Firms' Strategies and Markets
Advertising

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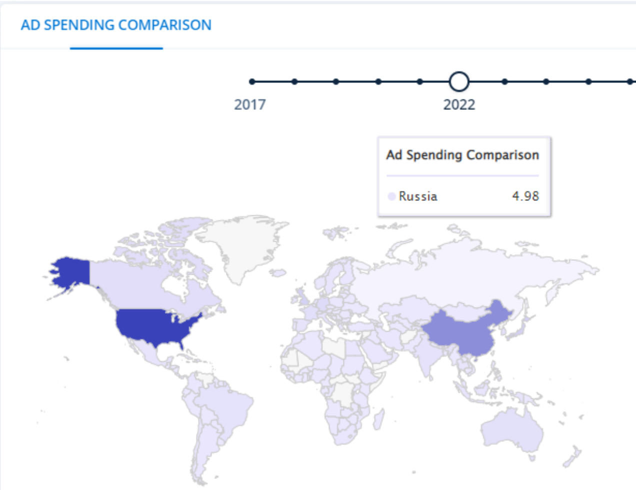
Introduction

- Worlwide amount of ad spending in 2022 is about 781 billlion \$;
- More than 60% of this amount are digital advertising and mobile phone (growing)–the rest are mainly TV and radio ($\approx$ 30%) or print medias (newspapers and magazine $<$5%);
  - Google (alphabet) is the largest digital ad seller in the world;
  - Google and Facebook (Meta) have more than 60% market share of online advertising.
  - CMA report in 2020 / role of consumer data in digital market ads.
- The largest advertisers in 2022 are Amazon, Alibaba, L'Oréal and Procter & Gamble (in $[10, 20]$ billions US \$ in 2022.)

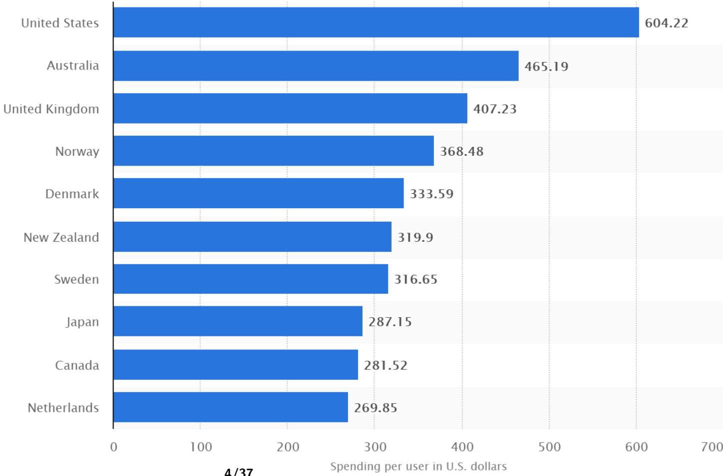
Countries with highest advertising spending in 2022



Top 5 (2022) in billion USD (US$)

1. United States	365.00
2. China	195.80
3. Japan	47.51
4. United Kingdom	45.27
5. Germany	26.46

Countries with highest advertising spending per person in 2016 (US $)



Advertising boosts demand

Assumptions:

- The demand $Q(p, A)$ is such that $Q_p < 0$ and $Q_A > 0$.
- The firm faces a variable cost of production $C(Q)$ with $C_Q > 0$
- The cost of advertising is $-A$.

The monopoly maximizes its profit with respect to Q and A:

$$\max_{Q,A} \Pi(Q, A) = pQ(p, A) - C(Q(p, A)) - A$$

The First Order Conditions are:

$$\Pi_p = (p - C_Q)Q_p + Q = 0 \Rightarrow \frac{p - C_Q}{p} = \frac{-1}{\epsilon_{Q/p}}$$

$$\Pi_A = (p - C_Q)Q_A - 1 = 0 \Rightarrow \frac{p - C_Q}{p} = \frac{1}{\epsilon_{Q/A}} \frac{A}{pQ}$$

Result

The advertising intensity is equal to the ratio of the advertising elasticity of demand and the price elasticity of demand: $\frac{A}{pQ} = \frac{\epsilon_{Q/A}}{-\epsilon_{Q/p}}$: Dorfman-Steiner condition !

