Markups, Markdowns, and Bargaining in a Vertical Supply Chain

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Abstract

This article bridges monopoly, monopsony, and countervailing power theories to analyze the welfare effects of seller and buyer power in a vertical supply chain. We develop a bilateral monopoly setting with bargaining over a linear price, where the upstream firm sources input from an increasing supply curve, exerting monopsony power mirroring the downstream firm monopoly power. We leverage the short-side rule to endogenize which side sets the quantity traded in equilibrium. We show that welfare is maximized when each firm's bargaining power fully countervails the other's market power. Otherwise, double marginalization occurs: *double markupization* arises when the upstream firm holds excessive bargaining power, and *double markdownization* in the opposite case. Our analysis yields novel insights for policy intervention and empirical research.

Keywords: Markups, Markdowns, Bargaining, Countervailing buyer power, Monopsony power, Bilateral monopoly.

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1 Introduction

Two prominent theories offer contrasting perspectives on the welfare effects of buyer power in vertical supply chains. The countervailing power theory, introduced by Galbraith (1952), suggests that buyer power mitigates seller market power, leading to lower markups, higher output quantity, and greater welfare.[1] In contrast, the monopsony power theory, originating with Robinson (1933), argues that buyer power increases the market power of dominant buyers, resulting in greater markdowns, lower output quantity, and lower welfare.[2]

Both theories have been highly influential in academic research and policymaking. For instance, a stream of research on vertical supply chains examines the factors underlying countervailing buyer power, highlighting how it reduces double marginalization and benefit consumers (see, e.g., Snyder, 2008; Smith, 2016; Lee et al., 2021, for comprehensive surveys). Building on these insights, the concept of countervailing buyer power is frequently invoked in competition policy debates, either as an efficiency defense for downstream horizontal mergers or to justify the formation of buying alliances.[3] In parallel, a vast literature in labor economics documents the prevalence of monopsony power and examines the mechanisms to mitigate its adverse effects (see, e.g., Manning, 2021; Card, 2022; Azar and Marinescu, 2024, for reviews). Beyond the labor market, recent empirical work has highlighted that monopsony power is pervasive in various input markets (e.g., Morlacco, 2019; Avignon and Guigue, 2022; Treuren, 2022; Zavala, 2022; Rubens, 2023). Consequently, antitrust agencies have increasingly incorporated the concept of monopsony power into their analyses.[4] Thus, despite being grounded in different sets of assumptions, the countervailing and monopsony power theories conflict

[1]More precisely, Galbraith's (1952) argument states that retailers (or intermediaries) with buyer power should negotiate lower prices from manufacturers and pass these benefits on to consumers through reduced output prices.

[2]Specifically, Robinson (1933) formalizes the idea that large employers have the potential to reduce employment and pay workers below their marginal revenue.

[3]See, e.g., the Horizontal Merger Guidelines of the European Commission (2004) and the JRC policy report on buying alliances (Daskalova et al., 2020).

[4]For instance, the U.S. Department of Justice sued to block a merger between two of the largest book publishers in 2021, mentioning the potential harm to American authors as the primary concern (United States v. Bertelsmann SE & Co. KGaA et al., No. CV 21-2886-FYP). See also the recent Federal Trade Commission's lawsuit to block the merger between the supermarket giants Albertsons and Kroger (press release).

in shaping appropriate antitrust treatment of buyer power.[5]

In this article, we develop a unified framework that incorporates both theories to provide new insights into the welfare effects of buyer power in vertical supply chains. Specifically, we consider a setting where an upstream monopolist, U, sells its product to a downstream monopolist, D, which then resells it to final consumers. To examine monopsony power, we depart from the canonical model of vertical contracting (e.g., Spengler, 1950), which typically assumes that U operates with constant marginal costs. Instead, U sources its input from an upward-sloping supply curve, resulting in increasing marginal costs.[6] Mirroring D's exercise of monopoly power in the product market, U thus exercises monopsony power in the input market. We model the interactions between U and D as follows. First, U and D bargain over a linear wholesale price.[7] Second, given the agreed wholesale price, U and D simultaneously determine the quantity to offer on the market: U orders a quantity from its input suppliers, and D decides how much to purchase from U and sell to consumers. Importantly, our modeling approach departs from the canonical model of vertical relations where the equilibrium quantity is always determined by D. Although this assumption is innocuous when U's marginal costs are constant, it no longer holds with increasing marginal costs. Indeed, for a given wholesale price, buying an additional unit of input is not always profitable for U, as it raises its cost of acquiring all other units. Thus, we consider that the equilibrium quantity is determined by the short-side rule: at the agreed wholesale price, the quantity exchanged is the minimum between what U is willing to sell (i.e., supply) and what D is willing to purchase (i.e., demand).

We highlight that the balance of bargaining power between U and D affects both the magnitude and the nature of the double marginalization phenomenon. When D's

[5]As highlighted by Hemphill and Rose (2018), the Federal Trade Commission and the U.S. Department of Justice have adopted conflicting views on the treatment of buyer power in recent merger reviews.

[6]For instance, this increasing supply curve may result from the aggregation of individual price-taking input suppliers.

[7]We focus on simple linear tariffs to revisit the double marginalization problem. Although it is well known that nonlinear wholesale contracts can resolve the double marginalization issue (e.g., Stigler, 1954), it remains a concern when information asymmetries or market imperfections prevail (e.g., Rey and Tirole, 1986; Calzolari et al., 2020). Moreover, the use of simple linear wholesale tariffs has been documented in the Chilean coffee market (Noton and Elberg, 2018), the UK liquid milk market (Smith and Thanassoulis, 2015), and various other sectors (see, e.g., Mortimer, 2008; Crawford and Yurukoglu, 2012; Grennan, 2013; Gowrisankaran et al., 2015; Ho and Lee, 2017).

bargaining power vis-à-vis U is low, the equilibrium wholesale price and quantity move along D's marginal revenue curve (i.e., its demand for U's product). The intuition is as follows. As the wholesale price is high, U is willing to supply a quantity greater than D's demand. The short-side rule implies that the latter constrains the quantity exchanged in equilibrium. Hence, by internalizing D's downward-sloping demand, U exercises monopoly power when selling to D by charging a markup. This markup adds up to D's markup stemming from its monopoly power in the product market, resulting in a lower quantity and a higher output price compared to what a vertically integrated firm would set in equilibrium. This *double markupization* gives rise to the classical double marginalization problem highlighted by Spengler (1950).[8] In this case, Galbraith's (1952) countervailing buyer power argument applies: increasing D's bargaining power reduces U's markup, which improves welfare.

In contrast, when D's bargaining power vis-à-vis U is high, we find that the equilibrium wholesale price and quantity move along U's marginal cost curve. Again, this arises from the short-side rule: as the wholesale price is low, U is willing to supply a quantity smaller than D's demand, meaning that the former constrains the quantity exchanged in equilibrium. Thus, by internalizing U's upward-sloping marginal cost, D exercises monopsony power when purchasing from U by charging a markdown. This markdown adds up to U's markdown stemming from its monopsony power in the input market.[9] We show that this *double markdownization* mirrors the double markup scenario and constitutes a novel source of double marginalization. In this case, Galbraith's (1952) argument no longer applies: increasing D's bargaining power raises its markdown, which reduces welfare. Instead, enhancing U's bargaining power vis-à-vis D improves welfare by strengthening its ability to exercise *countervailing seller power*.[10]

We further characterize the level of D's bargaining power vis-à-vis U at which each firm effectively counteracts the other's market power, thereby eliminating double marginalization and achieving the vertically integrated outcome.This efficient level of

[8]As discussed by Linnemer (2022), the double marginalization phenomenon commonly attributed to Spengler (1950) is originally due to Cournot (1838) and Edgeworth (1925).

[9]More generally, D charges a markdown whenever U has increasing marginal costs, regardless of its underlying cause (e.g., monopsony power in the input market, decreasing returns to scale).

[10]This reasoning mirrors Galbraith's (1952) countervailing buyer power argument under double markupization, where D's bargaining power mitigates U's markup.

bargaining power depends on the underlying supply and demand primitives and includes two limiting cases: D should hold all the bargaining power when U faces a perfectly elastic supply curve (i.e., constant marginal cost), whereas U should hold all the bargaining power when D faces a perfectly elastic demand curve.

Contributions. We build and contribute to the extensive literature on vertical relations that, following the pioneering work of Spengler (1950), explores the sources of the double marginalization phenomenon and its potential remedies.[11] Specifically, a strand of this literature analyzing manufacturer-retailer bargaining typically assumes constant marginal costs of production.[12] Our main contribution is to relax this assumption by considering the presence of monopsony power in the input market. This allows us to identify a novel source of double marginalization, which we refer to as double markdownization. Importantly, we show that this result has significant welfare implications. Unlike the canonical model of vertical contracting where double marginalization arises exclusively through double markupization, increasing D's bargaining power under double markdownization reduces welfare. Instead, as U exercises countervailing seller power, increasing U's bargaining power mitigates D's markdown and improves welfare. Our findings thus suggest that the welfare-maximizing outcome arises under a balanced distribution of bargaining power in the vertical supply chain. Although intuitively appealing, this logic has not been formalized in the literature.[13]

The nature of the double marginalization phenomenon (i.e., double-markupization or double-markdownization) depends on whether U or D ultimately chooses the quantity to be traded in equilibrium, that is, which firm has the "right-to-manage". As underscored by Toxvaerd (2024), the allocation of the right-to-manage in bilateral monopolies with increasing marginal production costs and linear tariffs remains a

[11] See Tirole (1988) for a textbook exposition and Rey and Vergé (2008) for a literature review. Recent contributions to this topic include Janssen and Shelegia (2015); Crawford et al. (2018); Luco and Marshall (2020); Choné et al. (2024); Ghili and Schmitt (2024), among others.

[12] See, e.g., Horn and Wolinsky (1988); Dobson and Waterson (1997, 2007); Allain and Chambolle (2011); Iozzi and Valletti (2014); Gaudin (2016, 2018); Rey and Vergé (2020) in the industrial organization literature; and Grossman et al. (2024) in the trade literature.

[13] A notable exception is Falch and Strøm (2007). However, their firm-union bargaining model differs markedly from our setting, as it does not account for vertical relations (and, hence, double marginalization), and both total payroll and employment enter directly into the union's objective function.

long-standing and unresolved issue.[14] Confronted with this modeling challenge, recent work in labor economics and international trade has exogenously assigned the right-to-manage to one side of the market (e.g., Azkarate-Askasua and Zerecero, 2022; Wong, 2023; Alviarez et al., 2023). However, such an ad hoc assumption may be overly restrictive as it predetermines the welfare consequences of each firm's bargaining power, making it either always welfare-detrimental or welfare-improving.[15] We contribute to the literature by proposing a non-cooperative allocation of the right-to-manage, leveraging the subgame perfection criterion and the short-side rule. In the last stage, given the negotiated wholesale price, each firm chooses its profit-maximizing quantity subject to the constraint that it does not exceed what the other firm is willing to trade.[16]

In a bargaining framework, firms are unable to equate their marginal revenue to their marginal cost, implying that the classical expressions for firms' markups and markdowns are no longer applicable. We address this issue by offering a definition of markups and markdowns, which allows us to derive explicit expressions within a bargaining setting, including the classical expressions as a special case when prices are set unilaterally. Our approach thus provides a comprehensive framework for analyzing firms' markups and markdowns in a vertical supply chain.

Finally, our findings may have important implications for empirical research on bargaining in vertical supply chains. As reviewed by Lee et al. (2021), it is common practice to assume that upstream manufacturers operate with constant marginal costs.[17] We show that the welfare consequences of the balance of power in the vertical supply chain can vary substantially depending on the slope of the marginal cost

[14] In Fellner's (1947) pioneering analysis of bilateral monopolies, when the seller (resp. buyer) makes the wholesale price offer, the buyer (resp. seller) is assumed to freely determine the quantity it intends to purchase (sell) at the offered price. However, as Toxvaerd (2024) points out, no solution has been provided to the right-to-manage allocation: "it is not clear why either firm would want to cede the right to set output to the other firm, even if a wholesale price could be agreed upon". Several articles have circumvented this issue with efficient bargaining (e.g., the price and quantity are jointly negotiated as in McDonald and Solow, 1981; or the wholesale price is non-linear as in Chipty and Snyder, 1999).

[15] For instance, assuming that D always has the right-to-manage as in the canonical model of vertical contracting implies that welfare strictly increases (resp. decreases) with D's (resp. U's) bargaining power.

[16] In contemporaneous work, Houba (2024) instead relies on a cooperative solution where firms Nash bargain over both the wholesale price and the allocation of the right-to-manage.

[17] Among others, see Draganska et al. (2010); Crawford and Yurukoglu (2012); Ho and Lee (2017); Crawford et al. (2018); Noton and Elberg (2018); Sheu and Taragin (2021); Bonnet et al. (2023).

function. Our results thus call for a greater flexibility in modeling cost functions in empirical work. This is particularly relevant given the prevalence of convex supply curves in many industries (e.g., Shea, 1993; Boehm and Pandalai-Nayar, 2022). In this context, inferring whether upstream or downstream firms have the right-to-manage becomes an important issue for estimating markups and markdowns. A first step in this direction is developed by Atkin et al. (2024) who exploit an Argentinian import license policy that exogenously affects traded volumes to identify whether the importer or exporter determines the equilibrium quantity.

The remainder of this article is structured as follows. Section 2 provides definitions markups and markdowns accommodating both unilateral price setting and bargaining environments. Section 3 presents the vertical chain framework and a benchmark case where U and D are vertically integrated. Section 4 introduces the bargaining model and equilibrium, characterizing markup(s) and markdown(s) that emerge along the vertical supply chain. Section 5 analyzes welfare implications. Section 6 discusses our assumptions and results.

2 Markups and Markdowns

In this section, we introduce our primary objects of interest: markups and markdowns. The definitions we propose aim to accommodate the different *environments* a firm may face throughout our analysis: standard unilateral price settings but also bargaining environments where a firm's marginal cost or marginal revenue may not be defined. More generally, a firm's environment encapsulates all factors determining its behavior. Among other things, this includes the supply and demand primitives, the market structure (vertical relationship or integration), the firm's position in the vertical supply chain (upstream or downstream), and the distribution of bargaining power. We now turn to the markup and markdown definitions.

Definition 1 *A markup μ_i is firm i's surplus from selling the marginal output unit. Formally, it is the wedge between the price x_i at which the firm sells the marginal output unit and the minimum price $\hat{x}_i$ at which this marginal unit would be supplied, given the* *firm environment:*

$$\mu_i \equiv \frac{x_i}{\hat{x}_i}.$$

Definition 2 *A markdown ν_i is firm i's surplus from purchasing the marginal input unit. Formally, it is the wedge between the maximum price $\hat{z}_i$ at which the marginal input unit would be purchased and the price z_i at which the firm buys this marginal unit, given the firm environment:*

$$\nu \equiv \frac{\hat{z}_i}{z_i}.$$

As is standard in the literature, markups and markdowns measure output and input market power, respectively. While we follow these general definitions, we emphasize that their specific expressions are contingent on the firm's environment. Given that we consider different environments throughout the analysis, both $\hat{x}_i$ and $\hat{z}_i$, and thus markups and markdowns, take different expressions in equilibrium.

Definitions 1 and 2 yield the familiar markup and markdown expressions when each price is unilaterally set by a firm (e.g., vertical integration, vertical chain with take-it-or-leave-it offers). Specifically, each firm i equalizes its marginal cost and marginal revenue in equilibrium. Formally, $MC_i(q^*) = MR_i(q^*)$ for every i, with q^* the equilibrium quantity. Hence, the minimum price at which firm i is willing to supply the marginal unit corresponds to its marginal cost ($\hat{x}_i = MC_i(q^*)$), and the maximum price at which it is willing to purchase the marginal unit corresponds to its marginal revenue ($\hat{z}_i = MR_i(q^*)$).

The generality of Definitions 1 and 2 is essential when firms are involved in bargaining. In such cases, the equilibrium condition $MC_i(q^*) = MR_i(q^*)$ no longer holds for any firm i.[18] Instead, we show that either $\hat{x}_i = \hat{z}_i = MC_i(q^*)$ or $\hat{x}_i = \hat{z}_i = MR_i(q^*)$ holds for any firm i.[19]

[18] Intuitively, prices are not unilaterally set by firms facing a demand and a supply curve. As a result, the seller's marginal revenue and the buyer's marginal cost are not well-defined functions.

[19] For each case throughout, the reader will be referred to formal proofs provided in Appendix A.

3 Vertical Chain and Integration Benchmark

3.1 Vertical Chain

Consider a vertical chain in which an upstream firm, U, purchases an input at a price r to produce a good sold to consumers at a price p through a downstream firm, D. We assume that U operates with one-to-one production technology and incurs no costs beyond the input price. Likewise, D incurs no costs beyond the wholesale price w paid to U. The inverse supply function $r(q)$ faced by U and the inverse demand function $p(q)$ faced by D satisfy the following Assumption that ensures the existence of a profit-maximizing equilibrium:

Assumption 1 *The inverse supply curve $r(q)$ and the inverse demand curve $p(q)$ are continuous and three-times differentiable such that:*

i) $r'(q) \geq 0$ *and* $\sigma_r(q) > -2$*;*

ii) $p'(q) < 0$, $\varepsilon_p(q) > 1$, *and* $\sigma_p(q) < 2$*;*

iii) $p(0) > r(0)$ *and* $\lim_{q \to +\infty} p(q) = 0$,

where, for any function $f(\cdot)$, $\epsilon_f(q) \equiv \frac{f(q)}{q|f'(q)|}$ *is the elasticity of* $f(\cdot)$, *and* $\sigma_f(q) \equiv \frac{qf''(q)}{|f'(q)|}$ *is a measure of convexity of* $f(\cdot)$.

