Exercize: Competition between two vertical chains

Let us consider two vertical chains denoted 1 and 2. Each vertical chain is made of two firms, U_i which produces an input, and D_i who sells it to final consumers, with $i = 1, 2$.

- Production cost at each level are normalized to 0.
- Only D_i can sell the product made by U_i.
- Final goods are horizontally differentiated. p_1 (reps. p_2 denotes the final price of the good sold by chain 1 (resp. chain 2).

$$D_1 = 1 - p_1 + ap_2$$
$$D_2 = 1 - p_2 + ap_1$$

with $0 < a \leq 1$.

1. How to interpret a?

2. Vertical integration Assume first that U_i and D_i are merged to form

   a firm I_i who sells good i at price p_i to consumers. I_1 and I_2 compete and set their prices simultaneously.

   2.1. Consider that p_2 is given. Determine the profit of I_1 and the best reply $p_1^r(p_2)$ maximizing the profit of I_1 with p_2 given.

   The firm I_1 maximizes $\pi_{I1} = p_1(1 - p_1 + ap_2)$ which gives the best reply

   $$p_1^r(p_2) = \frac{(1 + ap_2)}{2}.$$

   2.2. Determine the best reply of firm I_2 for a given p_1, the Nash equilibrium prices p_1^{VI} et p_2^{VI} and profits of firms.

   The best reply of firm I_2 is symmetric and we obtain the following Nash equilibrium:

   $$p_1^{VI} = p_2^{VI} = \frac{1}{2 - a}$$

   and the corresponding profits $\frac{1}{(2-a)^2}$.

3. Vertical separation with two-part tariff. Assume now that each U_i et D_i are independent.

- In stage 1: each U_i simultaneously offers a TIOLI two-part tariff contract (w_i, F_i) to their D_i. Each D_i can accept or reject the offer. Contract are observable by all.
- In stage 2, D_1 and D_2 set simultaneously their final prices p_1 et p_2.

3.1. Stage 2, determine the profit of each D_i for a given (w_i, F_i). Compute the best reply of each firm D_i at a given p_j, $p_i^r(p_j)$. What is the effect of an increase in w_i on both best replies ? Comment.

The firm D_i maximizes $\pi_i = (p_i - w_i)(1 - p_i + ap_j) - F_i$ which gives the best reply

$$p_i^r(p_j) = \frac{(1 + w_i + ap_j)}{2}$$

whenever the profit is positive. An increase in w_1 raises $p_1^r(p_2)$ but has no direct effect on $p_2^r(p_1)$.

3.2. Determine the Nash equilibrium in prices as a function of w_1 and w_2. What is the effect of an increase in w_1 on equilibrium prices?

The intersection of the best replies gives the following Nash equilibrium in prices:

$$p_i^S = \frac{2 + a + 2w_i + aw_j}{4 - a^2}$$

An increase in w_1 raises both equilibrium prices. It moves the best reply $p_1^r(p_2)$ upward and does not affect $p_2^r(p_1)$ but the intersection is moved upward and both equilibrium prices are increased.

3.3. Stage 1: Write the profit of U_i who anticipates final prices p_1^S et p_2^S. Determine F_i such that D_i accepts the contract.

U_i maximizes

$$w_i(1 - p_i^S(w_i, w_j) + ap_j^S(w_j, w_i)) + F_i$$

subject to $(p_i^S - w_i)(1 - p_i^S + ap_j^S) - F_i \geq 0$.

3.4. Derive the profit of U_i as a function of w_i and sign it in $w_i = 0$.
Does U_i sets $w_i > 0$? Explain.
$\pi_{U_i} = p_i^S(w_i, w_j)(1 - p_i^S(w_i, w_j) + ap_j^S(w_j, w_i))$.
The derivative of π_{U_i} in $w_i = 0$ is $\frac{a^2(2+a+aw_2)}{(4-a^2)^2} \geq 0$

3.5. Assume that, before the beginning of the game, firm U_i can choose whether or not to vertically integrate with D_i. What is its decision?

Because firm U_i gets back the whole profit of the vertical chain through F_i, it is better off in vertical separation as it can raise w_i above 0 to raise both final prices (it relaxes downstream competition).