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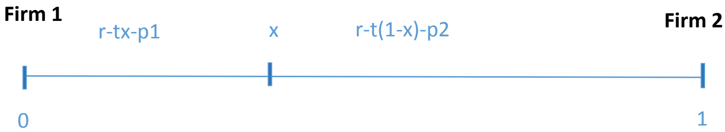
Typology of advertising

- **Persuasive Advertising** enhances consumers' tastes for a given product
  - Advertising increases consumers' willingness to pay.
  - Advertising changes the distribution of consumers' tastes.
  - Advertising increases perceived product difference.
- **Informative Advertising** provides consumers with information about the existence, prices and characteristics of products. Consumers make better informed decision.
  - Information about prices
  - Information about product's existence.
- **Signaling Quality**: the amount of ads spent or the price indirectly convey information about the quality of the products to consumers.

Persuasive Advertising

Assumptions

- Game: Stage 1- Advertising & Stage 2- price competition;
- Consumers are distributed according to $F(x)$ over $[0, 1]$
- The cost of advertising intensity λ_i is $a\lambda_i^2/2$.



- Advertising increases consumers' willingness to pay: $r_i(\lambda_i)$
- Advertising changes the distribution of consumers' tastes: $F(x, \lambda_i, \lambda_j)$
- Advertising increases perceived product difference : $t(\lambda_i, \lambda_j)$

Benchmark: Without advertising

Assumptions

- We assume that there is no advertising.

The indifferent consumer address $\hat{x}$ is such that:

$$r - t\hat{x} - p_1 = r - t(1-\hat{x}) - p_2$$

$$\hat{x} = \frac{1}{2} + \frac{p_2 - p_1}{2t}$$

$$\begin{aligned}\Pi_1 &= (p_1 - c)\hat{x}(p_1, p_2) \\ \Pi_2 &= (p_2 - c)(1 - \hat{x}(p_1, p_2))\end{aligned}$$

Firms maximize their profit with respect to p_i and the reaction functions are symmetric and increasing : Prices are strategic complement!

$$\underset{p_i}{Max}\Pi_i \Rightarrow p_i(p_j) = \frac{1}{2}(c + t + p_j)$$

Results

There is a symmetric equilibrium: $p_1^* = p_2^* = c + t$ and $\Pi_1^* = \Pi_2^* = \frac{t}{2}$.

Advertising increases consumers' willingness to pay

Assumptions

- We denote $r_i(\lambda_i) = r + \beta\lambda_i$

The indifferent consumer address $\hat{x}$ is such that:

$$r + \beta\lambda_1 - t\hat{x} - p_1 = r + \beta\lambda_2 - t(1 - \hat{x}) - p_2$$

$$\hat{x} = \frac{1}{2} + \frac{p_2 - p_1}{2t} + \beta\frac{\lambda_1 - \lambda_2}{2t}$$

$$\begin{aligned} \Pi_1 &= (p_1 - c)\hat{x}(p_1, p_2, \lambda_1, \lambda_2) - a\lambda_1^2/2 \\ \Pi_2 &= (p_2 - c)(1 - \hat{x}(p_1, p_2, \lambda_1, \lambda_2)) - a\lambda_2^2/2 \end{aligned}$$

Firms maximize their profit with respect to p_i and the reaction functions are symmetric and increasing : Prices are strategic complement!

$$\underset{p_i}{Max}\Pi_i \Rightarrow p_i(p_j) = \frac{1}{2}(c + t + p_j + \beta\lambda_i - \beta\lambda_j)$$