Assumption 1.i implies that U faces an increasing inverse supply curve $r(q)$ and that its marginal cost $MC_U(q)$ increases with quantity q. Note that the case of constant marginal cost is included as a special case. Assumption 1.ii implies that D faces a strictly decreasing inverse demand curve $p(q)$ and that its marginal revenue $MR_D(q)$ is positive and strictly decreasing in quantity. Finally, Assumption 1.iii implies that $MC_U(q)$ and $MR_D(q)$ intersect.

Before analyzing the case where U and D operate as separate entities, we first consider a vertically integrated structure, which serves as a useful benchmark for our analysis.

3.2 Vertical Integration Benchmark

Consider a benchmark case in which a monopolist, denoted I, purchases an input at a price r to produce a good that it sells to consumers at a price p. For simplicity, assume that I operates under a one-to-one production technology, requiring one unit of input to produce one unit of output, and incurs no additional costs beyond the input price.

Acting as a monopolist on both the output and the input market, the maximization problem of I is given by:

$$\max_{q} \Pi_I = \left(p(q) - r(q)\right) q.$$

which yields the following first-order condition:

$$\underbrace{p(q_I)\left(1 - \varepsilon_p^{-1}(q_I)\right)}_{MR_I(q_I)} = \underbrace{r(q_I)\left(1 + \varepsilon_r^{-1}(q_I)\right)}_{MC_I(q_I)}. \tag{1}$$

where q_I denotes the corresponding equilibrium quantity in which I's marginal revenue equals its marginal cost. The exercise of monopoly power over consumers implies that I's marginal revenue differs from the output price $p(q_I)$ by a *wedge* equal to $1 - \varepsilon_p^{-1}(q_I)$. Similarly, the exercise of monopsony power over input suppliers implies that I's marginal cost differs from the input price $r(q_I)$ by a *wedge* equal to $1+\varepsilon_r^{-1}(q_I)$. From (1), we obtain the following proposition:

Proposition 1 *In equilibrium, I's markup, markdown, and total margin are given by:*

$$\begin{aligned}
\mu_I &= \frac{p(q_I)}{MC_I(q_I)} = \frac{1}{1 - \varepsilon_p^{-1}(q_I)}, \\
\nu_I &= \frac{MR_I(q_I)}{r(q_I)} = 1 + \varepsilon_r^{-1}(q_I), \\
M_I &\equiv \frac{p(q_I)}{r(q_I)} = \nu_I \times \mu_I = \frac{1 + \varepsilon_r^{-1}(q_I)}{1 - \varepsilon_p^{-1}(q_I)}.
\end{aligned}$$

Proof. Appendix A.1 provides a formal characterization of markups and markdowns expressions based on Section 2 definitions. ■

As previously defined, μ_I measures the surplus I obtains from selling the marginal output unit, or simply here I's ability to set p above its marginal cost. The markup

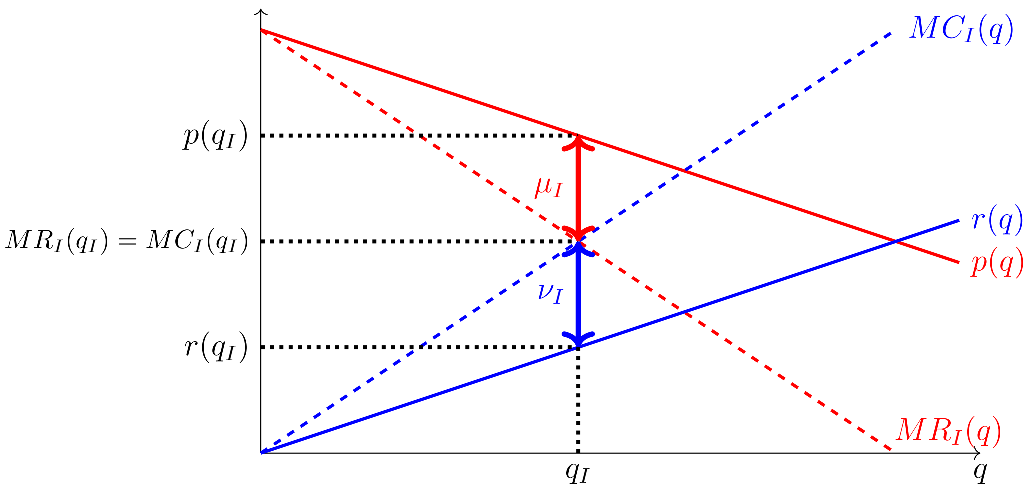

Figure 1: Monopoly and Monopsony Power in the Vertically Integrated Case

expression indicates a negative relationship with demand elasticity: as ε_p increases, μ_I decreases. Similarly, ν_I measures the surplus I obtains from purchasing the marginal input unit, or simply here I's ability to purchase the input at a price below its marginal revenue. The markdown expression indicates a negative relationship with supply elasticity: as ε_r increases, ν_I decreases. Note that $\mu_I = 1$ in the absence of monopoly power, and $\nu_I = 1$ in the absence of monopsony power. Finally, we introduce here the definition of the margin M_I of firm I, which measures the total surplus I obtains from both purchasing and selling the marginal unit. In this article's framework, a firm margin can be (i) trivially defined as the ratio of its output price (here, p) and input price (r) and (ii) written as the product of the firm markup (μ_I) and markdown (ν_I).[20] In this benchmark case, firm's I margin is thus negatively related to both demand and supply elasticities and equal to one in the absence of monopoly and monopsony power.

Figure 1 illustrates the economic forces in (1) and Proposition 1 by depicting the profit-maximizing equilibrium under linear demand and supply functions, that is $p(q) = a - bq$ and $r(q) = c + dq$.[21] The figure highlights the following mechanism. Due to I's monopoly and monopsony power, the equilibrium quantity q_I is lower than in the perfectly competitive outcome (i.e., the intersection between $p(q)$ and $r(q)$). This

[20]The expression of the margin as the product of the firm markup and markdown also extends to any production function with multiple outputs and substitutable inputs. In such frameworks, the margin M obtained from selling a given output quantity q at a price p and purchasing a given variable input quantity m at a price w would be defined as $M \equiv \theta_m \frac{pq}{wm}$ with $\theta_m = \frac{\partial q}{\partial m} \frac{m}{q}$.

[21]For simplicity, we set $c = 0$, $d = -b$, and $a > 0$.

stems from I's opportunity cost of selling (and buying) an additional output unit (and input). Given the downward-sloping inverse demand curve, I internalizes that selling one more output unit will lower the output price for all other units. Similarly, given the upward-sloping inverse supply curve, I internalizes that buying one more input unit will drive up the input price for all other units. Hence, these effects provide I with incentives to reduce the quantity exchanged in equilibrium, generating negative welfare consequences. This quantity reduction distorts prices, implying that consumers pay higher prices while input suppliers receive lower prices.

Building on insights from this benchmark case, we now examine our vertical chain framework with monopoly power, monopsony power, and a general distribution of bargaining power between the upstream firm U and downstream firm D.

4 Bargaining and Double Marginalization

We now analyze the bilateral monopoly setting introduced in Section 3.1, where U purchases an input at price $r(q)$ to produce a good sold to consumers at price $p(q)$ through D. Specifically, we consider that U and D interact in the market according to the following sequence of play:

- **Stage 1:** U and D engage in a bilateral negotiation to determine the linear wholesale price w.
- **Stage 2:** Given w, U orders a quantity q_U from its input suppliers, and D simultaneously chooses the quantity q_D to purchase from U and sell to consumers.

This bilateral monopoly setting nests the canonical model of Spengler (1950) and its extension to bargaining (e.g., Gaudin, 2016). We now discuss each stage and introduce our equilibrium notion. In stage 1, we use the Nash bargaining solution (Nash, 1950) to determine the terms of trade between U and D, where $\alpha \in [0, 1]$ denotes U's bargaining weight vis-à-vis D. In stage 2, each firm chooses its quantity, taking as given that it cannot exceed the quantity chosen by the other (i.e., no shortages or waste).[22] To offer

[22]We rationalize the absence of shortages ($q_D > q_U$) by the fact that D recognizes it cannot force U to produce quantities beyond what it is willing to supply. We also rule out excess supply ($q_D < q_U$) due

support for the reasonableness of our surplus division in the vertical supply chain, we provide a noncooperative formulation of our Nash bargaining solution. Specifically, we demonstrate in Appendix C that our equilibrium notion coincides with the subgame perfect Nash equilibrium of a variant of the noncooperative game developed by Rey and Vergé (2020). In what follows, we describe each stage in detail, proceeding in reverse order of timing.

4.1 Quantity Choice via the Short-Side Rule

In stage 2, given w, U orders a quantity q_U from its input suppliers, and D chooses the quantity q_D to purchase from U (and sell to consumers). If q_U and q_D do not match, we apply the short-side rule by considering that one firm's choice will restrict the quantity that the other can choose (i.e., no shortages or waste). Therefore, each firm chooses the quantity that maximizes its profit, taking into account this mutual constraint.

***D*'s quantity choice.** D chooses the quantity to purchase from U and sell to consumers.[23] Given w, D's maximization problem is as follows:

$$\max_{q_D} \Pi_D = (p(q_D) - w)q_D \quad \text{subject to} \quad q_D \leq q_U(w) \tag{2}$$

An interior solution to (2) arises when $\tilde{q}_D(w) \leq q_U(w)$, where $\tilde{q}_D(w)$ denotes the quantity satisfying the first-order condition from (2):

$$\underbrace{p(\tilde{q}_D(w))\left(1 - \varepsilon_p^{-1}(\tilde{q}_D(w)\right)}_{MR_D(\tilde{q}_D(w))} = w \tag{3}$$

which corresponds to D's inverse demand for U's product. Otherwise, we obtain a corner solution where $q_D = q_U(w)$, implying that $MR_D(q_U(w)) > w$ as MR_D is decreasing

to U's lack of incentive to produce more than what is necessary to cover demand, as waste generates losses.

[23]Note that setting either q_D or p leads to the same result because D operates as a monopolist.

in quantity (Assumption 1). Hence, the quantity chosen by D is given by:

$$q_D(w) = \begin{cases} \tilde{q}_D(w) & \text{if } \tilde{q}_D(w) \leq q_U(w), \\ q_U(w) & \text{otherwise.} \end{cases} \tag{4}$$

***U*'s quantity choice.** The problem U faces is symmetric to that of D. Specifically, U chooses the quantity to purchase from its input suppliers and resale to D.[24] Given w, U's maximization problem is as follows:

$$\max_{q_U} \Pi_U = (w - r(q_U))q_U \quad \text{subject to} \quad q_U \leq q_D(w) \tag{5}$$

An interior solution to (5) occurs when $\tilde{q}_U(w) \leq q_D(w)$, where $\tilde{q}_U(w)$ denotes the quantity satisfying the first-order condition from (5):

$$w = \underbrace{r(\tilde{q}_U(w))\left(1 + \varepsilon_r^{-1}(\tilde{q}_U(w))\right)}_{MC_U(\tilde{q}_U(w))}. \tag{6}$$

Otherwise, we obtain a corner solution where $q_U = q_D(w)$, implying that $w > MC_U(q_D(w))$ as MC_U is increasing in quantity (Assumption 1). Hence, the quantity chosen by U is given by:

$$q_U(w) = \begin{cases} \tilde{q}_U(w) & \text{if } \tilde{q}_U(w) \leq q_D(w), \\ q_D(w) & \text{otherwise.} \end{cases} \tag{7}$$

Subgame equilibrium quantity. As shown by (3) and (6), given w, D wants to determine the quantity to be traded according to its demand (MR_D), while U aims to set it based on its supply (MC_U). In equilibrium, the short-side rule applies: the quantity exchanged is the minimum of the quantities that each firm is willing to exchange. Formally, we have:

$$q(w) = \min\{\tilde{q}_D(w), \tilde{q}_U(w)\}. \tag{8}$$

Using (3) and (6), we can alternatively express the subgame equilibrium as follows:

[24] Again, setting either q_U or r is equivalent because U operates as a monopsonist.

Proposition 2 *The subgame equilibrium quantity follows a schedule $w(q)$ such that:*

$$w(q) = \begin{cases} MC_U(q) & \text{for any } w \leq w_I, \\ MR_D(q) & \text{otherwise.} \end{cases}$$

where w_I is the unique value of w such that $q(w_I) = q_I$.

Proof. See Appendix B.1. ■

Proposition 2 establishes a direct relationship between w and the allocation of the right-to-manage. The intuition is as follows. When w is high ($w \geq w_I$), U wants to produce a quantity greater than what D is willing to purchase to maximize its profit ($\tilde{q}_U(w) \geq \tilde{q}_D(w)$).[25] As D never purchases a quantity that exceeds $\tilde{q}_D(w)$, the quantity exchanged in equilibrium is determined by D (i.e., D has the right-to-manage). Conversely, the reverse holds when w is low ($w \leq w_I$). U prefers to produce a smaller quantity than what D seeks to purchase to maximize its profit ($\tilde{q}_D(w) \geq \tilde{q}_U(w)$), resulting in U dictating the equilibrium quantity (i.e., U has the right-to-manage).[26] Consequently, the firm that sets the equilibrium quantity is endogenously determined, depending on the level of w. This result stands in contrast to prior work in the bilateral monopoly literature, which typically assumes that, for any given w, either U or D unilaterally chooses the quantity to be traded in equilibrium (see Toxvaerd, 2024, for a review).[27]

4.2 Efficient Bargaining, Double Markupization, and Double Markdownization

We now turn to stage 1, where U and D bargain over w anticipating its effect on the quantity determined in stage 2. Using the (asymmetric) Nash bargaining solution, we

[25] That is, for a given $w \geq w_I$, D's marginal cost, w, exceeds its marginal revenue at U's (unconstrained) profit-maximizing quantity (i.e., $\tilde{q}_U(w)$).

[26] Again, taking $w \leq w_I$ as given, U's marginal revenue, w, is lower than its marginal cost at D's (unconstrained) profit-maximizing quantity (i.e., $\tilde{q}_D(w)$).

[27] It is worth noting that some articles have addressed this issue by considering that both q and w are jointly determined through bargaining, yielding the vertically integrated outcome described in Section 3 (e.g., McDonald and Solow, 1981; Manning, 1987; Björnerstedt and Stennek, 2007).

derive the equilibrium wholesale price from the following maximization problem:

$$\max_{w} {\Pi_U}^{\alpha} {\Pi_D}^{(1-\alpha)} \tag{9}$$

where $\Pi_U(w) = (w - r(q(w)))\, q(w)$ and $\Pi_D(w) = (p(q(w)) - w)\, q(w)$. The first-order condition from (9) is given by:

$$\underbrace{\alpha \left[\frac{\partial w(q) q}{\partial q} - MC_U(q) \right]}_{\frac{\partial \Pi_U(q)}{\partial q}} \Pi_D(q) + (1-\alpha) \underbrace{\left[MR_D(q) - \frac{\partial w(q) q}{\partial q} \right]}_{\frac{\partial \Pi_D(q)}{\partial q}} \Pi_U(q) = 0 \tag{10}$$

which characterizes the equilibrium wholesale price w^*. As U and D account for the impact of w on the equilibrium quantity (given by (8)), (10) depends on which firm has the right-to-manage. As emphasized in Proposition 2, the level of w (and, consequently, the bargaining weight α) determines whether U or D ultimately chooses the quantity to be traded. In what follows, we consider all possible cases. First, we analyze the scenario where α is such that the quantity set by U and D replicates the vertically integrated outcome ($w = MR_D(q_I) = MC_U(q_I)$). Second, we examine the case where α is high, implying that D determines the equilibrium quantity ($w = MR_D(q)$). Finally, we consider the case where α is low, meaning that U determines the equilibrium quantity ($w = MR_U(q)$).

