# ECO 650: Firms' Strategies and Markets
# Course 1: Multiproduct firms' pricing strategies

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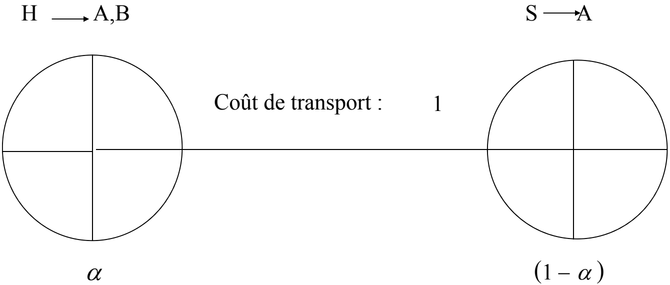
# Exercise 1

- Two stores H (Hypermarket) and S (Supermarket)
- H sells A and B – S sells A
- $\alpha \in [0, \frac{1}{2}]$ consumers are located at H and $1 - \alpha$ in S.
- Transportation cost among the stores is normalized to 1.
- $u_A = 1$ ; $u_B$ uniformly distributed over $[0, 1]$ around each store.
- $b \in [0, 1]$ is the unit cost for B. No cost for A.



1. Which consumers may travel from one store to the other?
2. We note $p^H = p_A^H + p_B^H$ the sum of prices for the two goods at store $H$; $p^S$ the price of A at store $S$.
   Determine the demand at each store.
3. Determine the two candidates Nash equilibria in pure strategy.
4. Assume $b \to 0$ and $\alpha = \frac{1}{9}$; show that the loss-leading equilibrium is the unique Nash equilibrium in pure strategy.
5. How do you explain the emergence of this loss-leading equilibrium?

We note $p^H = p^H_A + p^H_B$ the sum of prices for the two goods at store $H$; $p^S$ the price of A at store $S$.

1. Which consumers may travel from one store to the other?

No consumer in $H$ will travel to $S$ as $u_A = 1$.

In contrast, consumers located in $S$ may choose to travel to $H$ to buy the two goods $A$ and $B$ instead of $A$ alone in $S$, i.e. when:

$$1 + u_B - p^H - 1 > 1 - p^S \Rightarrow u_B > 1 + p^H - p^S$$

2 Determine the demand at each store.

- If $p^H > p^S$, no consumer travels:
  - $D_A^H = \alpha$
  - $D_B^H = \alpha(1 - p_B^H)$
  - $D^S = 1 - \alpha.$

- If $p^H < p^S$, some consumers travel from $S$ to $H$ to buy the two goods :
  - $D_A^H = \alpha + (1 - \alpha)(p^S - p^H)$
  - $D_B^H = \alpha(1 - p_B^H) + (1 - \alpha)(p^S - p^H).$
  - $D^S = (1 - \alpha)(1 + p^H - p^S).$

3 Determine the two candidates Nash equilibria in pure strategy.

- **If $p^H > p^S$**, the profit of H and S can be respectively written as:

$$\Pi^H = p_A^H \alpha + \alpha(1 - p_B^H)(p_B^H - b), \ \Pi^S = (1 - \alpha)p^S$$

Maximizing $\Pi^H$ with respect to $p_A^H$ and $p_B^H$, and $\Pi^S$ with respect to $p^S$, we have $\Pi^H$ strictly increases in $p_A^H$ and $\Pi^S$ strictly increases in $p^S$.

We obtain a local monopoly equilibrium candidate:

$$\hat{p}_A^H = 1, \hat{p}_B^H = \frac{1+b}{2}, \hat{p}^S = 1$$

3 Determine the two candidates Nash equilibria in pure strategy.

- **If $p^H < p^S$**, the profit of H and S can be written as:

$$\Pi^H = (p^H - b)[\alpha + (1-\alpha)(p^S - p^H)] - \alpha p_B^H(p_B^H - b)$$

$$\Pi^S = (1-\alpha)p^S(1 + p^H - p^S)$$

Maximizing $\Pi^H$ with respect to $p^H$ and $p_B^H$, and $\Pi^S$ with respect to $p^S$, we obtain the following best reactions: we obtain $p_B^H = \frac{b}{2} < b$ and $p^H(p^S) = \frac{\alpha+(1-\alpha)p^S}{2(1-\alpha)}$. $p^S(p^H) = \frac{1+p^H}{2}$.

We obtain the following loss-leading equilibrium candidate :

$$p^{H*} = \frac{1+\alpha}{3(1-\alpha)} + \frac{2b}{3}, p_B^{H*} = \frac{b}{2}, p^{S*} = \frac{2-\alpha}{3(1-\alpha)} + \frac{b}{3}$$

4 Assume $b \to 0$ and $\alpha = \frac{1}{9}$; show that the loss-leading equilibrium is the unique Nash equilibrium in pure strategy.