The Nash equilibrium in prices is:

$$p_i(\lambda_i, \lambda_j) = c + t + \frac{1}{3}\beta(\lambda_i - \lambda_j)$$

$$\Pi_i(\lambda_i, \lambda_j) = \frac{1}{18t}(3t + \beta(\lambda_i - \lambda_j))^2 - a\lambda_i^2/2$$

In stage 1, each firm i maximizes its profit with respect to λ_i anticipating the stage 2 competition in prices:

$$\underset{\lambda_i}{Max}\Pi_i(\lambda_i, \lambda_j) \Rightarrow \lambda_i(\lambda_j) = \frac{\beta(3t - \beta\lambda_j)}{9at - \beta^2}$$

The best reaction functions are symmetric and decreasing: advertising investments are strategic substitutes!

Results

$\lambda_1^* = \lambda_1^* = \frac{\beta}{3a}$, $p_1^* = p_2^* = c + t$ and $\Pi_1^* = \Pi_2^* = \frac{t}{2} - \frac{\beta^2}{18a} < \frac{t}{2}$. Firms are worse-off with advertising. If they could coordinate, they would refrain from investing.

Advertising changes the distribution of consumers' tastes

Assumptions

- We denote $F(x, \lambda_1, \lambda_2) = (1 + \lambda_1 - \lambda_2)x - (\lambda_1 - \lambda_2)x^2$ with a continuous density $f(x, \lambda_1, \lambda_2) = (1 + \lambda_1 - \lambda_2) - 2x(\lambda_1 - \lambda_2)$.
- If $\lambda_1 = \lambda_2$ we find a uniform distribution, $\lambda_1 = 1$ and $\lambda_2 = 0$ a distribution that favors firm 1. Distribution Function

The address of the indifferent consumer $\hat{x}$ is such that:

$$r - t\hat{x} - p_1 = r - t(1 - \hat{x}) - p_2 \Rightarrow \hat{x} = \frac{1}{2} + \frac{p_2 - p_1}{2t}$$

$$Q_1 = F(\hat{x}, \lambda_1, \lambda_2), Q_2 = 1 - F(\hat{x}, \lambda_1, \lambda_2)$$

$$\Pi_1 = (p_1 - c)Q_1 - a\lambda_1^2/2 \text{ and } \Pi_2 = (p_2 - c)Q_2 - a\lambda_2^2/2$$

Maximizing their profit **simultaneously** with respect to p_i and λ_i, and focusing on the symmetric equilibrium:

Results

$p_1^* = p_2^* = c + t$ and $\lambda_1^* = \lambda_2^* = \frac{t}{4a}$. $\Pi_1^* = \Pi_2^* = \frac{t}{2} - \frac{t^2}{32a} < \frac{t}{2}$. Firms are worse-off with advertising. If they could coordinate, they would refrain from investing.

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Advertising increases perceived product differences

Assumptions Differentiation

- We denote $t(\lambda_1, \lambda_2) = t + \beta\lambda_1 + \beta\lambda_2$.

It is immediate that in stage 2:

$$p_1(\lambda_1, \lambda_2) = p_2(\lambda_1, \lambda_2) = c + t + \beta\lambda_1 + \beta\lambda_2$$

$$\Pi_1 = (p_1 - c)\hat{x} - a\lambda_1^2/2 \text{ and } \Pi_2 = (p_2 - c)(1 - \hat{x}) - a\lambda_2^2/2$$

In stage 1, maximizing their profit with respect to λ_i, and focusing on the symmetric equilibrium:

$$\lambda_1^* = \lambda_2^* = \frac{\beta}{2a} \text{ and } p_1^* = p_2^* = c + t + \frac{\beta^2}{a}$$

$$\Pi_1^* = \Pi_2^* = \frac{t}{2} + \frac{3\beta^2}{8a} > \frac{t}{2}$$

Result

Advertising that increases perceived product difference relaxes competition and therefore firms' investment is profitable.
Public good: coordination raises investment.