4.2.1 Efficient Bargaining

Consider that α is such that the quantity determined in stage 2 is q_I, implying that $w = MC_U(q_I) = MR_D(q_I)$. In this case, (10) boils down to:

$$\alpha \Pi_D(q_I) - (1-\alpha)\Pi_U(q_I) = 0 \tag{11}$$

From (11), we obtain that $\alpha_I \equiv \frac{\Pi_U(q_I)}{\Pi_U(q_I)+\Pi_D(q_I)} = \frac{\varepsilon_p - 1}{\varepsilon_p + \varepsilon_r}$ is the level of U's bargaining power such that the equilibrium wholesale price $w^* = w(\alpha_I)$ yields the vertically integrated outcome $q^* = q_I$. Hence, we derive the following proposition:

Proposition 3 *The bargaining outcome is efficient when $\alpha = \alpha_I = \frac{\varepsilon_p - 1}{\varepsilon_p + \varepsilon_r}$. In this case,* *the equilibrium replicates the vertically integrated outcome, where the quantity traded is* $q^* = q_I$*, the wholesale price is* $w^* = w_I = MC_U(q_I) = MR_D(q_I)$*, the consumer price is* $p(q_I)$*, and the raw input price is* $r(q_I)$*.*

Proof. See Appendix B.2. ■

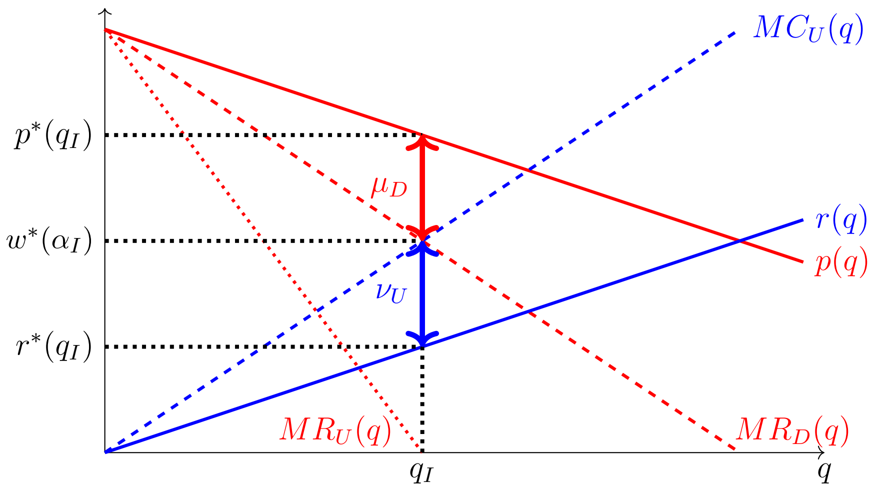

Figure 2: Equilibrium under Efficient Bargaining ($\alpha = \alpha_I$)

When U has constant marginal costs (i.e., $\varepsilon_r \to \infty$), a well-established result from the canonical model of vertical contracting is that efficiency requires U to have no bargaining power, thereby eliminating its markup and, consequently, double marginalization. Proposition 3 encompasses this result but also highlights that it no longer holds when $\varepsilon_r < \infty$. In this case, monopsony power arises, and as will become clearer in the subsequent analysis, efficiency requires a balanced distribution of bargaining to eliminate not only U's markup but also D's markdown. Figure 2 illustrates the equilibrium described in Proposition 3 using the linear demand and supply functions. In Figure 2, the symmetry between supply and demand results in $\alpha_I = \frac{1}{2}$ and equal levels of markup and markdown. More generally, the efficient bargaining power α_I, contingent on supply and demand primitives, falls below (or exceeds) $\frac{1}{2}$ when the elasticity of supply is relatively lower (or higher) than the elasticity of demand, for a quantity q_I.

4.2.2 Double Markupization

Consider that U's bargaining power is high ($\alpha_I < \alpha \leq 1$) such that D determines the equilibrium quantity in stage 2 ($\tilde{q}_D(w) < \tilde{q}_U(w)$). In this case, we have $w = MR_D(q)$ implying that (10) boils down to:

$$\alpha(MR_U(q^*) - MC_U(q^*))\Pi_D(q^*) + (1-\alpha)(MR_D(q^*) - MR_U(q^*))\Pi_U(q^*) = 0 \quad (12)$$

where q^* denotes the equilibrium quantity and $MR_U(q) \equiv \frac{\partial MR_D(q)q}{\partial q} = MR'_D(q)q + MR_D(q)$ corresponds to U's marginal revenue function in the case where it makes a take-it-or-leave-it offer to D (i.e., U faces a demand curve given by $MR_D(q)$). To ensure that (12) is derived from a well-defined Nash bargaining problem, we impose the following assumption:

Assumption 2 *D's marginal revenue satisfies the following conditions:*

i) $\varepsilon_{MR_D} > 1$;

ii) $\sigma_{MR_D} < 2$.

Assumption 2.i, which can also be written as $\varepsilon_p > 3 - \sigma_p$, imposes that consumer demand is supermodular (e.g., Mrázová and Neary, 2017).[28] This guarantees that $MR_U(q) > 0$, ensuring that (12) can be satisfied for any $\alpha \in [0,1]$.[29] Assumption 2.ii guarantees that the second-order condition of (9) when $w = MR_D(q)$ is satisfied (see Appendix B.3.1 for further details).

To gain further insight into the equilibrium outcome, (12) can be rearranged as follows:

$$MC_U(q^*) = \widetilde{MR}_U(q^*, \alpha) \quad (13)$$

where $\widetilde{MR}_U(q^*, \alpha) \equiv \beta_D(q^*, \alpha)MR_D(q^*) + (1 - \beta_D(q^*, \alpha))\, MR_U(q^*)$ can be interpreted as a "shadow" marginal revenue, with $\beta_D(q^*, \alpha) \equiv \frac{1-\alpha}{\alpha}\frac{\Pi_U(q^*)}{\Pi_D(q^*)}$ (see Appendix B.3.2 for

[28] Supermodular demand functions include, among others, the CES demand model, the translog demand model, or the AIDS demand model. Supermodularity also holds in the linear demand model when $\epsilon_p > 3$, and in the logit demand model for low values of q^*.

[29] Formally, $MR_U > 0 \Leftrightarrow \varepsilon_{MR_D} = \frac{\varepsilon_p - 1}{2 - \sigma_p} > 1 \Leftrightarrow \varepsilon_p > 3 - \sigma_p$.

details). When U has all the bargaining power ($\alpha = 1$), we have $\beta_D(q^*, \alpha) = 0$, implying that (13) boils down to $MC_U(q^*) = MR_U(q^*)$. This corresponds to the canonical model of vertical contracting, where the inefficient outcome $q^* < q_I$ arises because $w^* > w_I$ (see Appendix C.1 for further details). When $1 > \alpha > \alpha_I$, we have $\beta_D(q^*, \alpha) > 0$, which shifts $\widetilde{MR}_U(q^*, \alpha)$ towards $MR_D(q^*)$ such that $MR_D(q^*) > \widetilde{MR}_U(q^*, \alpha) > MR_U(q^*)$. As $MC_U(q)$ increases in q, the equilibrium quantity q^* characterized by (13) increases, thereby reducing the inefficiency. Finally, when $\alpha = \alpha_I$, we have $\beta_D(q^*, \alpha) = 1$, implying that (13) reduces to $MC_U(q^*) = MR_D(q^*)$. This corresponds to the vertically integrated outcome, where $q^* = q_I$. Based on this reasoning, we derive the following proposition:

Proposition 4 *When U is powerful ($\alpha_I \leq \alpha \leq 1$), the bargaining between U and D results in an inefficient outcome where, in equilibrium, the quantity is $q^* \leq q_I$, the wholesale price is $w^* = MR_D(q^*)$, the consumer price is $p(q^*) \geq p(q_I)$, and the raw input price is $r(q^*) \leq r(q_I)$. U's and D's markups are respectively given by:*

$$\begin{aligned}
\mu_U &= \tfrac{w^*}{MC_U(q^*)} = \frac{\varepsilon_{MR_D}}{\varepsilon_{MR_D} - (1 - \beta_D(q^*, \alpha))} = \frac{\alpha\varepsilon_{MR_D}(\varepsilon_r + 1) + (1-\alpha)(\varepsilon_p - 1)\varepsilon_r}{(\varepsilon_r + 1)(\alpha(\varepsilon_{MR_D} - 1) + (1-\alpha)(\varepsilon_p - 1))}, \\
\mu_D &= \tfrac{p(q^*)}{MR_D(q^*)} = \tfrac{\varepsilon_p}{\varepsilon_p - 1},
\end{aligned}$$

and U's and D's markdowns are respectively given by:

$$\begin{aligned}
\nu_U &= \tfrac{MC_U(q^*)}{r(q^*)} = \tfrac{\varepsilon_r + 1}{\varepsilon_r}, \\
\nu_D &= \tfrac{MR_D(q^*)}{w^*} = 1.
\end{aligned}$$

Consequently, U's margin is equal to $M_U = \frac{w^*}{r(q^*)} = \nu_U \times \mu_U$, D's margin is equal to $M_D = \frac{p(q^*)}{w^*} = \mu_D$, and the total margin of the vertical supply chain is given by $\mathcal{M} = \frac{p(q^*)}{r(q^*)} = \nu_U \times \mu_U \times \mu_D$.

Proof. Appendix A.2 provides a formal characterization of markup and markdown expressions based on their definition introduced in Section 2. Appendix B.3.3 shows the main derivations and Appendix B.3.4 delimitates the set of equilibria. ■

When U's bargaining power vis-à-vis D is high ($\alpha_I \leq \alpha \leq 1$), the equilibrium

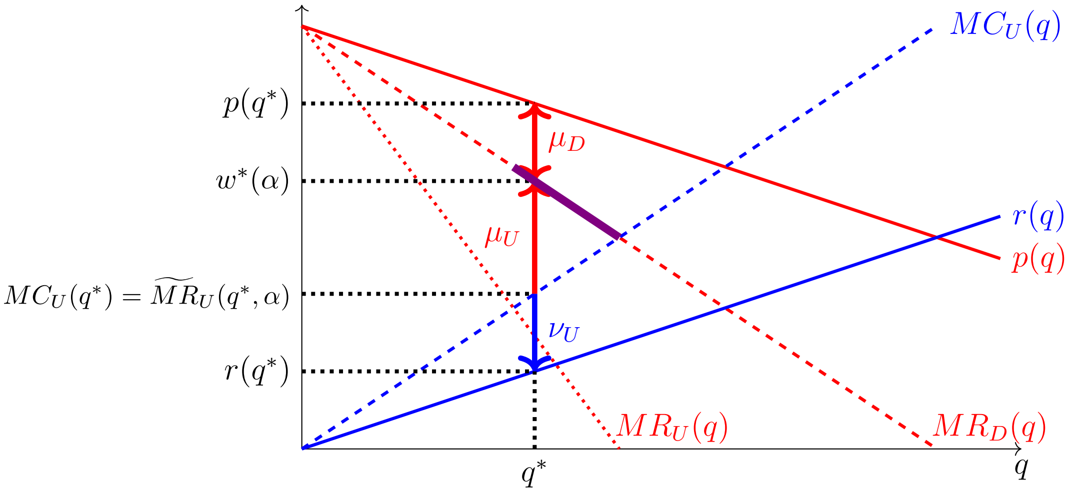

Figure 3: Equilibrium with Double Markupization ($\alpha_I < \alpha < 1$).

wholesale price satisfies $w^* \geq w_I$. In this case, the short-side rule implies that w^* and q^* co-move along D's demand, which is given by MR_D (Proposition 2). As MR_D decreases with q, Proposition 4 establishes that U exercises monopoly power by charging a markup over its marginal cost when selling to D. This markup adds up to D's markup due to its monopoly power in the product market. The resulting *double markupization* gives rise to the classical double marginalization phenomenon (Spengler, 1950), leading to an inefficient outcome ($q^* \leq q_I$).

Analogous to the vertically integrated outcome, U's markdown (ν_U) and D's markup (μ_D) are determined by the elasticities of supply and demand, respectively, reflecting U's monopsony power in the input market and D's monopoly power in the product market. Interestingly, U's markup (μ_U) depends on two factors. The first is D's demand elasticity (ε_{MR_D}), reflecting U's monopoly power. The second is β_D, which captures D's ability to exert countervailing buyer power, with μ_U decreasing as β_D increases. Finally, D charges no markdown as $\nu_D = 1$.

Figure 3 illustrates the equilibrium described in Proposition 4 using the linear demand and supply functions for a given $\alpha \in [\alpha_I, 1]$. In equilibrium, w^* lies on MR_D and satisfies the condition $MC_U(q^*) = \widetilde{MR}_U(q^*, \alpha)$, which then determines q^*. The set of equilibrium wholesale prices and quantities is represented by the purple segment.

4.2.3 Double Markdownization

Consider now that D's bargaining power is high ($0 \leq \alpha < \alpha_I$) such that U determines the equilibrium quantity in stage 2 ($\tilde{q}_U(w) < \tilde{q}_D(w)$). In that case, we have $w(q) = MC_U(q)$, implying that (10) boils downs to:

$$\alpha(MC_D(q^*) - MC_U(q^*))\Pi_D(q^*) + (1-\alpha)(MR_D(q^*) - MC_D(q^*))\Pi_U(q^*) = 0 \tag{14}$$

where q^* denotes the equilibrium quantity and $MC_D(q) \equiv \frac{\partial MC_U(q)q}{\partial q} = \underbrace{MC_U'(q)q}_{>0} + MC_U(q)$ corresponds to D's marginal cost function in the case where it makes a take-it-or-leave-it offer to U (i.e., D faces a supply curve given by $MC_U(q)$). To ensure that the second-order condition of (9) when $w = MC_U(q)$ is satisfied (see Appendix B.4.1 for further details), we impose the following assumption:

Assumption 3 *U's marginal cost satisfies* $\sigma_{MC_U} > -2$.

To gain further insight into the equilibrium outcome, (14) can be rearranged as follows:

$$MR_D(q^*) = \widetilde{MC}_D(q^*, \alpha) \tag{15}$$

where $\widetilde{MC}_D(q^*, \alpha) \equiv \beta_U(q^*, \alpha)MC_U(q^*) + (1 - \beta_U(q^*, \alpha))\, MC_D(q^*, \alpha)$ can be interpreted as a “shadow" marginal cost, with $\beta_U(q^*, \alpha) \equiv \frac{\alpha}{1-\alpha}\frac{\Pi_D(q^*)}{\Pi_U(q^*)}$ (see Appendix B.3.2 for details). When D has all the bargaining power, we have $\beta_U(q^*, \alpha) = 0$, implying that (13) boils down to $MC_D(q^*) = MR_D(q^*)$. The inefficient outcome $q^* < q_I$ arises because $w^* < w_I$ (see Appendix C.2 for further details). When $0 < \alpha < \alpha_I$, we have $\beta_U(q^*, \alpha) > 0$, which shifts $\widetilde{MC}_D(q^*, \alpha)$ towards $MC_U(q^*)$ such that $MC_D(q^*) > \widetilde{MC}_D(q^*, \alpha) > MC_U(q^*)$. As $MR_D(q)$ decreases in q, the equilibrium quantity q^* characterized by (13) increases, thereby reducing the inefficiency. Finally, when $\alpha = \alpha_I$, we have $\beta_U(q^*, \alpha) = 1$, implying that (13) reduces to $MC_U(q^*) = MR_D(q^*)$. This corresponds to the vertically integrated outcome, where $q^* = q_I$. Based on this reasoning, we derive the following proposition:

Proposition 5 *When D is powerful ($0 \leq \alpha \leq \alpha_I$), the bargaining between U and D results in an inefficient outcome where, in equilibrium, the quantity is $q^* \leq q_I$, the wholesale price is $w^* = MC_U(q^*)$, the consumer price is $p(q^*) \geq p(q_I)$, and the raw input price is $r(q^*) \leq r(q_I)$. U's and D's markups are respectively given by:*

$$\begin{aligned}\mu_U &= \tfrac{w(q^*)}{MC_U(q^*)} = 1,\\ \mu_D &= \tfrac{p(q^*)}{MR_D(q^*)} = \tfrac{\varepsilon_p}{\varepsilon_p - 1},\end{aligned}$$

and U's and D's markdowns are respectively given by:

$$\begin{aligned}\nu_U &= \tfrac{MC_U(q^*)}{r(q^*)} = \tfrac{\epsilon_r+1}{\epsilon_r},\\ \nu_D &= \tfrac{MR_D(q^*)}{w(q^*)} = \frac{\varepsilon_{MC_U} + (1-\beta_U(q^*,\alpha))}{\varepsilon_{MC_U}} = \frac{(\varepsilon_p - 1)(\alpha(\varepsilon_r+1) + (1-\alpha)(\varepsilon_{MC_U}+1))}{\alpha\varepsilon_p(\varepsilon_r+1) + (1-\alpha)(\varepsilon_{MC_U})(\varepsilon_p - 1)}.\end{aligned}$$

Consequently, U's margin is equal to $M_U = \frac{w^*}{r(q^*)} = \nu_U$, D's margin is equal to $M_D = \frac{p(q^*)}{w^*} = \nu_D \times \mu_D$, and the total margin of the vertical supply chain is given by $\mathcal{M} = \frac{p(q^*)}{r(q^*)} = \nu_U \times \nu_D \times \mu_D$.

Proof. Appendix A.2 provides a formal characterization of markup and markdown expressions based on their definitions introduced in Section 2. Appendix B.4.3 shows the main derivations and Appendix B.4.4 delimitates the set of equilibria. ■

When D's bargaining power vis-à-vis U is high ($0 \leq \alpha \leq \alpha_I$), the equilibrium wholesale price satisfies $w^* \leq w_I$. In this case, the short-side rule implies that w^* and q^* co-move along U's supply, which is given by MC_U (Proposition 2). As MC_U increases with q, Proposition 5 establishes that D exercises monopsony power by charging a markdown below its marginal revenue when purchasing from U. This markdown adds up to U's markdown due to its monopsony power in the input market, resulting in *double markdownization* which leads to an inefficient outcome ($q^* \leq q_I$). Although symmetric to double markupization, *double markdownization* had not yet been identified in the literature.

