ECO 650: Final Exam 2020 (2 hours)

1 Entry and switching costs (12. pts)

Consumers of mass 1 are uniformly distributed over a segment $[0, 1]$. We consider a two-period model.

- In $t = 1$, only an incumbent firm I is located in 0. We assume that a share $0 < K1 \leq 1$ of consumers buy from I in $t = 1$.
- In $t = 2$, an entrant E may enter at a fixed cost F and locate in 1. A consumer located in x incurs $-x$ as a desutility to buy in I and $-(1 - x)$ to buy in 1. Prices set by I and E are respectively denoted p_I and p_E. Among the two periods, consumers redraw their type and a consumer who has bought from I in $t = 1$ has to incur an additional switching cost z in $t = 2$ to buy from E.

Assume E has entered.

1. Determine the demand in $t = 2$ from consumers who have not bought from I in $t = 1$.

A consumer who has not bought from I in $t = 1$ and is located in x in $t = 2$ decides to buy from I if $p_I + x < p_E + (1 - x) \rightarrow \tilde{x} = \frac{1+p_E-p_I}{2}$.

2. Determine the demand in $t = 2$ from consumers who have bought from I in $t = 1$. A consumer who has bought from I in $t = 1$ and is located in x decides to buy from I if $p_I + x < p_E + (1 - x) + z \rightarrow \hat{x} = \tilde{x} + \frac{z}{2}$.

3. Assume that $\hat{x} < 1$ (small switching costs), determine the total demand and profit for firm I and E in $t = 2$ and the corresponding Nash equilibrium in prices and profits in $t = 2$.

   Total demand for firm I is $D_I(p_I, p_E, K1) = K1(\tilde{x} + \frac{z}{2}) + (1 - K1)\tilde{x}$ and a profit $\Pi_I(p_I, p_E, K1) = p_I(K1(\tilde{x} + \frac{z}{2}) + (1 - K1)\tilde{x})$. Total demand for firm E is $D_E(p_I, p_E, K1) = K1(1 - \tilde{x} - \frac{z}{2}) + (1 - K1)(1 - \tilde{x})$ and the corresponding profit is $\Pi_E(p_I, p_E, K1) = p_E(K1(1 - \tilde{x} - \frac{z}{2}) + (1 - K1)(1 - \tilde{x}))$ Best reaction functions are :

$$\frac{\partial \Pi_I}{\partial p_I} = 0 \Rightarrow p_I(p_E, K1) = \frac{1}{2}p_E + \frac{1}{2}(1 + K1z)$$

$$\frac{\partial \Pi_E}{\partial p_E} = 0 \Rightarrow p_E(p_I, K1) = \frac{1}{2}p_I + \frac{1}{2}(1 - K1z)$$

   Nash equilibrium prices and profits is:

$$p_I^* = 1 + \frac{K1z}{3}, p_E^* = 1 - \frac{K1z}{3}$$

$$\Pi_I^* = \frac{(3 + K1z)^2}{18}, \Pi_E^* = \frac{(3 - K1z)^2}{18}$$

4. Assume that $\hat{x} > 1$ (large switching costs), determine the total demand and profit for firm I and E in $t = 2$ and the corresponding Nash equilibrium in prices and profits in $t = 2$. Total demand for firm I

is $D_I(p_I, p_E, K1) = K1 + (1-K1)\tilde{x}$ and a profit $\Pi_I(p_I, p_E, K1) = p_I(K1 + (1-K1)\tilde{x})$. Total demand for firm E is $D_E(p_I, p_E, K1) = (1-K1)(1-\tilde{x})$ and the corresponding profit is $\Pi_E(p_I, p_E, K1) = p_E((1-K1)(1-\tilde{x}))$ Best reaction functions are :

$$\frac{\partial \Pi_I}{\partial p_I} = 0 \Rightarrow p_I(p_E, K1) = \frac{1}{2}p_E + \frac{1+K1}{2(1-K1)}$$

$$\frac{\partial \Pi_E}{\partial p_E} = 0 \Rightarrow p_E(p_I, K1) = \frac{1}{2}p_I + \frac{1}{2}$$

Nash equilibrium prices and profits are:

$$p_I^* = \frac{3+K1}{3(1-K1)}, p_E^* = \frac{3-K1}{3(1-K1)}$$

$$\Pi_I^* = \frac{(3+K1)^2}{18(1-K1)}, \Pi_E^* = \frac{(3-K1)^2}{18(1-K1)}$$

5. Assume now that a non strategic firm I, i.e. an incumbent who does not anticipate the entry of E in $t=2$ would set $K1 = \frac{1}{2}$ in $t=1$. This means that $K_1 = \frac{1}{2}$ is the maximum of the first period profit. When taking into account the second period profit, the sign of the derivative of the this profit with respect to K_1 indicates how I distort $K1$.

   More precisely:

   (a) When switching costs are small ($\hat{x} < 1$).

      - To deter E's entry?I wants to overinvest :$\frac{\partial \Pi_E^*}{\partial K_1} < 0$
      - To accomodate $E's$ entry?I wants to overinvest :$\frac{\partial \Pi_I^*}{\partial K_1} > 0$
      - What is the name of this strategy in the Fudenberg-Tirole taxonomy? Explain. Top dog- Top dog

(b) When switching costs are large ($\hat{x} > 1$). Does I overinvest or underinvest in $K1$

- To deter E's entry? I wants to underinvest : $\frac{\partial \Pi_E^*}{\partial K_1} > 0$
- To accomodate E's entry? I wants to overinvest:$\frac{\partial \Pi_I^*}{\partial K_1} > 0$
- What is the name of this strategy in the Fudenberg-Tirole taxonomy? Explain. Lean and hungry look- Top dog

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2 Bundling (8 pts)

Two consumers A and B have the following valuations for Sport tickets:

Consumers	5 Basket	5 Tennis
Type A	90	50
Type B	70	40

On an annual basis, SPORT 24 offer annual supscription for basketball and Tennis games. Each game costs 5 euros to the Company. Sport 24 cannot discriminate among consumers. To simplify, consider that there is 1 consumer of each type (A and B).

Questions:

1. Determine the best pricing strategy for SPORT 24 if it offers an annual card fee per sport type? (2 pts) $p_B = 70$ $\pi_B = (70-25).2 = 90$ $p_T = 40$ $\pi_B = (40 - 25).2 = 30$ Total profit is 120.

2. Determine the optimal price for SPORT 24 if it offers only a Gold card membership (Full access to all games- pure bundling)? (2 pts) $p_b = 110$ and $\Pi_b = 2.(110 - 50) = 120$. Bundling is not profitable.

3. Consumers now have the following valuations:

Consumers	5 Basket	5 Tennis
Type A	90	50
Type B	40	70

Answer to the same questions (1) and (2). (2 pts) $p_B = 90$, $\pi_B = (90-25) = 55$ and $P_T = 50$ with $\Pi_T = (50-25).2 = 50$. Total profit is 105 and $\pi_b = 2.(110-50) = 120$.

4. In which case bundling is the most profitable? Explain. (2 pts) Bundling is profitable in the second case because consumer's valuation are negatively correlated.