Remember

- Advertising creates or boosts the demand for a product.
- In a competition framework: different types of persuasive advertising lead to different outcomes
  - Increasing the consumers' willingness to pay, or changing consumers' taste for a good at the expense of rivals may lead to a business stealing effect and result in an efficient advertising race.
  - Advertising characteristics of the products may increase the perceived differentiation among products and soften competition !
- Heavy regulation of ads – in France:
  - Comparative ads are regulated (not authorized to depreciate/lie the product of a rival)!!
  - Law "Evin" (1991) forbids any ads on tobacco or alcool.
  - Since 2022, debates to ban ads on some products that are bad for environment (high GHG emissions- SUV) or for health (food products listed by PNNS).

Informative advertising on prices

Assumptions

- Consider a duopoly of homogenous products with marginal cost c.
- Consumers do not know the price charged by each firm.
- Consumers have a valuation $v > c$ for the good.
- Consumers have search cost: they can only discover one price (0 for one firm , $+\infty$ for two).

Without advertising on prices : consumers choose between the two firms randomly, check the price and buy if $p < v$. The two firms set $p = v$.

With advertising : Competition is Bertrand like, because the product is homogenous: $p = c$.

Result

Informative advertising on prices may intensify competition by reducing consumers' search costs.

- Argument often put forward in favor of "online" sales.

Informative advertising on product's existence

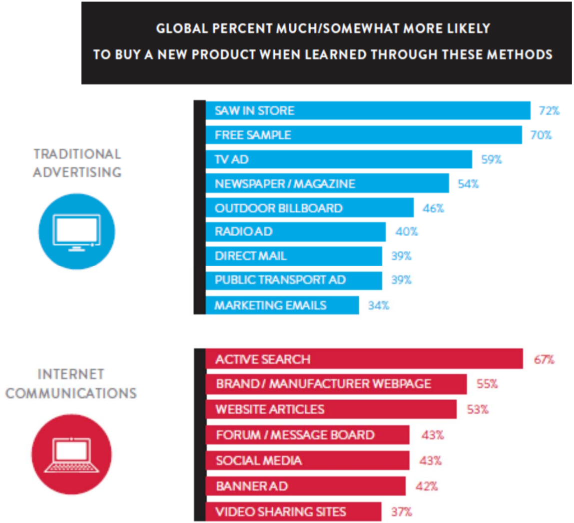
Grossman & Shapiro (1984)

- Consumers unaware of a new product's existence: no utility and no demand.
- Consumers aware of a new product's existence
  - $u(q) > 0$ with $u'(q) > 0$ and $u''(q) < 0$.
  - Maximising $u(q) - pq$ where p is the price, we derive a demand $q(p) > 0$, with $q'(p) < 0$.

Information about the existence of a product

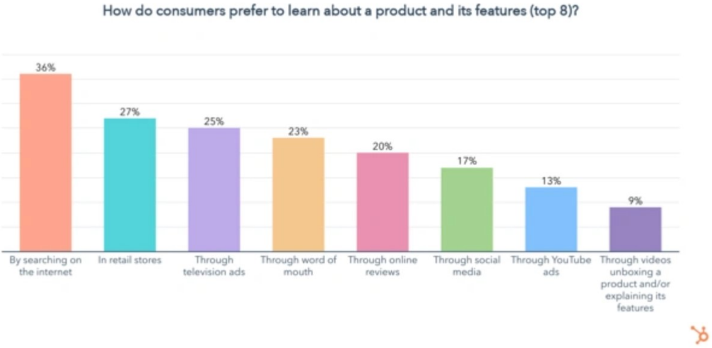
Advertising can inform consumers about the very existence of a product!

Advertising is key to launch a new product



Source: Nielsen Global Survey of New Product Purchase Sentiment, Q3 2012

Advertising is key to launch a new product in 2023



Remember

- In a competition framework: different types of informative advertising lead to different outcomes
  - It might increase competition when it vehicles information on prices.
  - Informative advertising is profitable when it reveals the product's existence (See Exercice 1).