Again, U's markdown (ν_U) and D's markup (μ_D) are shaped by the elasticities of supply and demand, respectively, reflecting U's monopsony power in the input market

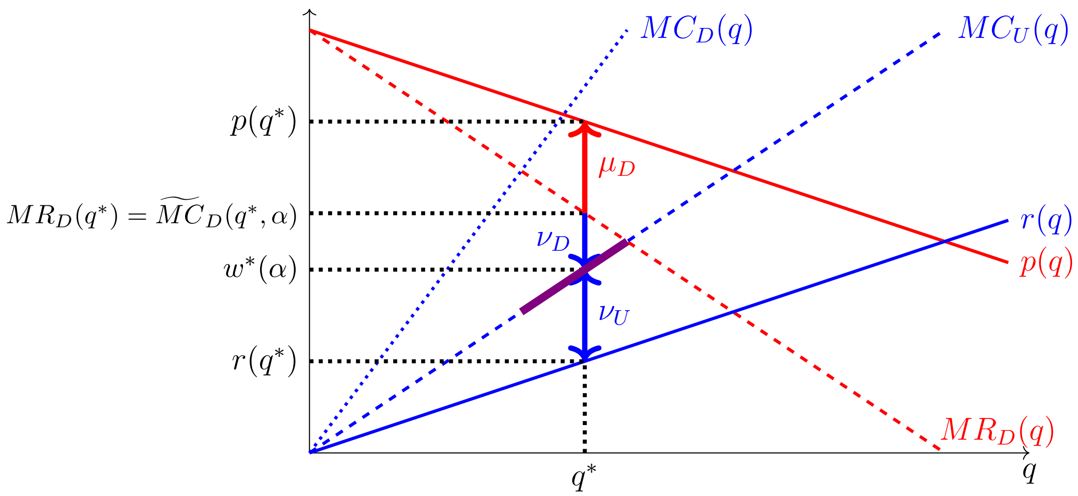

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Figure 4: Equilibrium with Double Markdownization ($0 < \alpha < \alpha_I$)

and D's monopoly power in the product market. Moreover, D's markdown ν_D depends on two factors. The first is U's supply elasticity (ε_{MC_U}), reflecting D's monopsony power. The second is β_U, which captures U's ability to exert countervailing seller power, with ν_D decreasing as β_U increases. Finally, U charges no markup as $\mu_U = 1$.

Figure 4 illustrates the equilibrium described in Proposition 5 using the linear demand and supply functions for a given $\alpha \in [\alpha_I, 1]$. In equilibrium, w^* lies on MC_U and satisfies the condition $MR_D(q^*) = \widetilde{MC}_U(q^*, \alpha)$, which then determines q^*. The set of equilibrium wholesale prices and quantities is represented by the purple segment.

5 Welfare Effects of Buyer and Seller Power

This section performs a static comparative analysis to discuss the impact of a variation in the bargaining weight parameter α on equilibrium outcomes. In this simple model, the variation in α is treated as exogenous, and we do not explore the underlying mechanisms driving this change.[30] The purple segment in Figure 5 represents the whole set of equilibrium wholesale price and quantity when α goes from 0 to 1, with

[30]In practice, the balance of power within the supply chain is also affected by various changes in the market structure, such as consolidation, entry, or exit at one or the other level. It could also be affected by a change in firms' strategies, such as constituting a buying alliance. Modelling these endogenous sources of change in the balance of power would require a model with competition at the industry and/or the retail level, which we leave as an avenue for further research.

the arrows indicating the direction of the variation, as formalized in the following corollary.

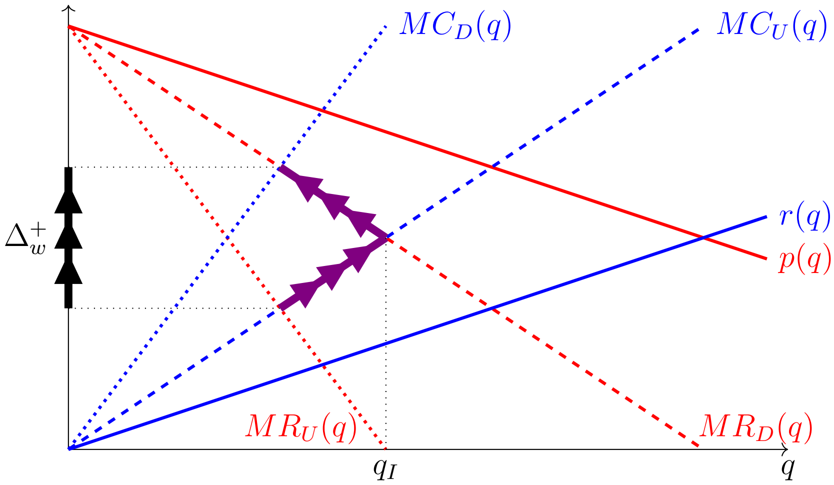

Figure 5: Effects of Increasing Seller Power (higher α).

Corollary 1 *Bargaining power has as the following welfare effects:*

- *When U is powerful ($\alpha_I < \alpha \leq 1$), the total welfare is decreasing in α, and the countervailing buyer power theory prevails.*
  Formally, $\frac{\partial q^*(\alpha)}{\partial \alpha} < 0$, $\frac{\partial r^*(\alpha)}{\partial \alpha} < 0$, $\frac{\partial w^*(\alpha)}{\partial \alpha} > 0$, *and* $\frac{\partial p^*(\alpha)}{\partial \alpha} > 0$.

- *When D is powerful ($0 \leq \alpha < \alpha_I$), the total welfare is increasing in α, and the countervailing seller power theory prevails.*
  Formally, $\frac{\partial q^*(\alpha)}{\partial \alpha} > 0$, $\frac{\partial r^*(\alpha)}{\partial \alpha} > 0$, $\frac{\partial w^*(\alpha)}{\partial \alpha} > 0$, *and* $\frac{\partial p^*(\alpha)}{\partial \alpha} < 0$.

Corollary 1 highlights that when U is powerful (as long as $\alpha > \alpha_I$) an increase in the bargaining power of D, i.e. a decrease in α, exerts a countervailing buyer power effect against U, which benefits consumers by decreasing $p^*(\alpha)$, and increasing $q^*(\alpha)$ as well as total welfare. This countervailing buyer power effect was widely discussed in the literature and mainly refers to the positive effect of increasing retail power to counteract the upstream firm market power, thus decreasing the wholesale input price ($w^*(\alpha)$), and in turn, retail final prices ($p^*(\alpha)$). In addition, we highlight that when the supplier purchases on a competitive labor /raw input market, the countervailing power also raises wages or prices paid to raw input suppliers by increasing $r^*(\alpha)$.

This phenomenon was not previously described as the countervailing power theory has been mainly formalized in a vertical supply chain where suppliers produce at constant marginal cost. Corollary 1 also highlights that, when D is powerful, that is, as long as $\alpha < \alpha_I$, an increase in the bargaining power of D, i.e. a decrease in α, exerts a monopsony power effect against U which reduces the raw input price paid to suppliers $r^*(\alpha)$ and is detrimental to consumers and welfare. In that case, an increase in α exerts a countervailing seller power effect which benefits welfare.

The welfare effects associated with a variation of α stem from its effect on the (in)efficiency of the relationship between U and D, as summarized in the following Corollary.

Corollary 2 *When U is powerful ($\alpha_I < \alpha \leq 1$), a change in α affects markups, markdowns, and margins in the value chain in the following way:*

(i) $\mathcal{M}$, the value-chain margin, increases in α.

(ii) M_U and μ_U, respectively the margin and the markup of U, increase in α,

(iii) under demand and supply subconvexity, which ensures that $\frac{\partial \varepsilon_f}{\partial q} < 0, \forall f \in \{p, r\}$,

- *ν_U, the markdown of U, decreases in α,*
- *M_D, the margin of D (here equal to its markup μ_D), decreases in α.*

Results in (iii) are reversed under demand and supply super-convexity $\left(\frac{\partial \varepsilon_f}{\partial q} > 0, \forall f \in \{p, r\}\right)$, and canceled under C.E.S demand and supply $\left(\frac{\partial \varepsilon_f}{\partial q} = 0, \forall f \in \{p, r\}\right)$.

Proof. See Appendix B.3.5. ■

When U is powerful, a rise in α increases the value-chain margin $\mathcal{M}$. This stems from the vertical relationship between U and D becoming less balanced and less efficient via the increase in U's markup μ_U. This exacerbated distortion reduces the exchanged quantity and welfare. The increase in inefficiency following a rise in α can be amplified or attenuated by the adjustments in the markdown imposed in the upstream market and the markup imposed in the downstream market. The direction of these adjustments depends on the shape of demand and supply. More specifically, it depends on how demand and supply elasticities, and thus markups and markdowns, respectively, evolve along demand and supply curves. Under demand subconvexity, the markup and thus the margin of D decreases following an increase in α, the bargaining power of U. Under supply subconvexity, the markdown of U also decreases with α.[31] However, it is more than compensated by the increase in the markup of U, so the margin of U increases. When U is powerful, an increase in α always exacerbates the total distortion in the value chain, despite reducing the upstream markdown and the downstream markup under supply and demand subconvexity. The distortionary effect is greater under demand and supply superconvexity, as both the upstream markdown and the downstream markup increase when α increases.

At the limit where U holds all the bargaining power, i.e. when $\alpha = 1$, we have $\mu_U = \frac{\varepsilon_{MR_D}}{\varepsilon_{MR_D}-1}$ as $\beta_D(q^*, 1) = 0$. In such a case, D has no countervailing buyer power and becomes a price taker. The standard Lerner index rule applies. At the opposite limit, i.e. when $\alpha = \alpha_I$, $\mu_U = 1$ as $\beta_D(q^*, \alpha_I) = 1$. In such a case, D's countervailing buyer power fully counteracts U's monopoly power. Double markupization is suppressed, and the chain reaches the vertical integration outcome.

Similarly, the welfare effects associated with a variation of α when D is powerful stem from its effect on the (in)efficiency of the relationship between U and D, as summarized in the following Corollary.

Corollary 3 *When D is powerful ($0 \leq \alpha < \alpha_I$), a change in α affects markups, markdowns, and margins in the value chain in the following way:*

(i) $\mathcal{M}$, the value-chain margin, decreases in α,

(ii) M_D and ν_D, respectively the margin and the markdown of D, decrease in α,

(iii) under demand and supply subconvexity, which ensures that $\frac{\partial \varepsilon_f}{\partial q} < 0, \forall f \in \{p, r\}$,

- *M_U, the margin of U (here equal to its markdown ν_U), increases in α,*

[31] We emphasize demand and supply convexity as they are consistent with much of the available empirical evidence. Demand subconvexity, often called Marshall's Second Law of Demand, aligns with most findings in the IO and Trade literature, as pointed out by Mrázová and Neary (2017). Supply subconvexity, although less studied, is consistent with recent findings of Boehm and Pandalai-Nayar (2022) for US industries or Avignon and Guigue (2022) in the context of the French milk industry.

- μ_D, *the markup of* D, *increases in* α.

Results in (iii) are reversed under demand and supply superconvexity $\left(\frac{\partial \varepsilon_f}{\partial q} > 0, \forall f \in \{p, r\}\right)$, *and canceled under C.E.S demand and supply* $\left(\frac{\partial \varepsilon_f}{\partial q} = 0, \forall f \in \{p, r\}\right)$.

Proof. See Appendix B.4.5. ■

When D is powerful, a rise in α decreases the value-chain margin $\mathcal{M}$. This stems from the fact that the vertical relationship between U and D becomes more balanced and more efficient via the decrease in D's markdown, ν_D, and the exchanged quantity and welfare increase.

The increase in efficiency following a rise in α can be amplified or attenuated by the adjustments in the upstream markdown and the downstream markup. Under demand subconvexity, the markdown and thus the margin of U increase following an increase in α. Under supply subconvexity, the markup of D also increases with α. However, it is more than compensated by the decrease in the markdown of D, and the margin of D decreases. When D is powerful, an increase in α always reduces the total distortion in the value chain despite increasing the markdown in the upstream market and the markup in the downstream market under supply and demand subconvexity. The efficiency effect is exacerbated under demand and supply super-convexity, as the upstream markdown and the downstream markup also decrease when α increases.

At the limit where D holds all the bargaining power ($\alpha = 0$), we have $\nu_D = \frac{\varepsilon_{MC_U}+1}{\varepsilon_{MC_U}}$ as $\beta_U(q^*, 0) = 1$. In such a case, U has no countervailing seller power and becomes price-taking. The symmetric of the standard Lerner index rule applies. At the opposite limit, i.e. when $\alpha = \alpha_I$, $\nu_D = 1$ as $\beta_U(q^*, \alpha_I) = 1$. In such a case, U's countervailing seller power fully counteracts D's monopsony power. Double-marginalization is suppressed, and the chain reaches the vertical integration outcome.

6 Discussion

6.1 Comparison with Exogenous Right-to-Manage Frameworks

This article provides a non-cooperative foundation for the "right-to-manage" (henceforth RTM) allocation in bilateral monopolies with increasing marginal costs (MC), decreasing marginal revenue (MR) and linear price (w). In contrast, recent work in industrial organization, labor economics, and international trade similarly incorporate such features, but exogenously assign the RTM to one side.[32] Two implications follow.

First, the ad hoc RTM allocation predetermines the welfare consequences of a firm's bargaining power. On the one hand, exogenously allocating the RTM to the demand side, i.e. the buying or hiring firm, implies that w and quantity q always move along a derived, decreasing, demand curve ($MR_D(q)$ in our setting). Consequently, welfare-detrimental monopoly power and welfare-improving countervailing buyer power inevitably arise in equilibrium. Indeed, increasing buyer power (lower α) implies a lower w and a higher q, thus always improving welfare. On the other hand, exogenously allocating the RTM to the supply side, i.e. the selling firm or workers/union, implies that w and q always move along an increasing supply curve.[33] Consequently, welfare-detrimental monopsony power and welfare-improving countervailing seller power inevitably arise in equilibrium. Indeed, increasing seller power (higher α) implies a higher w and q, thus always improving welfare.

Second, when the bargaining power is sufficiently tilted toward the side with RTM, such frameworks deliver an equilibrium quantity above the bilaterally efficient quantity q_I, misaligning bilateral efficiency and welfare. When D holds the RTM, the equilibrium w lies along MR_D but below MC_U, implying that U supplies a quantity at a price below

[32] In a bilateral monopoly model where the upstream firm faces increasing marginal costs, Mukherjee and Sinha (2024) maintains the RTM assigned to the downstream firm. In structural models with bargaining between unions and firms, Azkarate-Askasua and Zerecero (2022) and Wong (2023) assume firms internalize facing increasing labor supply curves, while workers/unions hold the RTM. In a structural model of bilateral oligopoly, Alviarez et al. (2023) assume that importers holding the RTM internalize facing foreign suppliers with possibly decreasing returns to scale technology. In Appendices B.3.3 and B.4.3, we present a decomposition of markups and markdowns similar to the approach taken in the referenced papers. We also discuss the extent to which our expressions align with or diverge from their findings.

[33] Such supply curve can be $r(q)$ or $MC_U(q)$ depending on whether or not the upstream side exerts a markdown in its input market.

its marginal cost. Similarly, when U holds the RTM, the equilibrium w lies along the marginal cost curve and above the marginal revenue curve, implying that D purchases a quantity at a price above its marginal revenue. This underlines that an exogenous RTM framework requires an ex-ante commitment that may be difficult to fully rationalize.

In contrast, our framework endogenizes the RTM and bargaining power welfare effects without requiring any ex ante commitment.

7 Conclusion

This article provides a comprehensive analysis of the interactions between monopsony, monopoly, and countervailing power theories within vertical supply chains. By introducing an upstream firm with increasing marginal costs into the standard chain of monopolies model, we demonstrate that the downstream firm no longer solely determines the quantity traded in the supply chain. These modifications to the canonical model of vertical relationships offer new perspectives on how the balance of bargaining power shapes welfare outcomes. Specifically, we highlight that the type of double marginalization—manifesting as either double markupization or markdownization—is intricately tied to the distribution of bargaining power between firms and the relative degree of supply and demand elasticity.

Appendix

A Proofs of Markups and Markdowns Expressions

A.1 Vertical Integration and Take-it-or-leave-it Offers

In an environment in which each price is set unilaterally by a firm, each firm faces a supply or demand curve. The firm making the take-it-or-leave-it offer internalizes that its price will affect the quantity supplied or purchased by its trade partner. The firm receiving the offer holds the right-to-manage because, given the proposed price, its trade partner would be willing to trade more. In equilibrium, each firm equalizes marginal revenue to marginal cost.

First, let us consider the vertically integrated benchmark. The proof directly extends to any firm i unilaterally choosing its prices or quantity facing demand and supply curves. In this context, the firm I faces an increasing inverse supply curve $r(q)$ and a decreasing inverse demand curve $p(q)$. In this environment, the equilibrium is given by the integrated firm profit maximization:

$$Max_q \ \Pi_I(q) = p(q)q - r(q)q.$$

The equilibrium quantity q_I is such that:

$$MR_I(q_I) = MC_I(q_I).$$

A.1.1 Markup

Consistently with our definition of μ_i, we consider the following equilibrium perturbation: the quantity deviates from q_I to $q_I+\varepsilon$ and the quantity $\varepsilon \to 0$ is sold at price $\overline{p}$. The environment otherwise remains identical. Again building on the definition of μ_i, we determine in what follows the minimum price $\hat{p}$ for which the marginal unit ε is supplied, i.e the minimum price at which the integrated firm is willing to offer the marginal unit. Formally, we look for the minimal value of $\overline{p}$ such that:

$$\overline{\Pi}_I(q_I, \varepsilon) \geq \Pi_I(q_I), \tag{16}$$

where $\overline{\Pi}_I(q_I, \varepsilon, \overline{p})$ is firm's I profit for the perturbed equilibrium, as defined by:

$$\overline{\Pi}_I(q_I, \varepsilon, \overline{p}) = p(q_I)(q_I) + \overline{p}\varepsilon - r(q_I + \varepsilon)(q_I + \varepsilon). \tag{17}$$

Using (17), the inequality (16) can thus be rewritten as:

$$\overline{p} \geq \frac{r(q_I + \varepsilon) - r(q_I)}{\varepsilon} q_I + r(q_I + \varepsilon).$$

As $\varepsilon \to 0$ and

$$lim_{\varepsilon \to 0} \frac{r(q_I + \varepsilon) - r(q_I)}{\varepsilon} q_I + r(q_I + \varepsilon) = r(q_I) q_I + r(q_I) = MC_I(q_I),$$

the minimum price at which the perturbed equilibrium is preferred by firm I is thus $\hat{p} = MC_I(q_I)$. It follows by definition that $\mu_I(q_I) = \frac{p(q_I)}{MC_I(q_I)}$.

