\overline{p}_B^B = \frac{1}{4}(2z - t)$.