Firms' Strategies and Markets
Course 4: Dynamic Pricing

Claire Chambolle

October, 23th, 2024

École Polytechnique
Université Paris-Saclay

Dynamic Pricing

- Repeated interactions among firms may enable collusive strategies (IO class M1)
  - High prices over time.
- Reputation or Signaling strategies can occur (Class / Advertising & Entry)
  - Either a low or a high price can signal a high quality to an uninformed consumer in a first period.
  - Fighting on one market can create the reputation of being tough.
- We focus here on “consumer inertia" which may come from different sources and imply various firm's dynamic pricing strategies.
  - Durable Goods
  - Search costs → generate temporal price dispersion.
  - Switching costs → Consumers are *locked-in* within the same firm

Durable goods: Goods that are not consumed or destroyed in use; Consumers derive the benefit of their purchase for a period of time (several years).

- Cars, Washing Machines, Computers, Smartphones ...

Insights: A durable good monopoly who cannot discriminate in a given period among heterogenous consumers can use intertemporal discrimination to extract more surplus from consumers.

- Some consumers buy in the first period;
- Others delay their purchase expecting a lower price.

Assumptions

- A durable monopoly with a production cost 0.
- A continuum of heterogenous consumers live two periods $t = \{1, 2\}$. Consumers buy either 0 or 1 unit and their valuation for the good v is uniformly distributed over $[0, 1]$.
- δ is the discount factor.
- The monopoly sets p_1 in $t = 1$ and p_2 in $t = 2$.

Consider first the benchmark case in which the monopoly can sell only in $t = 1$ at price p.

- A consumer is willing to purchase the good if $(1+\delta)v - p > 0$ in $t = 1$. The demand is $D(p) = 1 - \frac{p}{1+\delta}$.
- $\max_p p(1 - \frac{p}{1+\delta}) \Leftrightarrow p = \frac{1+\delta}{2}$.
- The corresponding profit $\Pi = \frac{1+\delta}{4}$.

Consider now the two period pricing strategy

- For a given couple of prices (p_1, p_2), we determine the consumer indifferent between purchasing in $t = 1$ and in $t = 2$.

$$\underbrace{(1+\delta)\tilde{v} - p_1}_{t=1} = \underbrace{\delta(\tilde{v} - p_2)}_{t=2} \Rightarrow \tilde{v}(p_1, p_2) = p_1 - \delta p_2$$

- Suppose that consumers with $v > \tilde{v}$ have purchased the good in $t = 1$. The residual demand for the good in $t = 2$ is

$$D_2(p_1, p_2) = \tilde{v}(p_1, p_2) - p_2.$$

In $t = 2$, the monopoly chooses p_2 to maximise $p_2 D_2(p_1, p_2)$ and this gives

$$p_2(p1) = \frac{p_1}{2(1+\delta)}$$

The price in the second period is lower than half of the price in the first period.

- in $t = 1$ now, the demand is

$$D_1(p_1, p_2) = 1 - \tilde{v}(p_1, p_2)$$

and the monopoly sets p_1 to maximise its intertemporal profit

$$\Pi_{1,2} = p_1 D_1(p_1, p_2) + \delta p_2 D_2(p_1, p_2)$$

under the constraint that $p_2(p_1) = \frac{p_1}{2(1+\delta)}$. This leads to

$$p_1 = \frac{2(1+\delta)}{(4+\delta)} < \frac{1+\delta}{2}$$

and the profit is:

$$\Pi_{1,2} = \frac{1+\delta}{(4+\delta)} < \Pi$$

The durable good monopolist

-Obtains lower profit in selling over the two periods than only in the first.
-Cannot prevent from competing with itself.

Remember

- A durable good monopolist may compete with itself throughout time
- Some business practices may limit this phenomenon
  - Renting the good instead of selling it! Here renting at price $p_1 = p_2 = \frac{1}{2}$ at each period brings Π.
  - Return policies, money back guarantees or repurchase agreements, ...Contracts that are offered by M to protect the consumers in $t = 1$ against any future price cut.
  - Reputation
  - Technology (capacity constraints, planned obsolescence, new version of the product...)
- If discrete classes of consumers can be identified, intertemporal discrimination can become profitable.