A.1.2 Markdown

Consistently with our definition of ν_i, we consider the following equilibrium perturbation: the quantity deviates from q_I to $q_I + \varepsilon$ where the quantity $\varepsilon \to 0$ is purchased at price $\overline{r}$. The environment otherwise remains identical. Again building on the definition of ν_i, we determine in what follows the maximum price $\hat{r}$ for which the marginal unit ε is purchased, i.e the maximum price at which the integrated firm is willing to purchase the marginal unit. Formally, we look for the maximum value of $\overline{r}$ such that:

$$\overline{\Pi}_I(q_I, \varepsilon, \overline{r}) \geq \Pi_I(q_I), \tag{18}$$

where $\overline{\Pi}_I(q_I, \varepsilon, \overline{r})$ is the firm's I profit for the perturbed equilibrium, as defined by:

$$\overline{\Pi}_I(q_I, \varepsilon, \overline{r}) = p(q_I + \varepsilon)(q_I + \varepsilon) - r(q_I)(q_I) - \overline{r}\varepsilon. \tag{19}$$

Using (19), the inequality (18) can thus be rewritten as:

$$\frac{p(q_I + \varepsilon) - p(q_I)}{\varepsilon} q_I + p(q_I + \varepsilon) \geq \overline{r}.$$

As $\varepsilon \to 0$ and

$$lim_{\varepsilon \to 0} \frac{p(q_I + \varepsilon) - p(q_I)}{\varepsilon} q_I + p(q_I + \varepsilon) = p(q_I) q_I + p(q_I) = MR_I(q_I),$$

the maximum price at which the perturbed equilibrium is preferred by firm I is thus $\hat{r} = MR_I(q_I)$. It follows by definition that $\nu_I(q_I) = \frac{MR_I(q_I)}{r(q_I)}$.

A.2 Bargaining

In an environment in which U and D bargain over a linear tariff w, the equilibrium quantity traded is determined by the maximization of the Nash product.

The equilibrium value of the Nash-Product $N(q^*)$ is the following:

$$N(q^*) = \max_q \Pi_U(q)^\alpha \Pi_D(q)^{(1-\alpha)} \quad \text{s.t} \quad w(q) = \begin{cases} MC_U(q) \text{ if } \tilde{q}_u(w) < \tilde{q}_D(w) \\ MR_D(q) \text{ if } \tilde{q}_u(w) > \tilde{q}_D(w) \end{cases} \tag{20}$$

where $\Pi_U(q) = (w(q) - r(q))q$ and $\Pi_D(q) = (p(q) - w(q))q$.

A.2.1 Markups

Consistently with our definition of μ_i, we consider the following equilibrium perturbation: the quantity deviates from q^* to $q^* + \varepsilon$ where the quantity $\varepsilon \to 0$ is sold by firm $i \in \{D, U\}$ at a price $\overline{x}$, with $x \in \{p, w\}$. The environment otherwise remains identical. Again building on the definition of μ_i, we determine in what follows the minimum price $\hat{x}$ for which the marginal unit ε is supplied, i.e the minimum price at which the perturbed equilibrium leads to a higher Nash product, denoted $\overline{N}(q^*, \varepsilon, \overline{x})$, than the equilibrium Nash product $N(q^*)$. Formally, we determine the minimal value of $\overline{x}$ such that:

$$\overline{N}(q^*, \varepsilon, \overline{x}) \geq N(q^*). \tag{21}$$

Markup of D

By definition, the perturbed equilibrium relevant for characterizing the markup of D yields the following Nash product:

$$\overline{N}(q^*, \varepsilon, \overline{p}) = [\underbrace{(w(q+\varepsilon)(q+\varepsilon) - r(q+\varepsilon)(q+\varepsilon)}_{\Pi_U(q^*+\varepsilon)}]^\alpha [\underbrace{(p(q)(q) + \overline{p}\varepsilon - w(q+\varepsilon)(q+\varepsilon))}_{\overline{\Pi}_{D(q,\varepsilon,\overline{p})}}]^{1-\alpha}. \tag{22}$$

Applying the Taylor formula to the equilibrium Nash product in (20) yields:

$$N(q^* + \varepsilon) = N(q^*) + \frac{\partial N(q^*)}{\partial q}\varepsilon + o_N(\varepsilon^2) = N(q^*) + o_N(\varepsilon^2), \tag{23}$$

where $o_N \to 0$ when $\epsilon \to 0$. Using (22) and (23), we can rewrite (21) as:

$$\begin{aligned}
& \overline{N}(q^*, \varepsilon, \overline{p}) \geq N(q^* + \varepsilon) - o_N(\varepsilon^2) \\
\Leftrightarrow \quad & \Pi_U(q^* + \varepsilon)^\alpha \overline{\Pi}_D(q^*, \varepsilon, \overline{p})^{1-\alpha} \geq \Pi_U(q^* + \varepsilon)^\alpha \Pi_D(q^* + \varepsilon)^{1-\alpha} - o_N(\varepsilon^2) \\
\Leftrightarrow \quad & \overline{\Pi}_D(q^*, \varepsilon, \overline{p}) \geq \Pi_D(q^* + \varepsilon) - \left(\frac{o_N(\varepsilon^2)}{\Pi_U(q^* + \varepsilon)^\alpha}\right)^{\frac{1}{1-\alpha}}.
\end{aligned}$$

Using again a Taylor formula,

$$\Pi_D(q^* + \varepsilon) = q^* p(q^*) + p'(q^*)\varepsilon q^* + \varepsilon(q^*) + \varepsilon^2 p'(q^*) + w(q^* + \varepsilon)(q^* + \varepsilon) + \varepsilon o_p(\varepsilon^2),$$

where $o_p \to 0$ when $\epsilon \to 0$, the inequality can be rewritten as:

$$\begin{aligned}
\Leftrightarrow \quad & \varepsilon\overline{p} \geq p(q^*)\varepsilon + p'(q^*)\varepsilon q^* + \varepsilon^2 p'(q^*) + o_p(\varepsilon^2) - \left(\frac{o_N(\varepsilon^2)}{\Pi_U(q^* + \varepsilon)^\alpha}\right)^{\frac{1}{1-\alpha}} \\
\Leftrightarrow \quad & \overline{p} \geq p'(q^*)q^* + p(q^*) + \frac{1}{\varepsilon}\left(\varepsilon^2 p'(q^*) + o_p(\varepsilon^2) - \left(\frac{o_N(\varepsilon^2)}{\Pi_U(q^* + \varepsilon)^\alpha}\right)^{\frac{1}{1-\alpha}}\right) \\
\Leftrightarrow \quad & \overline{p} \geq MR_D(q^*) + \frac{1}{\varepsilon}\left(\varepsilon^2 p'(q^*) + o_p(\varepsilon^2) - \left(\frac{o_N(\varepsilon^2)}{\Pi_U(q^* + \varepsilon)^\alpha}\right)^{\frac{1}{1-\alpha}}\right).
\end{aligned}$$

As $\varepsilon \to 0$, the inequality boils downs to $\overline{p} \geq MR_D(q^*)$. The minimum price at which D would supply the marginal unit when the bargaining with U leads to quantity q^* thus is $\hat{p} = MR_D(q^*)$. It follows by definition that $\mu_D(q^*) = \frac{p(q^*)}{MR_D(q^*)}$.

Markup of U

By definition, the perturbed equilibrium relevant for characterizing the markup of U yields the following Nash product:

$$\overline{N}(q^*, \varepsilon) = \left(w(q)q + \overline{w}\varepsilon - r(q+\varepsilon)(q+\varepsilon)\right)^\alpha \left(p(q+\varepsilon)(q+\varepsilon) - w(q) - \overline{w}\varepsilon\right)^{1-\alpha}. \tag{24}$$

We can rewrite (21) as:

$$\begin{aligned}
\Leftrightarrow \quad & \left(w(q^*)q^* + \overline{w}\varepsilon - r(q^* + \varepsilon)(q^* + \varepsilon)\right)^\alpha \left(p(q^* + \varepsilon)(q^* + \varepsilon) - w(q^*) - \overline{w}\varepsilon\right)^{1-\alpha} \geq \Pi_U(q^*)^\alpha \Pi_D(q^*)^{1-\alpha} \\
\Leftrightarrow \quad & [\varepsilon\overline{w} - (r(q^* + \varepsilon) - r(q^*))q^* + \Pi_U(q^*)]^\alpha \times \\
& [(p(q^* + \varepsilon) - p(q))q^* - \overline{w}\varepsilon + \Pi_D(q^*)]^{1-\alpha} \geq \Pi_U(q^*)^\alpha \Pi_D(q^*)^{1-\alpha} \\
\Leftrightarrow \quad & \left(\varepsilon(\overline{w} - MC_U(q^*)) + \Pi_U(q^*)\right)^\alpha \left(\varepsilon(MR_D(q^*) - \overline{w}) + \Pi_D(q^*)\right)^{1-\alpha} \geq \Pi_U(q^*)^\alpha \Pi_D(q^*)^{1-\alpha}. \qquad (25)
\end{aligned}$$

The simplification of inequality (25) depends on the value of $\overline{w}$, whether $\tilde{q}_D(\overline{w}) \leq \tilde{q}_U(\overline{w})$ or $\tilde{q}_D(\overline{w}) \geq \tilde{q}_U(\overline{w})$.

Assume first $\overline{w}$ such that $q^* + \varepsilon = \tilde{q}_D(\overline{w}) \leq \tilde{q}_U(\overline{w})$, the stage 2 of the game implies that $\overline{w} = MR_D(q^* + \varepsilon)$. Using the Taylor formula gives:

$$\begin{aligned}
& MR_D(q^* + \varepsilon) = MR_D(q^*) + \varepsilon MR'_D(q^*) + o_{MR_D}(\varepsilon^2) \\
\Leftrightarrow \quad & MR_D(q^*) - \overline{w} = -\varepsilon MR'_D(q^*) - o_{MR_D}(\varepsilon^2) \qquad (26)
\end{aligned}$$

Rewriting inequality (25) gives:

$$(\varepsilon(\overline{w} - MC_U(q^*)) + \Pi_U(q^*))^\alpha \left(-\varepsilon^2 MR'_D(q^*) - \varepsilon o_{MR_D}(\varepsilon^2) + \Pi_D(q^*)\right)^{1-\alpha} \geq \Pi_U(q^*)^\alpha \Pi_D(q^*)^{1-\alpha}. \tag{27}$$

As $\varepsilon \to 0$ terms involving ε^2 are negligible compared to terms linear in ε. Approximating equation (26) gives:

$$-\varepsilon^2 MR'_D(q^*) - \varepsilon o_{MR_D}(\varepsilon^2) + \Pi_D(q^*) \approx \Pi_D(q^*),$$

and the inequality (27) can thus be rewritten as:

$$\begin{aligned} & (\varepsilon(\overline{w} - MC_U(q^*)) + \Pi_U(q^*))^\alpha \geq \Pi_U(q^*)^\alpha \\ \Leftrightarrow \quad & \overline{w} \geq MC_U(q^*). \end{aligned}$$

If the minimum price at which the marginal unit is supplied by U is $\hat{w}$ such that $q^* + \varepsilon = \tilde{q}_U(\hat{w}) \leq \tilde{q}_D(\hat{w})$, then $\hat{w} = MC_U(q^*)$.
Assume now $\overline{w}$ such that $q^* + \varepsilon = \tilde{q}_U(\overline{w}) \leq \tilde{q}_D(\overline{w})$, the stage 2 of the game implies that $\overline{w} = MC_U(q^* + \varepsilon)$. Using the Taylor formula gives:

$$\begin{aligned} & MC_U(q^* + \varepsilon) = MC_U(q^*) + \varepsilon MC'_U(q^*) + o_{MC_U}(\varepsilon^2) \\ \Leftrightarrow \quad & \overline{w} - MC_U(q^*) = \varepsilon MC'_U(q^*) + o_{MC_U}(\varepsilon^2) \end{aligned} \tag{28}$$

Rewriting inequality (25) gives:

$$\left(\varepsilon^2 MC'_U(q^*) + \varepsilon o_{MC_U}(\varepsilon^2)) + \Pi_U(q^*)\right)^\alpha (\varepsilon(MR_D(q^*) - \overline{w}) + \Pi_D(q^*))^{1-\alpha} \geq \Pi_U(q^*)^\alpha \Pi_D(q^*)^{1-\alpha}. \tag{29}$$

As $\varepsilon \to 0$ terms involving higher powers of ε^2 are negligible compared to terms linear in ε. Approximating (28) gives:

$$\varepsilon^2 MC'_U(q^*) + \varepsilon o_{MC_U}(\varepsilon^2) + \Pi_U(q^*) \approx \Pi_U(q^*), \tag{30}$$

and the inequality (29) can thus be rewritten as:

$$\begin{aligned} & (\varepsilon(MR_D(q^*) - \overline{w}) + \Pi_D(q^*))^{1-\alpha} \geq \Pi_D(q^*)^{1-\alpha} \\ \Leftrightarrow \quad & MR_D(q) \geq \overline{w}. \end{aligned}$$

If the minimum price at which U would supply the marginal unit U is $\hat{w}$ such that $q^* + \varepsilon = \tilde{q}_U(\hat{w}) \geq \tilde{q}_D(\hat{w})$ then is no constraint coming from the first-order condition of the Nash product, thus the minimum value is given by the constraint of the bargaining $\hat{w} = MC_U(q^*)$.

Finally, the minimum price at U would supply the marginal unit by firm U when the bargaining with D leads to quantity q^* thus is $\hat{w} = MC_U(q^*)$. It follows by definition that $\mu_U(q^*) = \frac{p(q^*)}{MC_U(q^*)}$.

A.2.2 Markdowns

Consistently with our definition of ν_i, we consider the following equilibrium perturbation: the quantity deviates from q^* to $q^* + \varepsilon$ where the quantity $\varepsilon \to 0$ is sold by firm $i \in \{D, U\}$ at a price $\overline{x}$, with $x \in \{p, w\}$. The environment otherwise remains identical. Again building on the definition of ν_i, we determine in what follows the maximum price $\hat{x}$ for which the marginal unit ε is purchased, i.e the maximum price at which the perturbed equilibrium leads to a higher Nash product, denoted $\overline{N}(q^*, \varepsilon, \overline{x})$, than the equilibrium Nash product $N(q^*)$. Formally, we determine the minimal value of $\overline{x}$ such that:

$$\overline{N}(q^*, \varepsilon, \overline{x}) \geq N(q^*). \tag{31}$$

Markdown of D

As w is both the output price of U and the input price of D, the perturbed equilibrium associated with the analysis of D's markdown gives the same Nash product $\overline{N}(q^*, \varepsilon, \overline{w})$ as in the proof characterizing the expression of the markup of U given in equation (24). Simplification of inequality (31) is identical to Appendix A.2.1.

Again, two cases must be treated depending on the value of $\overline{w}$, whether $\tilde{q}_D(\overline{w}) \leq \tilde{q}_U(\overline{w})$ or $\tilde{q}_D(\overline{w}) \geq \tilde{q}_U(\overline{w})$.

Assume first $q^* + \varepsilon = \tilde{q}_D(\overline{w}) \leq \tilde{q}_U(\overline{w})$. Stage 2 of the game implies that $\overline{w} = MR_D(q^* + \varepsilon)$, inequality (31) is satisfied when:

$$\overline{w} \geq MC_U(q^*).$$

Thus, there is no constraint on the maximum value of $\overline{w}$ coming from the FOC of the bargaining, the maximum value is given by the constraint of the bargaining $\hat{w} = MR_D(q*)$.

Assume now, $q^* + \varepsilon = \tilde{q}_U(\overline{w}) \leq \tilde{q}_D(\overline{w})$. Stage 2 of the game implies that $\overline{w} = MR_D(q^* + \varepsilon)$, inequality (31) is satisfied when:

$$\overline{w} \leq MR_D(q^*).$$

Thus, the maximum price at which D would purchase purchased the marginal unit is $\hat{w} = MR_D(q^*)$.

Finally, the maximum price at which D would purchase the marginal unit when the bargaining with U leads to quantity q^* is $\hat{w} = MR_D(q^*)$. It follows by definition that $\nu_D(q^*) = \frac{MR_D(q^*)}{w(q^*)}$.

Markdown of U

By definition, the perturbed equilibrium relevant for characterizing the markdown of U yields the following Nash product:

$$\overline{N}(q^*,\varepsilon,\overline{r}) = [\underbrace{(w(q+\varepsilon)(q+\varepsilon)-r(q)(q)-\varepsilon\overline{r}}_{\overline{\Pi}_U(q^*,\varepsilon,\overline{r})}]^{\alpha}[\underbrace{(p(q+\varepsilon)(q+\varepsilon)-w(q+\varepsilon)(q+\varepsilon))}_{\Pi_D(q+\varepsilon)}]^{1-\alpha}. \tag{32}$$

The value $\hat{r}$ is the minimum value of $\overline{r}$ which satisfies equation (31) with $\overline{x} = \overline{r}$. The proof is symmetric to that characterizing the markup of D, thus one can show using Taylor approximation and $\varepsilon \to 0$ that equation (31) simplifies to:

$$\overline{r} \leq MC_U(q^*).$$

Thus, the maximum price at which U purchases the marginal when bargaining with D leads to quantity q^* is $\hat{w} = MC_U(q^*)$. It follows by definition that $\nu_U(q^*) = \frac{MC_U(q^*)}{r(q*)}$.

