

# Firms' Strategies and Markets Advertising

Claire Chambolle

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# Exercise 1

## Assumptions

- ▶ Consumers are uniformly distributed along a segment  $[0, 1]$ . A firm is localized in 0 and another firm in 1.
- ▶ A consumer who travels a distance  $x$  to buy one unit at price  $p$  has a utility  $U = v - p - tx$  if he buys and 0 if he does not buy. There is no utility for a second unit.
- ▶ A consumer buys only if he receives an ad. Let  $\Phi_i$  denote the share of consumers who have received an ad from  $i$ . The cost to reach this fraction of demand is  $A(\phi) = \frac{a\phi^2}{2}$  with  $a \geq \frac{t}{2}$ .

## Questions

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1. What is the demand of consumers who receive only an ad from  $i$ ?
  - ▶ The probability to receive an ad only from firm  $i$  is:  $\phi_i(1 - \phi_j)$ .
  - ▶ Consumers who buy are such that  $v - p_i - tx \geq 0$
  - ▶  $D_i = 1$  if  $x_0 = \frac{v-p_i}{t} > 1$  (covered market)!  $\Rightarrow$  We focus on this case for simplicity
  - ▶  $D_i = \frac{v-p_i}{t}$  otherwise (uncovered market).

2. What is the demand of consumers who receive an ad from  $i$  and  $j$ ?
- ▶ The probability to receive an ad from both firms is:  $\phi_i \phi_j$ .
  - ▶ Among them the address of the indifferent consumer  $\tilde{x}$  is such that  $v - p_i - tx = v - p_j - t(1 - x)$  or  $\tilde{x} = \frac{1}{2} + \frac{(p_j - p_i)}{2t}$ .
  - ▶  $\tilde{x}$  (resp.  $1 - \tilde{x}$ ) is the demand for  $i$  (resp.  $j$ ) when the gap in price is not too high.

3. What is the total demand for firm  $i$ ? How the price elasticity of demand varies in  $\phi$  in  $p_i = p_j = p$  and  $\phi_i = \phi_j = \phi$ ?

▶  $D_i = \phi_i[(1 - \phi_j) + \phi_j\tilde{x}]$

▶ At point  $p_i = p_j = p$  and  $\phi_i = \phi_j = \phi$ , the elasticity

$$\epsilon = \frac{-p_i \partial D_i / \partial p_i}{D_i} = \frac{p\phi}{t(2-\phi)} \text{ which increases in } \phi.$$

▶ A larger  $\phi$  implies a larger the probability that consumers are informed of the existence of both goods: They are thus more sensitive to price.

4. Firms choose simultaneously their price and their ad level.  
Determine the symmetric Nash equilibrium of this game.

- The profit of firm  $i$  is:

$$\Pi_i = (p_i - c)D_i - A(\phi_i)$$

- with  $D_i = \phi_i[(1 - \phi_j) + \phi_j \frac{p_i - p_j + t}{2t}] = \frac{\phi_i}{2t} [(1 - \phi_j)2t + \phi_j(p_i - p_j + t)]$

- The first order conditions are :

$$2p_i = c + t + p_j + \frac{2(1 - \phi_j)t}{\phi_j}$$

$$\phi_i = (p_i - c) \frac{(1 - \phi_j + \phi_j \tilde{x})}{a}$$

- At the symmetric equilibrium  $p_i = p_j = p^* = c + \sqrt{2at}$  and  $\tilde{x} = \frac{1}{2}$   
and  $\phi_i = \phi_j = \phi^* = \frac{2}{(1 + \sqrt{2a/t})}$ .