Durable Goods with discrete class of consumers

Assumptions

- A durable good monopoly, M, with a production cost c.
- Two consumers who live two periods $t = \{1; 2\}$. Two consumers buy either 0 or 1 unit. C1 has a valuation 1 and C2 v_l with $c < v_l < 1$.
- δ is the discount factor.
- M sets p_1 in $t = 1$ and p_2 in $t = 2$.
- We proceed in two steps:
  - We determine a benchmark if M only sells in $t = 1$.
  - We then determine the two period equilibrium and make the comparison.

Benchmark if if M only sells in $t = 1$

In a one period game, the problem boils down to a usual discrimination issue: M can choose either to sell only to C1 or to serve both consumers C1 and C2.

- If M sells only to C1, $p = 1 + \delta$ and its profit is $\Pi = 1 + \delta - c$.
- If M sells to C1 and C2 $p = v_l(1 + \delta)$ and its profit is $\Pi = 2(v_l(1 + \delta) - c)$.
- The first option is chosen if $c < v_l < \frac{1}{2}(1 + \frac{c}{1+\delta})$, i.e. a when the two types of consumers are sufficiently different.

The two period equilibrium

- Prices are (p_1, p_2) and profit $\Pi_{1,2}$ of M.
- M is willing to serve C1 in $t = 1$ and C2 in $t = 2$.
  To make sure C1 buys in $t = 1$: $1 + \delta - p_1 > \delta(1 - p_2) \Rightarrow$

$$p_1 < 1 + \delta p_2 \tag{1}$$

- Now, p_2 depends on the behavior of C1 in $t = 1$. If C1 has not purchased the good in $t = 1$,

  1). If $v_l < \frac{1}{2}(1 + c)$, M sets $p_2 = 1$ given the result of the benchmark. Therefore, given (1) M sets $p_1 = 1 + \delta$ and sells to C1. Then, M sets $p_2 = v_l$ and sells to C2.

  M obtains $\Pi_{1,2} = 1 + \delta - c + \delta(v_l - c)$.

2). If $v_l > \frac{1}{2}(1+c)$, M sets $p_2 = v_l$.
Thus, given (1), M sets $p_1 = 1 + \delta v_l$ and sells to C1. Then M sets $p_2 = v_l$ and sells to C2.

M obtains $\Pi_{1,2} = 1 + \delta v_l - c + \delta(v_l - c)$.

Comparison

- If $v_l < \frac{1}{2}(1 + \frac{c}{1+\delta}) < \frac{1+c}{2}$,

$$\Pi = 1 + \delta - c < \Pi_{1,2} = 1 + \delta - c + \delta(v_l - c).$$

  Intertemporal discrimination is profitable!

- The reverse is true when $v_l > \frac{1}{2}(1 + c)$!

$$\Pi = 2(v_l(1 + \delta) - c) > \Pi_{1,2} = 1 + \delta v_l - c + \delta(v_l - c)$$

Search Costs & The Diamond Paradox

Search costs: Consumers might be imperfectly informed about prices

- If getting information is costly, $p_1 = p_2 > c$ can be an equilibrium.
- Diamond Paradox: in a duopoly $p_1 = p_2 = p^M$ might be an equilibrium
  - All consumers are uninformed about prices
  - They have no cost to learn one price and a cost ϵ to learn the second price!
  - For any $p_1 = p_2 = p < p^M$, a firm has an incentive to deviate towards $p + \frac{\epsilon}{2}$!

Search Costs and Temporal Price Dispersion Varian (1980): A model of “sales".

Assumptions

- Monopolistic competition among n symmetric firms with free entry.
- I informed consumers and $U = \frac{M}{n}$ uninformed consumers per store.
- r is the reservation price of consumers.
- $C(q)$ is a firm cost function with strictly decreasing average cost (ex: $cq + f$).
- If a firm sets the lowest price, it obtains $I + U$ consumers.
- If the firm does not set the lowest price, it obtains U consumers.
- If several firms have the identical lowest price, there is a tie, and they share equally I consumers among them.