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The equilibrium profit in the loss-leading case is:

$$\Pi^{H*} = \frac{(1+\alpha - b(1-\alpha))^2}{9(1-\alpha)} + \frac{b^2\alpha}{4}, \Pi^{S*} = \frac{(2-\alpha)^2}{9(1-\alpha)} + \frac{b^2(1-\alpha)}{9}$$

In the local monopoly case:

$$\hat{\Pi}^H = \alpha + \frac{(1-b)\alpha}{4}, \hat{\Pi}^S = 1-\alpha$$

Assume $b \to 0$, when $\alpha = \frac{1}{9}$:

- In the loss-leading candidate, $H$ obtains $\Pi^{H*} = \frac{1}{2}.(\frac{5}{9})^2$ and $S$ gets $\Pi^{S*} = \frac{(17)^2}{(9)^2.8} \approx 0.44$.
- In the local monopoly candidate, $H$ obtains $\hat{\Pi}^H = \frac{5}{9}.\frac{1}{4}$ and $S$ gets $\hat{\Pi}^S = \frac{8}{9}$.

Which one is the equilibrium?

4 Show that the loss-leading equilibrium is the unique Nash equilibrium in pure strategy.

- Only $H$ could deviate unilaterally from the loss leading strategy by raising its price to the local monopoly level. No deviation here because $\Pi^{H*} > \hat{\Pi}^{H}$.
- $S$ cannot unilaterally deviate by raising her price as it would remain in the competition situation.

Conversely when $\alpha = \frac{1}{3}$, the deviation becomes profitable.

5. How do you explain the emergence of this loss-leading equilibrium?

The logic under the result here is complementarity.

- A complementarity between the two independent products arises through the transportation cost.
- $H$ has an incentive to sell $B$ below cost because this is the product which has an elastic demand, and therefore lowering this price below cost can attract consumers from $S$.
- If instead $\alpha = \frac{1}{3}$ there is a local monopoly equilibrium. $H$ has no incentive to compete to attract consumers from $S$.

# Exercice 2

Food for life makes health food for active, outdoor people. They sell 3 basics products (Whey powder, high protein Strenght bar, a meal additive(Sawdust))

Consumers fall into two types:

| Consumers | Whey | Strenght | Sawdust |
|---|---|---|---|
| Type A | 10 | 16 | 2 |
| Type B | 3 | 10 | 13 |

**Question**: Each product costs 3 to produce and the bundle of 3 products costs 9. What is the best pricing strategy for the firm? Separate selling, Pure bundling (only bundles of 3 products must be considered)? or mixed bundling?
The firm cannot discriminate among consumers. We assume there is 1 consumer of each type (A and B) and he wants one unit of each product.

# Exercice 2

**Separate selling**: for each product, the firm must choose either to sell the product at high price only to one type of consumers or at a lower price to the two types.

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- **Whey**: (10-3)>2(3-3) $\rightarrow p^W = 10$ and $\pi^W = 7$.
- **Strenght**: (16-3)<2(10-3) $\rightarrow p^{St} = 10$ and $\pi^{St} = 14$.
- **Sawdust**: (13-3)>2(2-3) $\rightarrow p^{Sa} = 13$ and $\pi^{Saw} = 10$.
- Total profit with separate selling strategy is $7 + 14 + 10 = 31$.

| Consumers | Whey | Strenght | Sawdust |
|---|---|---|---|
| Type A | 10 | 16 | 2 |
| Type B | 3 | 10 | 13 |

**Pure bundling**:
Highest price for type A: 28! Highest price for type B: 26!

$$2(26-9) > (28-9)$$

The best price for the bundle is 26 and the profit with a pure bundling strategy is: $34 > 31$

| Consumers | Whey | Strenght | Sawdust |
|---|---|---|---|
| Type A | 10 | 16 | 2 |
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**Mixed bundling**: Highest price for the bundle is 28! Mixed bundling may enable to raise the price of the bundle without loosing entirely type B consumers. The firm sets $p = 28$ and as type A consumers have no surplus, separate prices for each good must be such that:

$$p^W \geq 10, p^{St} \geq 16, p^{Sa} \geq 2.$$

Under this constraint, the best prices the firm can offer are:

$$p^W = 10, p^{St} = 16, p^{Sa} = 13.$$

Type A buys the bundle and Type B only buy Sawdust. Total profit with mixed bundling is

$$(28 - 9) + (13 - 3) = 29 < 34!$$

| Consumers | Whey | Strenght | Sawdust |
|---|---|---|---|
| Type A | 10 | 16 | 2 |
| Type B | 3 | 10 | 13 |

**Authorizing bundles of two products**, we compare all combinations of bundles of two goods and separate pricing and the best strategy is :

- Offer a bundle of Sawdust and Strenght at 23, while offering a price for separate sales $p^W = 10$, $p^{St} = 16$ and $p^{Sa} = 13$.
- Type $B$ buys the bundle only whereas Type $A$ buys Whey and Strenght separately.
- The firms makes: (23-6)+(10-3)+(16-3)=37!