ECO 650: Firms' Strategies and Markets
Vertical Relationships and Bargaining(II)

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Exercise 1

Assumptions:

- A manufacturer produces a good at a unit cost c.
- A retailer faces a demand $D(p) = 1 - p$.
- The game:
  1. The manufacturer and the retailer bargain over a two-part tariff contract (w, F);
  2. The retailer sets a final price p to consumers.

Questions:

1. Given the contract (w, F), determine the optimal price set by the retailer in stage 2. Determine the stage-2 equilibrium profits of firms $\pi_U(w) + F$ and $\pi_D(w) - F$.
2. Write down the Nash program and determine the optimal contract (w, F). Is it efficient?

Exercise 1: Solution

1. In stage 2, the retailer maximizes $\max_p (p-w)(1-p) - F$; The FOC is: $1 - 2p + w = 0 \Rightarrow p = \frac{1+w}{2}$; $\pi_U(w) = (w-c)(\frac{1-w}{2})$ and $\pi_D(w) = (\frac{1-w}{2})^2$.

2. The Nash program in stage 1 is

$$\max_{(w,F)} (\pi_U(w) + F)(\pi_D(w) - F)$$

FOCS are:

$$-(\pi_U(w) + F) + (\pi_D(w) - F) = 0 \quad (1)$$

$$\frac{\partial \pi_U(w)}{\partial w}(\pi_D(w) - F) + \frac{\partial \pi_D(w)}{\partial w}(\pi_U(w) + F) = 0 \quad (2)$$

(1) is the split the difference rule, F is used to share profits equally. Plugging (1) into (2): $\underbrace{(\frac{\partial \pi_U(w)}{\partial w} + \frac{\partial \pi_D(w)}{\partial w})}_{0} \underbrace{(\pi_D(w) - F)}_{>0} = 0$. w is set to maximize joint profits $w^* = c$: Efficiency!

Exercise 1: Solutions

$\pi_U(w) + \pi_D(w) = (\frac{1-w}{2})(\frac{1+w-2c}{2})$.
Deriving this joint profit w.r.t w gives:
$-(1+w-2c) + (1-w) = 0 \Rightarrow w^* = c$. $\pi_U(c) = 0, \pi_D(c) = \frac{(1-c)^2}{4}$
$F^* = \frac{\pi_D(c) - \pi_U(c)}{2} = \frac{(1-c)^2}{8}$.
In equilibrium both firms obtain a profit $\frac{(1-c)^2}{8}$.

3. The outside option profit $\bar{\Pi}$ is such that $\max_p (p - \bar{c})(1-p)$.

   $\bar{\Pi} = (\frac{1-\bar{c}}{2})^2$. The Nash program in stage 1 is

$$\max_{(w,F)} (\pi_U(w) + F)(\pi_D(w) - F - \bar{\Pi})$$

   FOCS are:

$$-(\pi_U(w) + F) + (\pi_D(w) - F - \bar{\Pi}) = 0 \quad (3)$$
$$\frac{\partial \pi_U(w)}{\partial w}(\pi_D(w) - F - \bar{\Pi}) + \frac{\partial \pi_D(w)}{\partial w}(\pi_U(w) + F) = 0 \quad (4)$$

   Plugging (3) into (4), again w maximizes the joint profit $w^* = c$: unchanged!

Exercise 2: Buyer size and buyer power

Assumptions:

- A manufacturer U produces a good at a unit cost $C(Q)$, with $C'(Q) > 0$ and $C''(Q) > 0$.
- Two retailers D_1 and D_2 are active on separate markets and face an inverse demand $P(Q)$ with $P'(Q) < 0$.
- The two retailers must buy from the manufacturer to offer the product to consumers.
- We consider the following one-stage game: Each manufacturer-retailer pair bargain simultaneously and secretly over a quantity forcing contract (q, F);
- Use $P(Q) = 1 - Q$ and $C(Q) = \frac{Q^2}{2}$ for numerical application.
    1. Determine the optimal contracts (q_1, F_1) and (q_2, F_2). Compute the equilibrium profit of each firm
    2. D_1 and D_2 merge and the new entity bargain with U over a new contract (q, F). Determine the new equilibrium profits.
    3. Compare the profits obtained in (1) and (2) and comment.

Solutions

1. Determine the optimal contracts (q_1, F_1) and (q_2, F_2). Compute the equilibrium profit of each firm

- Nash-bargaining with separate firms
  - $\Pi_U = F_1 + F_2 - C(q_1 + q_2)$, $\Pi_1 = P(q_1)q_1 - F_1$, and the status quo profit of firm U is $\Pi_U^{sq} = F_2 - C(q_2)$.
  - $U - D_1$ maximizes the Nash product: $\max_{q_1, F_1} [\Pi_U - \Pi_U^{sq}][\Pi_1]$
  - FOCS are:

$$F_1 - C(q_1 + q_2) + C(q_2) = P(q_1)q_1 - F_1$$

and

$$C'(q_1 + q_2) = P'(q_1)q_1 + P(q_1)$$

  - Numerical application : $q_1^* = q_2^* = \frac{1}{4}$, $F_1^* = F_2^* = \frac{9}{64}$, $\Pi_U^* = \frac{5}{32}$, $\Pi_1^* = \Pi_2^* = \frac{3}{64}$.

2. D_1 and D_2 merge and the new entity bargains with U over a new contract (q, F). Determine the new equilibrium profits.

- Nash bargaining with the merged entity
  - $U - D_1$ maximizes the Nash product: $\max_{q_1,q_2,F}[\Pi_U][\Pi_M]$ with $\Pi_M = P(q_1)q_1 + P(q_2)q_2 - F$
  - FOCS are:

    $$F - C(q_1 + q_2) = P(q_1)q_1 + P(q_2)q_2 - F$$

    and

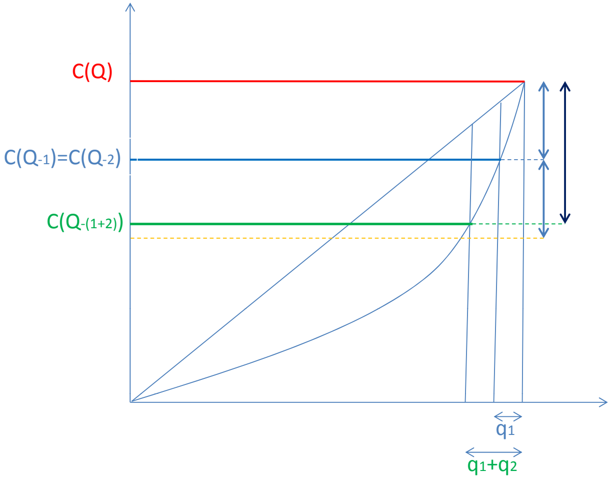
    $$C'(q_1 + q_2) = P'(q_1)q_1 + P(q_1)$$

    The second condition is unchanged which implies that quantity sold is the same.
  - Numerical application : $q_1^M = q_2^M = \frac{1}{4}$,$F = \frac{1}{4}$,$\Pi_U^M = \frac{1}{8}$, $\Pi_1^M = \Pi_2^M = \frac{1}{16} = \frac{4}{64} > \frac{3}{64}$.

3. Compare the profits obtained in (1) and (2) and comment.

Each retailer obtains a higher profit thanks to the merger. Buyer size leads to a discount!

- This is because of the convex cost function! No effect with a linear cost and reverse effect with a concave cost.
- When separated, each retailer bargains for the marginal quantity on the highest portion of the cost function.
- The merge unit bargain for the whole quantity, that is both the marginal quantity and the infra marginal quantity (less costly).



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