14/35

There exists no symmetric pure strategy Nash equilibrium

- First, the relevant range of prices is $[p^*, r]$. If$p > r$, there is no demand and if $p < p^* = \frac{C(I+U)}{I+U}$ the firm obtains a negative profit even in the best case, i.e. when serving all consumers.
- If all firms set $p = p^*$, there is a tie and then profits are negative: $p^* x(U + \frac{I}{n}) - C(U + \frac{I}{n}) < 0$.
- If all firms set $p \in]p^*, r]$, a slight price cut by one of the firms enables to capture all informed customers and realize a positive profit.

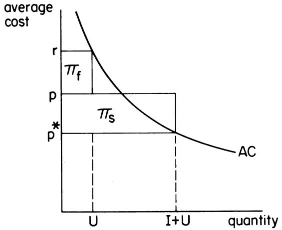
There is a symmetric equilibrium in mixed strategy.

- Each firm randomly chooses a price according to the same density of probability $f(p)$ ($F(p)$ is the distribution function) $\Rightarrow$ Temporal price dispersion arises!

Assume that all firms have the same distribution $F(p)$.

We build the expected profit function for a firm for any price p

- With probability $(1-F(p))^{n-1}$, p is the lowest price and then the firm earns $\pi_s(p) = p(U+I) - C(U+I)$ (Success).
- With probability $1-(1-F(p))^{n-1}$, p is not the lowest price and it obtains $\pi_f(p) = pU - C(U)$.



- The expected profit of the firm therefore is:

$$\int_{p^*}^{r} [\pi_s(p)(1-F(p))^{n-1} + \pi_f(p)(1-(1-F(p))^{n-1})]f(p)dp$$

- Maximizing the above profit with respect to p, the FOC is:

$$\pi_s(p)(1-F(p))^{n-1}+\pi_f(p)(1-(1-F(p))^{n-1})=0$$

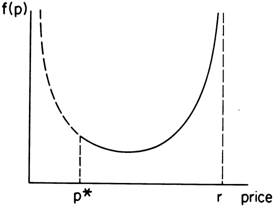
Rearranging, we obtain:

$$F(p)=\begin{cases} 0 & p<p^* \\ 1-\left(\frac{\pi_f(p)}{\pi_f(p)-\pi_s(p)}\right)^{\frac{1}{n-1}} & p\in[p^*,r] \\ 1 & p>r \end{cases}$$

- If firms compete in a market with both informed and uninformed consumers, temporal price dispersion may arise in equilibrium. In equilibrium firms alternate (ramdomly) relatively high prices and periods of sales.

An example with $c(q) = f$

- $\pi_f(r) = rU - f = 0 \Rightarrow U = \frac{f}{r}$
- $\pi_s(p^*) = p^*(I + U) - f = 0 \Rightarrow p^* = \frac{f}{I + \frac{f}{r}}$
- The corresponding $f(p)$ has the following shape:



- Firms tend to charge extreme prices with higher probability.
- Prices are lower as I increases and f is low (more competitive) but high prices are always charged with positive probability.

- This model also applies to competition among stores that have a base of *loyal consumers* and other *consumers that tend to switch among stores* when the store cannot distinguish among these consumers (see Narasimhan, 1988).
    - There is a tension between an incentive to set the monopoly price to loyal consumers and a competitive price for those who may go to the rival → Mixed strategy equilibrium
- These results on temporal price dispersion are robust if consumers can endogenously decide whether they want to acquire additional information through costly search.
- Empirical evidence for search costs - online *vs* offline.

Switching costs

Definition: The presence of switching costs give consumers an incentive to purchase repeatedly from the same supplier.

- *Transaction costs*: Time and effort to change supplier (e.g. changing bank accounts, insurances, telephone company, etc...)
- *Contractual costs* : Mobile phone company that offers a contract with a phone at low price for a 24 month lock-in contract.
- *Shopping costs* : Purchasing several goods from one supplier rather than shopping around for different products.
- *Search costs*
- ...

Imperfect competition and switching costs

Assumptions

- Two-period model with imperfect competition.
- Consumers are uniformly distributed along a Hotelling line $[0,1]$ with a linear transportation cost $-x$ for a distance x. Two firms A and B are located at the extremes.
- **Switching costs**
  - After $t = 1$, a share λ of consumers leaves the market and is replaced by new consumers.
  - The remaining share of consumers $(1-\lambda)$ who has bought from firm $K = A, B$ in $t = 1$ incurs a cost z to switch to the other firm in $t = 2$.
  - Old consumers keep their preference from one period to the next.
- Consumers have a reservation price r such that the market is fully covered.
- Consumers are **myopic**.