B Proofs

B.1 Proof of Proposition 2

Assumptions 1 imply that MR_D is decreasing and MC_U is increasing. These properties have multiple implications. First, there is a unique quantity $q = q_I$ such that $MR_D(q_I) = MC_U(q_I)$ (see Proposition 1). Second, in stage 2, there is a unique $w = w_I$ such that $w_I = MR_D(\tilde{q}_D(w_I)) = MC_U(\tilde{q}_U(w_I)) = MR_D(q_I) = MC_U(q_I)$. Third, for $w < w_I$, $\tilde{q}_D(w) > \tilde{q}_U(w)$, and for $w > w_I$, $\tilde{q}_D(w) < \tilde{q}_U(w)$.
The timing assumption of stage 2 stipulates that the quantity exchanged is the minimum of the two quantities each player is willing to exchange, $q(w) = \min\{q_U(w), q_D(w)\}$. The resolution of stage 2 leads to $q(w) = \min\{\tilde{q}_U(w), \tilde{q}_D(w)\}$, and thus:

$$q(w) = \begin{cases} \tilde{q}_U(w), \text{ for } w \leq w_I \\ \tilde{q}_D(w), \text{ for } w \leq w_I \end{cases}$$

For $w \leq w_I$, we have: $q = \tilde{q}_U = MC_U^{-1}(w) \Rightarrow MC_U(q) = w$. For $w \geq w_I$, we have: $q = \tilde{q}_D = MR_D^{-1}(w) \Rightarrow MR_D(q) = w$.

B.2 Proof of Proposition 3

When $\alpha = \alpha_I = \frac{\Pi_U(q_I)}{\Pi_U(q_I)+\Pi_D(q_I)}$, the bargaining is efficient and q_I is such that $MC_U(q_I) = MR_D(q_I)$. We can rewrite α_I as follows:

$$\begin{aligned}\alpha_I &= \frac{(MR_D(q_I) - r(q_I))q_I}{(p(q_I) - r(q_I))q_I} \\ &= \frac{MR_D(q_I) - r(q_I)}{p(q_I) - r(q_I)}.\end{aligned}$$

Using $p(q_I) = MR_D(q_I)\frac{\varepsilon_p(q_I)}{\varepsilon_p(q_I)-1}$, $r(q_I) = MC_U(q_I)\frac{\varepsilon_r(q_I)}{\varepsilon_r(q_I)+1}$ and $MC_U(q_I) = MR_D(q_I)$, we obtain:

$$\begin{aligned}\alpha_I &= \frac{MR_D(q_I) - MR_D(q_I)\frac{\varepsilon_r(q_I)}{\varepsilon_r(q_I)+1}}{MR_D(q_I)\frac{\varepsilon_p(q_I)}{\varepsilon_p(q_I)-1} - MR_D(q_I)\frac{\varepsilon_r(q_I)}{\varepsilon_r(q_I)+1}} \\ &= \frac{\varepsilon_p(q_I) - 1}{\varepsilon_p(q_I) + \varepsilon_r(q_I)}.\end{aligned}$$

B.3 Bargaining when U is Powerful ($\alpha_I < \alpha < 1$)

B.3.1 Second-Order Condition

The first-order condition given by (12) can be rearranged as follows:

$$\alpha(MR_U(q) - MC_U(q))(p(q) - MR_D(q))q + (1-\alpha)(MR_D(q) - MR_U(q))(MR_D(q) - r(q))q = 0. \quad (33)$$

We show that the first-order condition is strictly decreasing in q if $\sigma_r > -2$ and $\sigma^{MR} < 2$.

The second-order condition yields:

$$\begin{aligned}&\alpha(MR'_D(2 - \sigma^{MR}) - (\sigma_r + 2)r'(q))(p(q) - MR_D(q))q + (MR_U(q) - MC_U(q))(MR_D - MR_U)) \\ &\quad +(1-\alpha)(MR'_D(\sigma^{MR} - 1)(MR_D - r(q))q + (MR_D - MR_U)(MR_U - MC_U)) < 0. \quad (34)\end{aligned}$$

Using $-MR'_Dq = (MR_D - MR_U)$ and $a \equiv MR_U(q) - MC_U(q) < 0$, $b \equiv p(q) - MR_D(q) > 0$, $c \equiv MR_D(q) - MR_U(q) > 0$ and $d \equiv MR_D(q) - r(q) > 0$. Using also $a + c - d = -r'(q)q$, the first-order condition (33) simplifies as follows:

$$\alpha ab + (1-\alpha)cd = 0 \Leftrightarrow d = \frac{\alpha ab}{-(1-\alpha)c}, \quad (35)$$

and the second-order condition (34) becomes:

$$\alpha((\sigma^{MR} - 2)cb + (a + c - d)(\sigma_r + 2)b + ca) + (1-\alpha)(-(\sigma^{MR} - 1)cd + ca) < 0.$$

Using that $-c < a \Leftrightarrow MC_U < MR_D$, we find that the second-order condition (34) holds for any

$\sigma_r > -2$ and $\sigma^{MR} < 2$.

B.3.2 Weight β_D

We show here that $\beta_D(q^*, \alpha)$ increases in α. Equation (13) in the main text defines $\beta_D(q^*, \alpha)$:

$$\beta_D(q^*, \alpha) = \frac{MC_U(q^*) - MR_U(q^*)}{MR_D(q^*) - MR_U(q^*)}$$

and we note that $0 \leq \beta_D(q^*, \alpha) \leq 1$, as $MC_U(q^*) \geq MR_U(q^*)$, $MR_D(q^*) \geq MR_U(q^*)$ and $MC_U(q^*) \leq MR_D(q^*)$.

We now determine how β_D is affected by changes in α. The chain rule implies that:

$$\frac{\partial \beta_D}{\partial \alpha} = \frac{\partial \beta_D}{\partial q} \frac{\partial q}{\partial w} \frac{\partial w}{\partial \alpha}$$

with $\frac{\partial q}{\partial w} = MR'(w) < 0$ and $\frac{\partial w}{\partial \alpha} > 0$ (see proof of Proposition 6). To determine the sign of $\frac{\partial \beta_D}{\partial q}$ and hence $\frac{\partial \beta_D}{\partial \alpha}$, we totally differentiate (13), yielding:

$$dMC_U = (MR_D - MR_U)d\beta_D + [(1-\beta_D)MR'_U + \beta_D MR'_D]\, dq$$

Dividing both sides by dq and rearranging yields:

$$\frac{d\beta_D}{dq} = \frac{1}{MR_D - MR_U}\left[(1-\beta_D)MR'_U + \beta_D MR'_D - MC'_U\right].$$

We thus have $\frac{d\beta_D}{dq} > 0$ as $MR_D - MR_U < 0$ since $MR_U(q) \equiv MR'_D(q)q + MR_D(q) < MR_D(q)$, and $(1-\beta_D)MR'_U + \beta_D MR'_D - MC'_U < 0$ since $MR'_U(q) = (2 - \sigma_{MR_D})MR'_D(q) < 0$ from Assumption 3, $MR'_D(q) < 0$ and $MC'_U(q) > 0$ from Assumption 1, and $\beta_D(q^*, \alpha) \geq 0$ as proved above.

Putting pieces together:

$$\frac{\partial \beta_D}{\partial \alpha} = \underbrace{\frac{\partial \beta_D}{\partial q}}_{>0} \underbrace{\frac{\partial q}{\partial w}}_{<0} \underbrace{\frac{\partial w}{\partial \alpha}}_{>0} < 0.$$

B.3.3 Markup μ_U

To determine the markup $\mu_U = \frac{MR_D(q)}{MC_U(q)}$, we divide each term of the first-order condition given by (33) by $MC_U(q)$:

$$\alpha \left(\frac{MR_U}{MC_U} - 1\right)\left(\frac{p}{MC_U} - \frac{MR_D}{MC_U}\right) + (1-\alpha)\left(\frac{MR_D}{MC_U} - \frac{MR_U}{MC_U}\right)\left(\frac{MR_D}{MC_U} - \frac{r}{MC_U}\right) = 0.$$

We then use the following simplifications:

- $\frac{MR_U}{MR_D} = \frac{MR'_D q}{MR_D} + 1 = 1 - \frac{1}{\varepsilon_{MR_D}} = \frac{\varepsilon_{MR_D} - 1}{\varepsilon_{MR_D}}$,

- $MR_D = p\left(1-\frac{1}{\varepsilon_p}\right) \Leftrightarrow \frac{p}{MR_D} = \frac{1}{1-\frac{1}{\varepsilon_p}} = \frac{\varepsilon_p}{\varepsilon_p-1}$,
- $MC_U = r'q + r = r\left(\frac{1}{\varepsilon_r}+1\right) \Leftrightarrow \frac{r}{MC_U} = \frac{\varepsilon_r}{\varepsilon_r+1}$,

to rewrite:

$$
\begin{aligned}
& \alpha\left(\frac{MR_D}{MC_U}\left(\frac{\varepsilon_{MR_D}-1}{\varepsilon_{MR_D}}\right)-1\right)\left(\frac{MR_D}{MC_U}\frac{1}{\varepsilon_p-1}\right)+(1-\alpha)\left(\frac{MR_D}{MC_U}\frac{1}{\varepsilon_{MR_D}}\right)\left(\frac{MR_D}{MC_U}-\frac{\varepsilon_r}{\varepsilon_r+1}\right)=0 \\
\iff\quad & \alpha\left(\frac{MR_D}{MC_U}\frac{\varepsilon_{MR_D}-1}{\varepsilon_{MR_D}}-1\right)\left(\frac{1}{\varepsilon_p-1}\right)+(1-\alpha)\left(\frac{1}{\varepsilon_{MR_D}}\right)\left(\frac{MR_D}{MC_U}-\frac{\varepsilon_r}{\varepsilon_r+1}\right)=0 \\
\iff\quad & \frac{MR_D}{MC_U}\left(\alpha\left(\frac{\varepsilon_{MR_D}-1}{\varepsilon_{MR_D}\left(\varepsilon_p-1\right)}\right)+(1-\alpha)\left(\frac{1}{\varepsilon_{MR_D}}\right)\right)=\frac{\alpha}{\varepsilon_p-1}+(1-\alpha)\frac{\varepsilon_r}{\varepsilon_{MR_D}\left(\varepsilon_r+1\right)} \\
\iff\quad & \frac{MR_D}{MC_U}\left(\alpha\left(\varepsilon_{MR_D}-1\right)\left(\varepsilon_r+1\right)\right)+(1-\alpha)\left(\left(\varepsilon_r+1\right)\left(\varepsilon_p-1\right)\right)=\alpha\left(\varepsilon_r+1\right)\varepsilon_{MR_D}+(1-\alpha)\,\varepsilon_r\left(\varepsilon_p-1\right) \\
\iff\quad & \mu_U=\frac{MR_D}{MC_U}=\frac{\alpha\varepsilon_{MR_D}\left(\varepsilon_r+1\right)+(1-\alpha)\left(\varepsilon_p-1\right)\varepsilon_r}{\alpha\left(\varepsilon_{MR_D}-1\right)\left(\varepsilon_r+1\right)+(1-\alpha)\left(\varepsilon_r+1\right)\left(\varepsilon_p-1\right)}.
\end{aligned}
$$

Other Expression of μ_U. When U is powerful, its markup can also be rewritten as:

$$
\mu_U = \omega_D(\alpha)\frac{\varepsilon_{MR_D}}{\varepsilon_{MR_D}-1} + (1-\omega_D(\alpha))\frac{\varepsilon_r}{\varepsilon_r+1}, \tag{36}
$$

where $\omega_D(\alpha) \equiv \frac{\alpha(\varepsilon_{MR_D}-1)}{\alpha(\varepsilon_{MR_D}-1)+(1-\alpha)(\varepsilon_p-1)}$ and with $\dfrac{\partial\omega_D(\alpha)}{\partial\alpha} > 0$. In the case we consider here, namely $\alpha_I < \alpha < 1$, we have $(0 <) \frac{\varepsilon_{MR_D}-1}{\varepsilon_{MR_D}+\varepsilon_r} \leq \omega_D(\alpha) \leq 1$.[34] The weight $\omega_D(\alpha)$ is a function of α, and of demand primitives ε_p and σ_p (as $\varepsilon_{MR_D} = \frac{\varepsilon_p-1}{2-\sigma_p}$).

Equation (36) echoes expressions delivered by the exogenous right-to-manage models of Alviarez et al. (2023); Azkarate-Askasua and Zerecero (2022) and Wong (2023), whereby the bilateral markup (or markdown, see also Appendix B.4.3) is a weighted average between a monopoly (or oligopoly) markup term and a monopsony (or oligopsony) markdown term. Indeed, if $\alpha = 1$, then $\omega_D(\alpha) = 1$ and $\mu_U = \frac{\varepsilon_{MR_D}}{\varepsilon_{MR_D}-1}$, as U can make a take-it-or-leave-it offer to D. Similarly, if $\alpha = 0$ then $\omega_D(\alpha) = 0$, and (36) would state $\mu_U = \frac{\varepsilon_r}{\varepsilon_r+1} = \nu_U^{-1}$ and thus $p = w$ and $M_U = 1$. However, by sugbgame perfection criterion and short-side rule application, such equilibrium is ruled out in our framework, as D endogenously concedes the right-to-manage if becoming too powerful, i.e, when $\alpha < \alpha_I$. Instead, at this limit, i.e., if $\alpha = \alpha_I = \frac{\varepsilon_p-1}{\varepsilon_p+\varepsilon_r}$, then $\omega_D(\alpha_I) = \frac{\varepsilon_{MR_D}-1}{\varepsilon_{MR_D}+\varepsilon_r}$ and $\mu_U = 1$, yielding the vertical integration outcome q_I.

B.3.4 Set of Equilibria

Using the first order condition (12), we introduce the following function:

$$
\Phi(q_U,\alpha) \equiv \alpha(MR_U(q_U) - MC_U(q_U))\Pi_D(q_U) + (1-\alpha)(MR_D(q_U) - MR_U(q_U))\Pi_U(q_U) = 0.
$$

[34] $\omega_D(\alpha) > 0$ holds when the demand function is supermodular, which we assumed above.

Under Assumption 2, applying the implicit function theorem, we have that:

$$Sign\left(\frac{\partial q_U}{\partial \alpha}\right) = Sign\left(\frac{\partial \Phi(q_U,\alpha)}{\partial \alpha}\right),$$

with

$$\begin{aligned}\frac{\partial \Phi(q_U,\alpha)}{\partial \alpha} &= (MR_U(q_U) - MC_U(q_U))\Pi_D(q_U) - (MR_D(q_U) - MR_U(q_U))\Pi_U(q_U) \\ &= (MR_U(q_U) - MR_D(q_U))\Pi_U(q_U) < 0.\end{aligned}$$

Therefore, the equilibrium quantity q_U decreases in α. We know that $q_U = q_I$ when $\alpha = \alpha_I$ and therefore $q_U > q_I$ when $\alpha < \alpha_I$. As shown in Section 4.1, in that case $\tilde{q}_D(w) > \tilde{q}_U(w)$ and the initial assumption that $w = MR_D(q)$ no longer holds. The set of equilibria defined by (12) and $w_U = MR_D(q_U)$ thus only exists for $\alpha \in [\alpha_I, 1]$.

B.3.5 Proof of Corollary 2

(i) As $\mathcal{M} \equiv \frac{p}{r}$, with $\frac{\partial p}{\partial \alpha} = \underbrace{\frac{\partial p}{\partial q}}_{-}\underbrace{\frac{\partial q}{\partial \alpha}}_{-} > 0$ and $\frac{\partial r}{\partial \alpha} = \underbrace{\frac{\partial r}{\partial q}}_{+}\underbrace{\frac{\partial q}{\partial \alpha}}_{-} < 0$ from Corollary 1 and by assumption, we necessarily have $\frac{d\mathcal{M}}{d\alpha} > 0$.
(ii) As $\mu_U \equiv \frac{w}{MC_U} = \frac{MR_D}{MC_U}$, with $\frac{\partial MR_D}{\partial \alpha} = \underbrace{\frac{\partial MR_D}{\partial q}}_{-}\underbrace{\frac{\partial q}{\partial \alpha}}_{-} > 0$ and $\frac{\partial MC_U}{\partial \alpha} = \underbrace{\frac{\partial MC_U}{\partial q}}_{+}\underbrace{\frac{\partial q}{\partial \alpha}}_{-} < 0$ by assumption, we necessarily have $\frac{d\mu_U}{d\alpha} > 0$. Again similarly, as $M_U \equiv \frac{w}{r} = \frac{MR_D}{r}$, with $\frac{\partial MR_D}{\partial \alpha} = \underbrace{\frac{\partial MR_D}{\partial q}}_{-}\underbrace{\frac{\partial q}{\partial \alpha}}_{-} > 0$ and $\frac{\partial r}{\partial \alpha} = \underbrace{\frac{\partial r}{\partial q}}_{+}\underbrace{\frac{\partial q}{\partial \alpha}}_{-} < 0$ by assumption, we necessarily have $\frac{dM_U}{d\alpha} > 0$.
(iii) In the subconvex demand and supply case, where $\frac{\partial \varepsilon_f}{\partial q} < 0$ for every $f \in \{p, r\}$, we have $\frac{dM_D}{d\alpha} = \frac{d\mu_D}{d\alpha} = \underbrace{\frac{\partial \mu_D}{\partial \varepsilon_p}}_{-}\underbrace{\frac{\partial \varepsilon_p}{\partial q}}_{-}\underbrace{\frac{\partial q}{\partial \alpha}}_{-} < 0$, and similarly, $\frac{d\nu_U}{d\alpha} = \underbrace{\frac{\partial \nu_U}{\partial \varepsilon_r}}_{-}\underbrace{\frac{\partial \varepsilon_r}{\partial q}}_{-}\underbrace{\frac{\partial q}{\partial \alpha}}_{-} < 0$. Derivations follow a similar logic for the superconvex case, where $\frac{\partial \varepsilon_f}{\partial q} > 0$, and CES case where $\frac{\partial \varepsilon_f}{\partial q} = 0$, for every $f \in \{p, r\}$.

