Exercice 2: Poaching

Assumptions

- Two firms $k \in \{A, B\}$ are located at the extremes of a Hotelling line and compete during two periods, $t \in \{1, 2\}$. Prices are denoted p_k^t.
- Consumers with a reservation price r uniformly distributed along the line, incur a linear transportation cost $-x$ to travel distance x
- No production cost.

Questions

1. Determine the equilibrium of the two period game.
2. Firms now observe consumer's identities and can set personalized prices p_{kA}^2 and p_{kB}^2 respectively for consumers who bought from A and from B in $t = 1$. Assuming that α is the market share of firm A in $t = 1$, determine the second period equilibrium.
3. Consumers are forward looking. Determine the address of the indifferent consumer α in $t = 1$.
4. Determine the first period equilibrium prices.

Solutions: Exercice 2

1. The one shot game is repeated twice: no dynamic effect here! $p_A^t = p_B^t = 1$ in both periods and each firm gets a market share $\frac{1}{2}$, the equilibrium profit is 1.
2. The indifferent consumers addresses are $\hat{x}_A = \frac{1}{2} + \frac{(p_{BA}^2 - p_{AA}^2)}{2}$ and $\hat{x}_B = \frac{1}{2} + \frac{(p_{BB}^2 - p_{AB}^2)}{2}$. Firms A and B's maximization problems are:

$$\begin{array}{ll} \max_{p_{AA}^2, p_{AB}^2} & p_{AA}^2 \hat{x}_A + p_{AB}^2(\hat{x}_B - \alpha) \\ \max_{p_{BA}^2, p_{BB}^2} & p_{BA}^2(\alpha - \hat{x}_A) + p_{BB}^2(1 - \hat{x}_B) \end{array}$$

   The solution is:

$$\begin{aligned} p_{AA}^2 &= \tfrac{1}{3}(2\alpha + 1), p_{BA}^2 = \tfrac{1}{3}(4\alpha - 1) \\ p_{AB}^2 &= \tfrac{1}{3}(4(1-\alpha) - 1), p_{BB}^2 = \tfrac{1}{3}(2(1-\alpha) + 1) \\ \hat{x}_A &= \tfrac{1}{2} - \tfrac{1}{3}\alpha, \hat{x}_B = \tfrac{1}{2} + \tfrac{1}{3}\alpha \end{aligned}$$

Solutions: Exercice 2

3. In $t = 1$, anticipating a symmetric equilibrium the indifferent consumer anticipates that it will switch in $t = 2$ and therefore its address is such that:

$$r - \alpha - p_A^1 + (r - (1 - \alpha) - p_{BA}^2) = r - (1 - \alpha) - p_B^1 + (r - \alpha - p_{AB}^2)$$

$$\alpha = \frac{3(1 + p_B^1 - p_A^1) + 1}{4}$$

4. Firm A's intertemporal profit is:

$$\max_{p_A^1} p_A^1 \alpha + p_{AA}^2 \hat{x}_A + p_{AB}^2 (\hat{x}_B - \alpha)$$

$p_A^1 = p_B^1 = \frac{4}{3}$, $\alpha = \frac{1}{2}$, $p_{AA}^2 = p_{BB}^2 = \frac{2}{3}$, $p_{BA}^2 = p_{AB}^2 = \frac{1}{3}, \hat{x}_A = \frac{1}{3}, \hat{x}_B = \frac{2}{3}$.
Poaching relaxes competition in period 1 and intensifies it in the second. Total profit is $\Pi_A = \Pi_B = \frac{5}{6}$. Firms would be better off if they could refrain from poaching.