ECO 650: Final Exam 2024

December, 18th 2024

1 Exercice 1 : Innovation - 12 pts

There is one incumbent seller, I, who produces one unit at a cost $\overline{c_I}$.
There is a single buyer, B, who withdraws a surplus, r, from one unit of product. This value is the same if the product was bought from I or E. This surplus can come from the consumption of the product or its resale.
There is a potential entrant seller E, who can produce one unit at a cost c_E.
I and B can decide to sign an exclusive dealing agreement. If such exclusive dealing agreement is signed, B cannot deal with E. E has no bargaining power (implicitly, it is as if there were a competitive fringe of entrants with the same ability to offer a unit at cost c_E and that B could buy from any of them). In contrast, I and B have even bargaining powers.

We solve the following 3-stage game:

1. I and B may sign an exclusive dealing agreement.
2. I may decide to realize an investment that lowers its cost to a level $c_I = \overline{c_I} - x$: it chooses its level of investment x at a cost $\lambda(x)$ where $\lambda'(x) > 0$ and $\lambda''(x) > 0$. E cannot invest.
3. I and B bargain à la Nash over a fixed fee T.

Note that, the incumbent is always more efficient than the entrant ($c_E > \overline{c_I}$).

Questions:

1. Suppose an exclusive dealing agreement has been signed.
   (a) Determine the Nash bargaining product, the equilibrium tariff T^{ED} and profits Π_B^{ED} and Π_I^{ED}. (2 pts)

$$\max_T (r - T)(T - c_I + x)$$

$$r - T = T - c_I + x$$
$$T^{ED} = \frac{r + c_I - x}{2}$$
$$\Pi_I^{ED} = T^{ED} - (c_I - x) = \frac{r - c_I + x}{2}$$

and

$$\Pi_B^{ED} = r - T^{ED} = \frac{r - c_I + x}{2}$$

(b) Write down the FOC that gives the equilibrium investment level c_I chosen by I. (1 pts)

$$\max_x \frac{r - c_I + x}{2} - \lambda(x)$$
$$\lambda'(x) = \frac{1}{2}$$

2. Suppose now that no exclusive dealing agreement has been signed.

(a) Determine the Nash bargaining product, the equilibrium tariff T and profits Π_B and Π_I. (2 pts)

$$\max_T (r - T - (r - c_E))(T - c_I + x)$$
$$c_E - T = T - c_I + x$$
$$T^* = \frac{c_E + c_I - x}{2}$$
$$\Pi_I^* = T^* - (c_I - x) = \frac{c_E - c_I + x}{2}$$

and

$$\Pi_B^* = r - T^* = r - (\frac{c_E + c_I - x}{2})$$

(b) Write down the FOC that gives the equilibrium investment level c_I by I. (1 pts)

$$\max_x \Pi_I^* - \lambda(x)$$

$$\max_x \frac{c_E - c_I + x}{2} - \lambda(x)$$
$$\lambda'(x) = \frac{1}{2}$$

3. Are the incentives to invest of I affected by the exclusive dealing agreement? Comment. NO. The incentives to invest is not affected by ED. This is counter-intuitive as we could think that the ED agreement protects I against hold-up.

4. Assume now that, instead of I, B can invest in stage 2 to boost the value it withdraws from the product $r + \delta$. It can invest $\gamma(\delta)$ with $\gamma'(\delta) > 0$ and $\gamma''(r) > 0$.

(a) Using the previous profit expressions, determine the FOCS giving B's investment under an exclusive dealing agreement or not. (2pts) B would obtain

$$\Pi_B^{ED} = \frac{r + \delta - c_I}{2}$$

and therefore $\max_x \Pi_B^{ED} - \lambda(x)$ the FOC is

$$\frac{1}{2} = \gamma'(\delta)$$

in the case of ED. In the case without ED, B obtain

$$\Pi_B^* = r + \delta - \frac{c_E - c_I}{2}$$

and therefore

$$\max_x \Pi_B^* - \lambda(x)$$
$$1 = \gamma'(\delta)$$

.

(b) Are the incentives to invest of B affected by the exclusive dealing agreement? Comment. (1 pts) Yes B has more incentives to invest without ED. This make sense as there is no hold up on B's investment when B can buy from the entrant at the competitive price c_E (without ED).

5. Now assume that when the incumbent invests x, it also exerts a spillover on E's cost which becomes $c_E - \alpha x$ with $\alpha \in [0, 1]$.

(a) Interpret the role of α. (1 pts) it is a spillover. When $\alpha = 1$, the spillover is complete and when $\alpha = 0$ there is no spillover.

(b) Write down the FOCs with and without exclusive dealing and compare the incentives of I to invest in both cases. Comment. (2 pts)

$$\Pi_I^{ED} = \frac{r - c_I + x}{2}$$

and therefore the FOC is

$$\frac{1}{2} = \gamma'(\delta)$$

under ED.

$$\Pi_I^* = \frac{c_E - \alpha x - c_I + x}{2}$$

and therefore

$$\frac{(1 - \alpha)}{2} = \gamma'(\delta)$$

. The presence of a spillover does not affect I's investment when it is under ED. However, it reduces its incentive to invest without ED because its own investment benefits its rival entrant. In that case an ED agreement protects the investment of I.

2 Bundling (8. pts)

Consider a monopoly firm producing two goods A and B at zero cost. A unit mass of consumers have preferences over the two goods: each consumer is identified by a couple (θ_A, θ_B) uniformly distributed over $[0,1]$. The valuations for the two goods are independent; a consumer valuation for the bundle is $\theta_A + \theta_B$.

1. Represent consumers preferences in a square. Explain. (1pt) See class 1.
2. Assume that the two goods are sold separately. What is the profit of the firm? (1pt)

   $$\pi = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}.$$

3. Assume that the two goods are sold in a bundle only. Determine the demand of consumers and the equilibrium profit of the firm. Comment. (3pts)

   $$\pi^b = p(1 - \frac{p^2}{2}).$$

   Maximizing this profit we obtain $1 - \frac{3}{2}p^2 = 0$ and therefore $p = \sqrt{\frac{2}{3}}$ for a profit $\pi^b = \frac{2}{3}\sqrt{\frac{2}{3}}$

4. Show in which areas consumers lose or win with pure bundling? (3pt) See class 1.