Benchmark without switching cost

- Both periods are identical and independent.
- Old and new consumers behave in the same way:
  - A consumer x buys from A in $t = 1, 2$ if:

$$r - x - p_A^t \geq r - (1 - x) - p_B^t \Rightarrow x \geq \tilde{x} = \frac{1}{2}(1 + p_B^t - p_A^t)$$

- In each $t = 1, 2$ firm A (resp. firm B) maximizes :

$$p_A^t \tilde{x} \Rightarrow p_A^t = p_B^t = 1$$

- Equilibrium profits are $\Pi_K^t = \frac{1}{2}$ for each firm.

Competition in $t = 2$

- Assume that in $t = 1$, each firm A and B has obtained respectively a share α and $1 - \alpha$ of the market.
- A fraction $(1 - \lambda)$ of consumers remain
  - A consumer x who bought from A in $t = 1$ buys again from A if:

$$r - x - p_A^2 \geq r - (1 - x) - p_B^2 - z \Rightarrow x \leq \hat{x}_A = \frac{1}{2}(1 + p_B^2 - p_A^2 + z)$$

- A fraction λ are new consumers
  - A new consumer x buys from A in $t = 2$ if:

$$r - x - p_A^2 \geq r - (1 - x) - p_B^2 \Rightarrow x \leq \hat{x} = \frac{1}{2}(1 + p_B^2 - p_A^2)$$

- Assume $\hat{x}_A > \alpha$ (we check *ex post* this condition), the demand is:

$$q_A^2(p_A^1, p_B^1, p_A^2, p_B^2) = (1 - \lambda)\alpha(p_A^1, p_B^1) + \lambda\frac{1}{2}(1 + p_B^2 - p_A^2)$$

- The same reasoning applies for B.

Competition in $t = 2$

The FOC writes as:

$$\frac{\partial \pi_A^2}{\partial p_A^2} = q_A^2 + p_A^2 \frac{\partial q_A^2}{\partial p_A^2} = 0$$

We obtain :

$$p_A^2(p_B^2) = \frac{1-\lambda}{\lambda}\alpha + \frac{1}{2}(1 + p_B^2)$$

- Firms compete more aggressively to gain new costumers: $\frac{\partial p_A^2(p_B^2)}{\partial \lambda} < 0$
- Firms compete less aggressively as the share of "captive consumer" increases: $\frac{\partial p_A^2(p_B^2)}{\partial \alpha} > 0$
- In $t = 2$ equilibrium, $\pi_A^2(\alpha(p_A^1, p_B^1)) = \frac{1}{2\lambda}(1 + \frac{1}{3}(2\alpha - 1)(1-\lambda))^2$ with $\alpha(p_A^1, p_B^1) = \frac{1}{2}(1 + p_B^1 - p_A^1)$.

Competition in $t = 1$

In $t = 1$ firms take into account their intertemporal profit over the two periods.

$$\pi_A(p_A^1, p_B^1) = \pi_A^1(p_A^1, p_B^1) + \pi_A^2(\alpha(p_A^1, p_B^1))$$

The FOC is:

$$\frac{\partial \pi_A(p_A^1, p_B^1)}{\partial p_A^1} = \frac{\partial \pi_A^1(p_A^1, p_B^1)}{\partial p_A^1} + \underbrace{\frac{\partial \pi_A^2(\alpha(p_A^1, p_B^1))}{\partial \alpha}}_{+} \underbrace{\frac{\partial \alpha(p_A^1, p_A^2)}{\partial p_A^1}}_{-} = 0$$

- For $\lambda > \frac{2}{5}$, in equilibrium $\alpha = \frac{1}{2}$, and $p_K^1 = \frac{5\lambda - 2}{3}$ and $p_K^2 = \frac{1}{\lambda}$. For $\lambda \leq \frac{2}{5}$, in equilibrium $\alpha = \frac{1}{2}$, and $p_K^1 = 0$ and $p_K^2 = \frac{1}{\lambda}$. BOUTON
- In the benchmark case without switching costs: $p_K^1 = p_K^2 = 1$.
- In the first period $p_K^1 < 1$ is lower to lock in as much consumers as possible (second period profit effect).
- In the second period though, $p_K^2 > 1$ the equilibrium price is higher because firms compete only for new consumers.