B.4 Bargaining when D is Powerful $(0 < \alpha < \alpha_I)$

B.4.1 Second-Order Condition

The first-order condition given by (14) can be rearranged as follows:

$$\alpha(MC_D(q) - MC_U(q))\left(p(q) - MC_U(q)\right)q + (1-\alpha)(MR_D(q) - MC_D(q))(MC_U(q) - r(q))q = 0. \quad (37)$$

We show that the first-order condition is strictly decreasing in q if $\sigma_p < 2$ and $\sigma^{MC} > -2$. The second-order condition yields:

$$\alpha(MC_U'(\sigma^{MC}+1))(p(q)-MC_U(q))q + (MC_D - MC_U)(MR_D - MC_D))$$
$$+(1-\alpha)((2-\sigma_p)p'(q) - MC_U'(\sigma^{MC}+2))(MC_U(q)-r(q))q + (MR_D - MC_D)(MC_D - MC_U)). \quad (38)$$

Using $MC_U'q = (MC_D - MC_U) \equiv e > 0$, $f \equiv p(q) - MC_U(q) > 0$, $g \equiv MR_D(q) - MC_D(q) < 0$ and $h \equiv MC_U(q) - r(q) > 0$. Using also $g + e - f = p'(q)q$, the first-order condition (33) simplifies as follows:

$$\alpha ef + (1-\alpha)gh = 0 \Leftrightarrow h = \frac{\alpha ef}{-(1-\alpha)g},$$

and the second-order condition (38) becomes:

$$\alpha((\sigma^{MC}+1)ef + eg) + (1-\alpha)((2-\sigma_p)(g+e-f) - e(\sigma^{MC}+2)h + eg) < 0.$$

Using that $-g < e \Leftrightarrow MC_U < MR_D$, we find that the second-order condition (38) holds for any $\sigma_p < 2$ and $\sigma^{MC} > -2$.

B.4.2 Weight β_U

We show here that $\beta_U(q^*, \alpha)$ increases in α. Equation (15) in the main text defines $\beta_U(q^*, \alpha)$:

$$\beta_U(q^*, \alpha) = \frac{MC_D(q^*) - MR_D(q^*)}{MC_D(q^*) - MC_U(q^*)}$$

with $0 \leq \beta_U(q^*, \alpha) \leq 1$, as $MC_D(q^*) \geq MC_U(q^*)$, $MC_D(q^*) \geq MR_D(q^*)$, and $MC_U(q^*) \leq MR_D(q^*)$.

Studying how $\beta_U(q^*, \alpha)$ is affected by changes in α similarly to what we did for $\beta_D(q^*, \alpha)$ in Appendix B.3.2, we can show that $\frac{\partial \beta_U}{\partial \alpha} = \underbrace{\frac{\partial \beta_U}{\partial q}}_{>0} \underbrace{\frac{\partial q}{\partial w}}_{>0} \underbrace{\frac{\partial w}{\partial \alpha}}_{>0} > 0$.

B.4.3 Markdown ν_D

To determine the markdown $\nu_D = \frac{MR_D(q)}{MC_U(q)}$, we divide each term of the first-order condition given by (40) by $MC_U(q)$:

$$\alpha\left(\frac{MC_D(q)}{MC_U(q)} - 1\right)\left(\frac{p(q)}{MC_U(q)} - 1\right) + (1-\alpha)\left(\frac{MR_D(q)}{MC_U(q)} - \frac{MC_D(q)}{MC_U(q)}\right)\left(1 - \frac{r(q)}{MC_U(q)}\right) = 0. \quad (39)$$

We then use the following simplifications:

- $\frac{MC_D(q)}{MC_U(q)} = \frac{MC_U'(q)q + MC_U}{MC_U} = \left(\frac{1}{\varepsilon_{MC_U}} + 1\right)$,
- $MR_D(q) = p(q)\left(1 - \frac{1}{\varepsilon_p}\right) \Leftrightarrow \frac{p(q)}{MC_U(q)} = \frac{MR_D}{MC_U}\left(\frac{\varepsilon_p}{\varepsilon_p - 1}\right)$,

- $MC_U = r'(q)\,q + r(q) = r(q)\left(\frac{1}{\varepsilon_r}+1\right) \Leftrightarrow r(q) = MC_U(q)\,\frac{\varepsilon_r}{\varepsilon_r+1}$,

to rewrite:

$$\begin{aligned}
&\alpha\left(\frac{1}{\varepsilon_{MC_U}}\right)\left(\frac{MR_D}{MC_U}\left(\frac{\varepsilon_p}{\varepsilon_p-1}\right)-1\right)+(1-\alpha)\left(\frac{MR_D}{MC_U}-\left(\frac{\varepsilon_{MC_U}+1}{\varepsilon_{MC_U}}\right)\left(\frac{1}{\varepsilon_r+1}\right)\right)=0\\
\Longleftrightarrow\quad &\frac{MR_D}{MC_U}\left(\alpha\left(\frac{1}{\varepsilon_{MC_U}}\right)\left(\frac{\varepsilon_p}{\varepsilon_p-1}\right)+(1-\alpha)\left(\frac{1}{\varepsilon_r+1}\right)\right)=\alpha\left(\frac{1}{\varepsilon_{MC_U}}\right)+(1-\alpha)\left(\frac{\varepsilon_{MC_U}+1}{\varepsilon_{MC_U}}\right)\left(\frac{1}{\varepsilon_r+1}\right)\\
\Longleftrightarrow\quad &\frac{MR_D}{MC_U}\left(\alpha\varepsilon_p(\varepsilon_r+1)+(1-\alpha)(\varepsilon_{MC_U})(\varepsilon_p-1)\right)=\alpha(\varepsilon_p-1)(\varepsilon_r+1)+(1-\alpha)(\varepsilon_{MC_U}+1)(\varepsilon_p-1)\\
\Longleftrightarrow\quad &\nu_D=\frac{MR_D}{MC_U}=\frac{\alpha(\varepsilon_p-1)(\varepsilon_r+1)+(1-\alpha)(\varepsilon_{MC_U}+1)(\varepsilon_p-1)}{\alpha\varepsilon_p(\varepsilon_r+1)+(1-\alpha)(\varepsilon_{MC_U})(\varepsilon_p-1)}.
\end{aligned}$$

Other Expression of ν_D When D is powerful, its markdown can be rewritten as:

$$\nu_D = \omega_U(\alpha)\frac{\varepsilon_{MC_U}+1}{\varepsilon_{MC_U}} + (1-\omega_U(\alpha))\frac{\varepsilon_p-1}{\varepsilon_p}, \tag{40}$$

where $\omega_U(\alpha) \equiv \frac{\alpha(\varepsilon_r+1)\varepsilon_p}{\alpha(\varepsilon_r+1)\varepsilon_p+(1-\alpha)(\varepsilon_p-1)\varepsilon_{MC_U}}$, with $\dfrac{\partial\omega_U(\alpha)}{\partial\alpha} < 0$. In the case we consider here, namely $0 < \alpha < \alpha_I$, we have $(0 <)\ \frac{\varepsilon_p}{\varepsilon_{MR_U}+\varepsilon_p} \leq \omega_U(\alpha) \leq 1$.[35] This weight $\omega_U(\alpha)$ is a function of α, and of demand primitives ε_r and σ_r (as $\varepsilon_{MC_U} = \frac{\varepsilon_r+1}{\sigma_r+2}$).

As (36), (40) echoes expressions delivered by the exogenous right-to-manage models of Alviarez et al. (2023); Azkarate-Askasua and Zerecero (2022) and Wong (2023), whereby the bilateral markdown (or markup, see also Appendix B.3.3) is a weighted average between a monopoly (or oligopoly) markup term and a monopsony (or oligopsony) markdown term. Indeed, if $\alpha = 1$, then $\omega_U(\alpha) = 1$ and $\nu_D = \frac{\varepsilon_{MC_U}+1}{\varepsilon_{MC_U}}$, as D can make a take-it-or-leave-it offer to U. Similarly, if $\alpha = 0$ then $\omega_U(\alpha) = 0$ and (40) would state $\nu_D = \mu_D^{-1}$ and thus $r = w$ and $M_D = 1$. However, by sugbgame perfection criterion and short-side rule application, such equilibrium is ruled out in our framework, as U endogenously concedes the right-to-manage if becoming too powerful, i.e. when $\alpha > \alpha_I$. Instead, at this limit, i.e., if $\alpha = \alpha_I = \frac{\varepsilon_p-1}{\varepsilon_p+\varepsilon_r}$, then $\omega_U(\alpha_I) = \frac{\varepsilon_p}{\varepsilon_{MR_U}+\varepsilon_p}$, and $\nu_D = 1$, yielding the vertical integration outcome q_I.

B.4.4 Set of Equilibria

Using the first order condition (14), we introduce the following function:

$$\Psi(q_D,\alpha) \equiv \alpha(MC_D(q_D) - MC_U(q_D))\Pi_D(q_D) + (1-\alpha)(MR_D(q_D) - MC_D(q_D))\Pi_U(q_D) = 0.$$

Under Assumption 3, applying the implicit function theorem, we have that

$$Sign(\frac{\partial q_D}{\partial\alpha}) = Sign(\frac{\partial\Psi(q_D,\alpha)}{\partial\alpha}),$$

[35] $\omega_U(\alpha) > 0$ holds when the demand function is supermodular, which we assumed above.

and

$$\begin{aligned}\frac{\partial \Phi(q_D,\alpha)}{\partial \alpha} &= (MC_D(q_D)-MC_U(q_D))\Pi_D(q_D)-(MR_D(q_D)-MC_D(q_D))\Pi_U(q_D) \\ &= (MC_D(q_D)-MR_D(q_U))\Pi_U(q_U)>0.\end{aligned}$$

Therefore, the equilibrium quantity q_D increases in α. We know that $q_D(\alpha_I)=q_I$ when $\alpha=\alpha_I$ and therefore $q_D>q_I$ when $\alpha>\alpha_I$. As shown in Section 4.1, in that case $\tilde{q}_D(w)<\tilde{q}_U(w)$ and the initial assumption that $w=MC_U(q)$ no longer holds. The set of equilibria defined by (12) and $w_D=MC_U(q_D)$ thus only exists for $\alpha\in[0,\alpha_I]$.

B.4.5 Proof of Corollary 3

(i) As $\mathcal{M}\equiv\frac{p}{r}$, with $\frac{\partial p}{\partial \alpha}=\underbrace{\frac{\partial p}{\partial q}}_{-}\underbrace{\frac{\partial q}{\partial \alpha}}_{+}<0$ and $\frac{\partial r}{\partial \alpha}=\underbrace{\frac{\partial r}{\partial q}}_{+}\underbrace{\frac{\partial q}{\partial \alpha}}_{+}>0$ from Corollary 1 and by assumption, we necessarily have $\frac{d\mathcal{M}}{d\alpha}<0$.
(ii) As $\nu_D\equiv\frac{MC_U}{w}=\frac{MC_U}{MR_D}$, with $\frac{\partial MC_U}{\partial \alpha}=\underbrace{\frac{\partial MC_U}{\partial q}}_{+}\underbrace{\frac{\partial q}{\partial \alpha}}_{+}>0$ and $\frac{\partial MR_D}{\partial \alpha}=\underbrace{\frac{\partial MR_D}{\partial q}}_{-}\underbrace{\frac{\partial q}{\partial \alpha}}_{+}<0$ by assumption, we necessarily have $\frac{d\nu_D}{d\alpha}<0$. Again similarly, as $M_D\equiv\frac{p}{w}=\frac{p}{MC_U}$, with $\frac{\partial p}{\partial \alpha}=\underbrace{\frac{\partial p}{\partial q}}_{-}\underbrace{\frac{\partial q}{\partial \alpha}}_{+}<0$ and $\frac{\partial MC_U}{\partial \alpha}=\underbrace{\frac{\partial MC_U}{\partial q}}_{-}\underbrace{\frac{\partial q}{\partial \alpha}}_{+}>0$ by assumption, we necessarily have $\frac{dM_D}{d\alpha}<0$.
(iii) In the subconvex demand and supply case, where $\frac{\partial \varepsilon_f}{\partial q}<0$ for every $f\in\{p,r\}$, we have $\frac{dM_U}{d\alpha}=\frac{d\nu_U}{d\alpha}=\underbrace{\frac{\partial \nu_U}{\partial \varepsilon_r}}_{-}\underbrace{\frac{\partial \varepsilon_r}{\partial q}}_{-}\underbrace{\frac{\partial q}{\partial \alpha}}_{+}>0$, and similarly, $\frac{d\mu_D}{d\alpha}=\underbrace{\frac{\partial \mu_D}{\partial \varepsilon_p}}_{-}\underbrace{\frac{\partial \varepsilon_p}{\partial q}}_{-}\underbrace{\frac{\partial q}{\partial \alpha}}_{+}>0$. Derivations follow a similar logic for the superconvex case, where $\frac{\partial \varepsilon_f}{\partial q}>0$, and CES case where $\frac{\partial \varepsilon_f}{\partial q}=0$, for every $f\in\{p,r\}$.

C Microfoundation

C.1 Take-it-or-Leave-it Offer from U

Assuming U has all the bargaining power (i.e. $\alpha=1$), the equilibrium solution is equivalent to the solution of a game in which, in stage 1, U makes a take-it-or-leave-it offer to D.

Lemma 1 *When U makes a take-it-or-leave-it to D, the equilibrium wholesale price is $\overline{w}=MC_U(\overline{q})\left(\frac{\varepsilon_{MR_D(\overline{q})}}{1-\varepsilon_{MR_D(\overline{q})}}\right)$ with $\overline{q}\equiv q(\overline{w})$ the equilibrium quantity and $\overline{w}=MR_D(\overline{q})>w_I$.*

Proof. The maximization problem faced by U is the following:

$$\overline{w}\equiv\underset{w}{\operatorname{argmax}}\ \Pi_U(w)\quad\text{subject to } w=\begin{cases} MC_U(q(w)),\ \text{for } w\leq w_I \\ MR_D(q(w)),\ \text{for } w\geq w_I\end{cases}$$

By definition,

$$\Pi_U(w) = wq(w) - r(q(w))q(w),$$

and

$$\frac{\partial \Pi_U}{\partial w} = q(w) + q'(w)[w - MC_U(q)].$$

Assuming first that $w \leq w_I$, we have from Stage 2 that $w = MC_U(q)$, and thus that $\frac{\partial \Pi_U}{\partial w} = q(w) > 0$, implying that $w \leq w_I$ is not profit-maximizing for U. The optimal w chosen by U is thus such that $w > w_I$, which implies that $w(q) = MR_D(q)$ from Stage 2.[36]

U maximization problem thus simplifies to:

$$\overline{w} \equiv \underset{w}{\operatorname{argmax}}\ wq(w) - r(q(w))q(w) \quad \text{subject to } w = MR_D(q)$$

The first-order condition yields:

$$\overline{w}\left(1 + \frac{q(\overline{w})}{q'(\overline{w})\overline{w}}\right) = MC_U(q(\overline{w})). \tag{41}$$

Using the constraint and $\overline{q} \equiv q(\overline{w})$, the first-order condition can be rewritten:

$$\overline{w} = MC_U(\overline{q})\left(\frac{\varepsilon_{MR_D}(\overline{q})}{\varepsilon_{MR_D}(\overline{q}) - 1}\right),$$

with $\overline{w} = MR_D(\overline{q}) > w_I$.

The second-order condition yields:

$$\begin{aligned} \frac{\partial^2 \Pi_U(w)}{\partial w^2} &= q''(w)[w(q) - MC_U(q)] + q'(w)[2 - q'(w)MC'_U(q)] < 0 \\ &\iff \sigma_{MR_D} < \frac{2 - \frac{MC'_U}{MR'_D}}{\left(\frac{MC_U}{MR_D} + 1\right)\epsilon_{MR_D}} \end{aligned} \tag{42}$$

where we used that $w(q) = MR_D(q)$, and thus $q'(w) = \frac{1}{MR'_D(q)} < 0$ and $q''(w) = \frac{\sigma_{MR_D}}{q[MR'_D(q)]^2}$. We also have $\varepsilon_{MR_D} \equiv \frac{MR_D(q)}{q|MR'_D(q)|}$ and $\sigma_{MR_D} \equiv \frac{qMR''_D(q)}{|MR'_D(q)|}$ as defined in the main text. Using (41), which implies $\left(\frac{MC_U}{MR_D} + 1\right)\epsilon_{MR_D} = 1$, (42) simplifies to:

$$\sigma_{MR_D} < 2 - \frac{MC'_U}{MR'_D}.$$

As $MC'_U(q) > 0$ and $MR'_D(q) < 0$, it is straightforward that Assumption 3, which stipulates that $\sigma_{MR_D} < 2$, is sufficient to guarantee the second-order condition validity.[37] ■

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[36] Stage 2 results are summarized in Proposition 2.

[37] Assumption 3 imposes that U's marginal revenue is decreasing when making a take-it-or-leave-it offer to D. It guarantees U's profit concavity even if upstream supply is perfectly elastic.

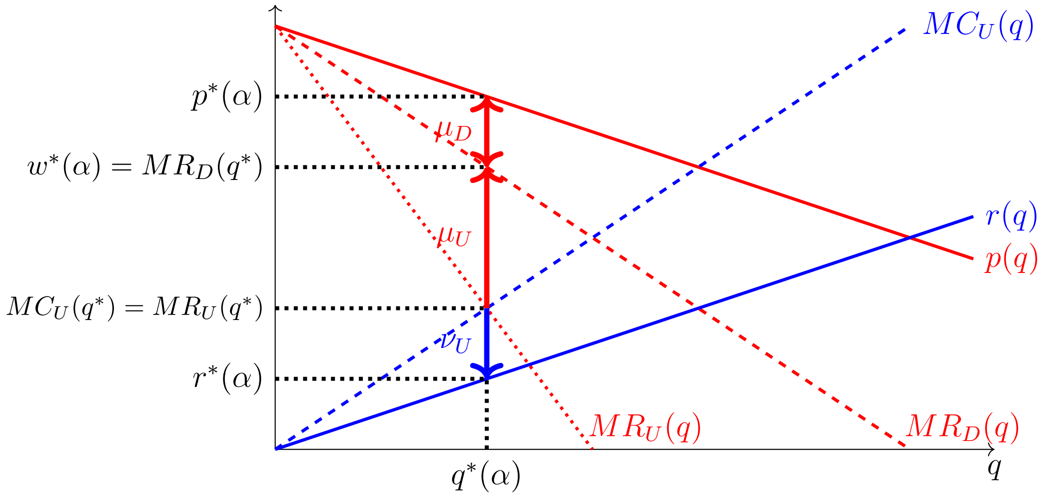

Figure 6: Take-it-or-Leave-it Offer from U ($\alpha = 1$)

In this extreme case, the expression of the equilibrium markup of U simplifies, so that:[38]

$$\mu_U \equiv \frac{w(q^*)}{MC_U(q^*)} = \frac{\varepsilon_{MR_D}}{\varepsilon_{MR_D} - 1} = \frac{\varepsilon_p - 1}{\varepsilon_p + \sigma_p - 3}.$$

Here, U makes a take-it-or-leave-it offer to D, and can thus equalize its marginal revenue and marginal cost, as visible in Figure 6. As a result, the markup of U only depends on demand primitives $(\varepsilon_p, \sigma_p)$. It contrasts with the intermediate case where U is $\alpha_I < \alpha < 1$, and where the supply elasticity (ε_r) affects the markup of U, as D exercises countervailing power via the Nash-bargaining.

C.2 Take-it-or-Leave-it Offer from D

Assuming D has all the bargaining power (i.e. $\alpha = 0$), the equilibrium solution is equivalent to the solution of a game in which, in stage 1, D makes a take-it-or-leave-it offer to U.

Lemma 2 *When D makes a take-it-or-leave-it to U, the equilibrium wholesale price is $\underline{w} = MR_D(\underline{q})\left(\frac{\varepsilon_{MC_U(\underline{q})}}{1+\varepsilon_{MC_U(\underline{q})}}\right)$ with $\underline{q} \equiv q(\underline{w})$ the equilibrium quantity and $\underline{w} = MC_U(\underline{q}) < w_I$.*

Proof. The maximization problem faced by D is the following:

$$\underline{w} \equiv \underset{w}{\text{argmax}}\ \Pi_D(w) \quad \text{subject to } w = \begin{cases} MC_U(q(w)), \text{ for } w \leq w_I \\ MR_D(q(w)), \text{ for } w \geq w_I \end{cases}$$

By definition,

$$\Pi_D = p(q(w))q(w) - wq(w),$$

[38] Apart from the endogenous quantity adjustment, other markup and markdown expressions in terms of primitives remain similar in this polar case to the ones derived in Proposition 4.

and

$$\frac{\partial \Pi_D}{\partial w} = q'(w)[MR_D(q) - w] - q(w).$$

Assuming first that $w \geq w_I$, we have from Stage 2 that $w = MR_D(q)$ and thus that $\dfrac{\partial \Pi_D}{\partial w} = -q(w) < 0$, implying that $w \geq w_I$ is not profit-maximizing for D. The optimal w chosen by D is thus such that $w < w_I$ which implies that $w(q) = MC_U(q)$ from Stage 2.[39]

D maximization problem thus simplifies to:

$$\underline{w} \equiv \underset{w}{\operatorname{argmax}}\ p(q(w))q(w) - wq(w) \quad \text{subject to } w = MC_U(q).$$

The first-order condition yields:

$$MR_D(\underline{q}) = \underline{w}\left(1 + \frac{q(\underline{w})}{q'(\underline{w})\underline{w}}\right). \tag{43}$$

Using the constraint and $\underline{q} = q(\underline{w})$, the first-order condition can be rewritten as:

$$\underline{w} = MR_D(\underline{q})\left(\frac{\varepsilon_{MC_U}(\underline{q})}{\varepsilon_{MC_U}(\underline{q}) + 1}\right), \tag{44}$$

with $\underline{w} = MC_U(\underline{q}) < w_I$.

The second-order condition yields:

$$\begin{aligned} \frac{\partial^2 \Pi_D(w)}{\partial w^2} &= q''(w)[MR_D(q) - w(q)] + q'(w)[q'(w)MR'_D(q) - 2] < 0 \\ \iff \sigma_{MC_U} &> \frac{\frac{MR'_D}{MC'_U} - 2}{\left(\frac{MR_D}{MC_U} - 1\right)\epsilon_{MC_U}} \end{aligned} \tag{45}$$

where we used that $w(q) = MC_U(q)$, and thus $q'(w) = \frac{1}{MC'_U(q)} > 0$ and $q''(w) = -\frac{\sigma_{MC_U}}{q[MC'_U(q)]^2}$. We also have $\varepsilon_{MC_U} \equiv \frac{MC_U(q)}{q|MC'_U(q)|}$ and $\sigma_{MC_U} \equiv \frac{qMC''_U(q)}{|MC'_U(q)|}$ as defined in the main text. Using (44), which implies $\left(\frac{MR_D}{MC_U} - 1\right)\epsilon_{MC_U} = 1$, (45) simplifies to:

$$\sigma_{MC_U} > \frac{MR'_D}{MC'_U} - 2.$$

As $MR'_D(q) < 0$ and $MC'_U(q) > 0$, it is straightforward that Assumption 2, which stipulates that $\sigma_{MC_U} > -2$, is sufficient to guarantee the second-order condition validity.[40] ■

[39] Stage 2 results are summarized in Proposition 2.

[40] Assumption 2 imposes that D's marginal cost is increasing when making a take-it-or-leave-it offer to U. It guarantees D's profit concavity even if the final demand is perfectly elastic.

In this extreme case, the expression of the equilibrium markdown of D simplifies, so that:[41]

$$\nu_D \equiv \frac{MR(q^*)}{w(q^*)} = \frac{\varepsilon_{MC_U}+1}{\varepsilon_{MC_U}} = \frac{\sigma_r+\varepsilon_r+3}{\varepsilon_r+1}.$$

Here, D makes a take-it-or-leave-it offer to U, and can thus equalize its marginal revenue and marginal cost, as visible in Figure 7. As a result, the markdown of D only depends on the supply primitives $(\varepsilon_r, \sigma_r)$. It contrasts with the intermediate case where $0 < \alpha < \alpha_I$, and where the demand elasticity (ε_p) affects the markdown of D, as U exercises countervailing power via the Nash-bargaining.

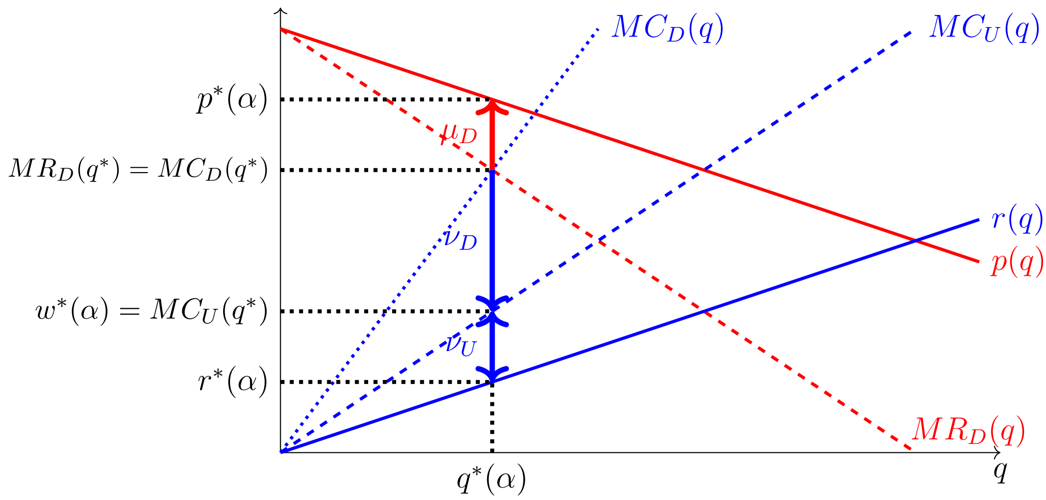

Figure 7: Take-it-or-Leave-it Offer from D ($\alpha = 0$)

C.3 Bargaining

We demonstrate that the equilibrium outcome of our model introduced in Section 4 coincides with the subgame perfect Nash equilibrium of the non-cooperative game developed in Rey and Vergé (2020). Specifically, consider that U and D play the following game:

- Stage 1: Wholesale negotiation.

  1.1 U makes a take-it-or-leave-it (TIOLI) offer to D, which either accepts or rejects.

  1.2 If D rejects the offer, Nature selects one side to make an ultimate TIOLI offer. It selects U with probability ϕ and D with probability $1-\phi$.

  1.3 The selected firm makes the ultimate TIOLI offer to its counterpart, which either accepts or rejects.

  Stage 2: Quantity setting (similar to Section 4).

[41] Apart from the endogenous quantity adjustment, other markup and markdown expressions in terms of primitives remain similar in this polar case to the ones derived in Proposition 5.

We proceed backward to look for the subgame perfect Nash equilibrium of this game. Stage 2 is similar to our model, Stage 1.3 is solved in appendices C.2, C.1, and Stage 1.2 involves no choice from strategic players, so we focus on the resolution of Stage 1.1.

In Stage 1.1, U solves the following maximization problem:

$$\max_{w} \Pi_U(w) \quad \text{subject to} \begin{cases} \Pi_D(w) \geq \phi\Pi_D(\overline{w}) + (1-\phi)\Pi_D(\underline{\mathrm{w}}) \\ w = \begin{cases} MC_U(q(w)) \text{ if } w \leq w_I \\ MR_D(q(w)) \text{ if } w \geq w_I \end{cases} \end{cases}$$

where $\Pi_D(\overline{w})$ and $\Pi_D(\underline{w})$ are the profits of D when U makes the TIOLI offer and when D makes the TIOLI offer, respectively.

Proposition 6 *For any Nash-bargaining solution $w^* \in [\underline{w}, \overline{w}]$ there exists a unique $\phi \in [0,1]$ such that the non-cooperative game solution $w^{**} = w^*$.*

Proof. It can first be shown that there exists a unique $w = w^{**} \in [\underline{\mathrm{w}}, \overline{w}]$ which (i) satisfies D's participation constraint and (ii) solves U's maximization problem, and (iii) satisfies $w^{**} = MC_U(q^{**})$ if $w^{**} \leq w_I$, or $MR_D(q^{**})$ if $w^{**} \geq w_I$, where $q^{**} \equiv q(w^{**})$. Indeed, we know from Appendix C.1 and Appendix C.2 that:

$$\begin{cases} \frac{\partial \Pi_U(w)}{\partial w} < 0 \text{ and } \frac{\partial \Pi_D(w)}{\partial w} < 0 \text{ for } w > \overline{w} \\ \frac{\partial \Pi_U(w)}{\partial w} > 0 \text{ and } \frac{\partial \Pi_D(w)}{\partial w} < 0 \text{ for } \underline{w} < w < \overline{w} \\ \frac{\partial \Pi_U(w)}{\partial w} > 0 \text{ and } \frac{\partial \Pi_D(w)}{\partial w} > 0 \text{ for } w < \underline{w} \end{cases}$$

First, note that it implies $\underline{w} \leq w^{**} \leq \overline{w}$ because for $w \leq \underline{w}$ the two firms prefer a higher w and for $w \geq w$ the two firms prefer a lower w. Second, note that, in $[\underline{w}, \overline{w}]$, profit derivative signs are invariant to which case of (iii) prevails in equilibrium and that (iii) has thus no bite in the rest of the proof. Third, suppose that $w^{**} \in [\underline{w}, \overline{w}]$ and satisfying (i), (ii), (iii). Considering deviations, any $w > w^{**}$ violates D's participation constraint whereas, any $w < w^{**}$ brings a lower profit to U.

We show now that there exists a unique $\phi \in [0,1]$ allowing to reach any $w^{**} \in [\underline{w}, \overline{w}]$. Defining $C(\phi) \equiv \phi\Pi_D(\overline{w}) + (1-\phi)\Pi_D(\underline{\mathrm{w}})$, D's participation constraint can be rewritten as $\Pi_D(w) \geq C(\phi)$. In equilibrium, D's participation constraint is binding, and $\Pi_D(w^{**}) = C(\phi)$. The value of $\phi \in [0,1]$ thus governs the value of w^{**}. As $C'(\phi) = \Pi_D(\overline{w}) - \Pi_D(\underline{\mathrm{w}}) < 0$ and $\Pi'_D(w) < 0$ (for $w > \underline{w}$), w^{**} is increasing in ϕ. For $\phi = 0$, D's participation constraint implies that $C(0) = \Pi_D(\overline{w})$, and thus $w^{**} = \underline{w}$. Similarly for $\phi = 1$, $C(1) = \Pi_D(\overline{w})$ and $w^{**} = \overline{w}$.

For any Nash-bargaining solution $w^* \in [\underline{w}, \overline{w}]$ there thus exists a unique $\phi \in [0,1]$ such that $w^{**} = w^*$. ■

Corollary 4 *The non-cooperative game solution is:*

$$w^{**} = \begin{cases} MC_U(q^{**}), \ \textit{for } \phi \leq \phi_I, \\ w_I = MC_U(q_I) = MR_D(q_I), \ \textit{for } \phi = \phi_I, \\ MR_D(q^{**}), \ \textit{for } \phi \geq \phi_I, \end{cases}$$

where

$$\phi_I = \frac{\Pi_D(\underline{w}) - \Pi_D(w_I)}{\Pi_D(\underline{w}) - \Pi_D(\overline{w})}.$$

Proof. Let us assume first that $w^{**} \geq w_I$, so that we must have $w^{**} = MR_D(q^{**})$. As D's participation constraint is binding in equilibrium from Proposition 6 (see proof), we necessarily have:

$$w^{**} = MR_D(q^{**}) = p(q^{**}) - \frac{C(\phi)}{q^{**}}$$

The condition $w^{**} \geq w_I$ implies that $MR_D(q^{**}) = p(q^{**}) - \frac{C(\phi)}{q^{**}} \geq w_I$, and at the limit:

$$w^{**} = MR_D(q^{**}) = p(q^{**}) - \frac{C(\phi)}{q^{**}} = w_I,$$

which holds true if and only if $q^{**} = q_I$. Such an equilibrium case thus prevails for a threshold value ϕ_I defined by:

$$\begin{aligned} & MR_D(q_I) && = p(q_I) - \frac{C(\phi_I)}{q_I} \\ \iff \quad & C(\phi_I) && = \underbrace{p(q_I)q_I - MR_D(q_I)q_I}_{\Pi_D(w_I)} \\ \iff \quad & \phi_I && = \frac{\Pi_D(\underline{w}) - \Pi_D(w_I)}{\Pi_D(\underline{w}) - \Pi_D(\overline{w})}. \end{aligned}$$

Note that $\phi_I \in (0,1)$ as $\Pi_D(\underline{w}) > \Pi_D(w_I) > \Pi_D(\overline{w}) > 0$. Indeed, for $w^{**} = w_I$, D's binding participation constraint implies that $\Pi(w_I)$ is a convex combination of $\Pi_D(\overline{w})$ and $\Pi_D(\underline{w})$, with $\Pi_D(\overline{w}) < \Pi_D(\underline{w})$.

This case where $w^{**} = MR_D(q^{**})$ and $w^{**} \geq w_I$ prevails in equilibrium for $\phi \in [\phi_I, 1]$. A similar reasoning straightforwardly applies to the case where $w^{**} = MC_U(q^{**})$ and $w^{**} \leq w_I$, which prevails in equilibrium for $\phi \in [0, \phi_I]$ (with ϕ_I uniquely defined as $MR_D(q_I) = MC_U(q_I)$ from Proposition 3).

■

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