25/35

- In terms of profit, each firm loses in $t = 1$ but earns more in $t = 2$ than absent switching costs.
- In equilibrium the intertemporal profit with switching costs is:

$$\pi_A = \begin{cases} \frac{1}{6}\left(\frac{1}{\lambda} + 5\right) & \text{for } \lambda > \frac{2}{5}, \\ \frac{1}{2\lambda} & \text{for } \lambda < \frac{2}{5} \end{cases}$$

- In equilibrium, the intertemporal profit without switching cost is 1.
- Here firms are always better off when they can lock-in consumers and the effect on consumers surplus is negative.

Endogenous switching cost: Coupons

- **Coupons** are discount offered on the price of the product at the next purchase.
- The oldest "Coupon" by TheCCC

Assumptions

- Consumers redraw their types in $t = 2$.
- In $t = 1$ firms can offer coupons $c_K \geq 0$ to their loyal consumers. In $t = 2$ the consumer will pay $p_A^2 - c_A$ if he buys again from A.
- Consumers are **forward looking**.

Competition in period 2

- A consumer who purchased from A in $t = 1$, buys from A again if its new address x is such that
  $r - x - (p_A^2 - c_A) > r - (1 - x) - p_B^2 \Rightarrow x < \hat{x}_A = \frac{1}{2}(1 + p_B^2 - p_A^2 + c_A)$
- Similarly, consumers who purchased from B in $t = 1$ buys from B again if $x > \hat{x}_B = \frac{1}{2}(1 + p_B^2 - p_A^2 - c_B)$
- We assume that $0 < \hat{x}_B \leq \hat{x}_A < 1$ i.e., that there is switching in equilibrium. (We check *ex post* this condition)

28/35

- In $t = 2$, A sells to consumers who had bought from A in $t = 1$ (α) and do not switch $(x < \hat{x}_A)$, and those who bought from B $(1-\alpha)$ and switch $(x < \hat{x}_B)$.
- The maximization program is:

$$\max_{p_A^2} \alpha \hat{x}_A (p_A^2 - c_A) + (1-\alpha)\hat{x}_B p_A^2$$

- The best reaction function is:

$$p_A^2(p_B^2) = \frac{1}{2}(1 + p_B^2 + 2\alpha c_A - (1-\alpha)c_B)$$

- Conversely, we obtain: $p_B^2(p_A^2) = \frac{1}{2}(1 + p_A^2 - \alpha c_A + 2(1-\alpha)c_B)$
- In equilibrium,

$$p_A^2 = 1 + \alpha c_A, p_B^2 = 1 + (1-\alpha)c_B$$

  Prices paid by switching (resp. loyal) consumers are higher (resp. lower).
- Equilibrium profit in $t = 2$ is: $\pi_A^2 = \frac{1}{2} - \frac{1}{2}\alpha(1-\alpha)c_A(c_A + c_B) < \frac{1}{2}$

Competition in $t = 1$

- In $t = 1$, A maximizes its intertemporal profit:

$$\max_{p_A^1, c_A} p_A^1 \alpha + \pi_A^2(\alpha, c_A)$$

- To determine α we need to find the address of the indifferent consumer. Assuming consumers are forward looking, we compute the difference in consumer's surplus in $t = 1$:

$$\Delta_s^1 = (r - \alpha - p_A^1) - (r - (1 - \alpha) - p_B^1) = 1 - 2\alpha + p_B^1 - p_A^1$$

and the difference in consumer's surplus in $t = 2$:

$$\begin{aligned} \Delta_s^2 &= \int_0^{\hat{x}_A} (r - (p_A^2 - c_A) - x)dx + \int_{\hat{x}_A}^1 (r - p_B^2 - (1 - x))dx \\ &- \int_0^{\hat{x}_B} (r - p_A^2 - x)dx + \int_{\hat{x}_B}^1 (r - (p_B^2 - c_B) - (1 - x))dx \\ &= \frac{1}{4}((c_A + c_B)^2 + 2(c_A - c_B)) - \frac{1}{2}(c_A + c_B)^2 \alpha \end{aligned}$$

Competition in $t = 1$

- $\Delta_s^1 + \Delta_s^2 = 0$ gives:

$$\alpha = \frac{4(1 + p_B^1 - p_A^1) + (c_A + c_B)^2 + 2(c_A - c_B)}{2(4 + (c_A + c_B)^2)}$$

- Deriving the intertemporal profit $\max_{p_A^1, c_A} p_A^1 \alpha + \pi_A^2(\alpha, c_A)$ for A and B and focusing on a symetric equilibrium, we find:

$$c_A = c_B = \frac{2}{3}, p_A^1 = p_B^1 = \frac{13}{9} > 1, p_A^2 = p_B^2 = \frac{4}{3} > 1, \pi_A = \pi_B = \frac{10}{9} > 1.$$

$$\alpha = \frac{1}{2}, \hat{x}_A = \frac{5}{6}, \hat{x}_B = \frac{1}{6}$$

BOUTON

- Without coupons, prices would be equal to 1 in both periods and the intertemporal profit would be 1.
- Prices with coupon are $p_A^2 - c_A = \frac{2}{3} < 1$
- Firms are better off with coupons, it enables them to relax competition and all consumers (except loyal costumers in $t = 2$ who pay $\frac{2}{3}$) pay a higher price.

Exercice : Poaching

Assumptions

- Two firms $k \in \{A, B\}$ are located at the extremes of a Hotelling line and compete during two periods, $t \in \{1, 2\}$. Prices are denoted p_k^t.
- Consumers with a reservation price r uniformly distributed along the line, incur a linear transportation cost $-x$ to travel distance x
- No production cost.

Questions

1. As a benchmark, determine the equilibrium of the two period game (the one shot game is repeated twice: no dynamic effect here!).
2. Firms now observe consumer's identities (and consumers keep their address through time) and set in $t = 2$ personalized prices p_{kA}^2 and p_{kB}^2 for consumers who respectively bought from A and B in $t = 1$.
   - 2.1 Assuming that α is the market share of firm A in $t = 1$, determine the second period equilibrium.
   - 2.2 Consumers are forward looking. Determine the address of the indifferent consumer α in $t = 1$.
   - 2.3 Determine the first period equilibrium prices.

Références

Bulow, J., 1982, “Durable-Goods Monopolists", *Journal of Political Economy*, Vol. 90, No. 2 , pp. 314-332.

Caminal, R., and C. Matutes, (1990), “Endogenous Switching Costs in a duopoly Model", *International Journal of Industrial Organization*, 8, pp 353-373.

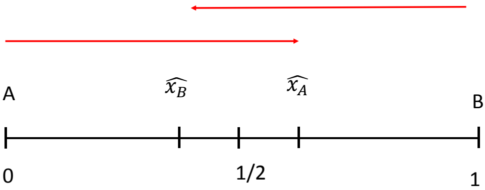
Narasimhan, C., 1988, “Competitive Promotional Strategies", *The Journal of Business*, Vol. 61, pp.427-450.

Varian, H., 1980, “A Model of Sales", *The American Economic Review*, Vol. 70, No. 4, pp. 651-659.

Initial Condition

back

- We check here that, in equilibrium, the initial condition is met, i.e. that all old consumers who have bought from A buy again from A in $t = 2$.
- Formally we had assume that $\hat{x}_A = \frac{1}{2}(1 + z) > \alpha = \frac{1}{2}$.

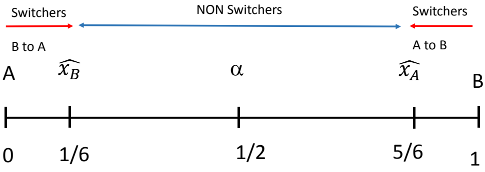


Consumers do not switch.

Initial Condition

back

- We check here that, in equilibrium, the initial condition is met, i.e. that all old consumers who have bought from A buy again from A in $t = 2$.
- Formally we had assume that $\hat{x}_A = \frac{1}{2}(1 + z) > \alpha = \frac{1}{2}$.



Consumers that do not switch.