Advertising Signals

Assumptions

- One consumer with a valuation for a high quality good v_H and for the low quality $v_L < v_H$.
- Production cost is the same, $c < v_L$, for a high or a low quality good.
- Two period game. The consumer wants one unit in each period. Experience good!
- Firms can choose to spend an advertising amount A which is observed by the consumer before he chooses to purchase in period 1.

Full Information

Consumers know the quality and thus firms do not advertise.
A high quality firm sets $p_H = v_H$ and gets $\Pi_H = 2(v_H - c)$;
A low quality firm sets $p_L = v_L$ and gets $\Pi_L = 2(v_L - c)$.

Asymmetric Information

We look for a separating equilibrium BOUTON. We assume that only advertising amounts (not price) can convey a signal about quality.

Advertising Signals

Assume that there exists a separating equilibrium such that if a firm spends A in advertising, consumers believe that it is a high quality firm with probability 1.

In such separating equilibrium: $\Pi_H = 2(v_H - c) - A$, and $\Pi_L = 2(v_L - c)$.

Participation constraint

- Check that a high quality firm makes a positive profit i.e. $\Pi_H > 0$, that is $A < 2(v_H - c)$.

Incentive constraints

- Check that a high quality firm is better off advertising! Its deviation profit is $\Pi'_H = v_L + v_H - 2c < \Pi_H \Rightarrow A \leq v_H - v_L$
- Check that a low quality firm is better off not advertising! Its deviation profit is $\Pi'_L = v_H + v_L - 2c - A < \Pi_L \Rightarrow A \geq v_H - v_L$

Advertising Signals

Assume now that if a consumer was cheated in the first period, the firm is boycotted in the next period. The incentive constraint for the low quality firm becomes:

- A low quality firm is better off not advertising! Its deviation profit is $\Pi'_L = v_H - c - A < \Pi_L \Rightarrow A \geq v_H - v_L - (v_L - c)$
- A separating equilibrium exists for $A \in [v_H - v_L - (v_L - c), v_H - v_L]$.
- In equilibrium the high quality firm chooses the minimum advertising amount $A^* = v_H - v_L - (v_L - c)$ and obtains a profit $\Pi^*_H = v_H - c + 2(v_L - c) > \Pi'_H$

Result

Burning money through advertising can be a credible means for a firm to signal a high quality in particular in the case of experience good with repeated purchases.

Price and Advertising signals

Milgrom and Roberts (1986)

Assumptions

- A firm has a new product of quality H or L and knows its quality.
- Consumers do not know the quality.
- Repeated purchase game. Consumers discover the quality after one purchase.
- $\pi(P, q, Q) - A$, expected present value of the profit of a firm where :
  - q true quality;
  - the introductory price is P;
  - the introductory advertising spending A
  - consumers believe the product is of quality Q.
- $\pi(P, q, Q)$ increases in Q (initial sales)

- We define $P_Q^q = \arg\max_P \pi(P, q, Q)$. P_L^L and P_H^H are full information optimal prices.
- We are looking for a SE such that there exists a couple (P, A) that makes consumers believe the quality is H (with proba 1) and L otherwise.

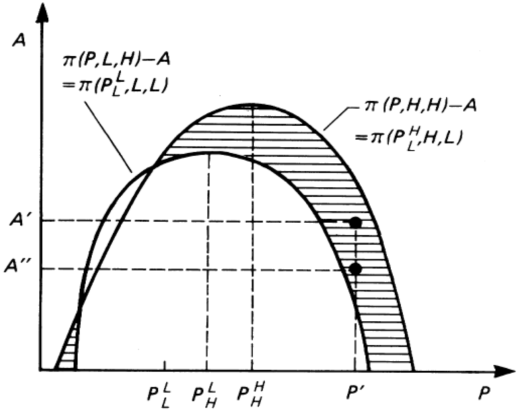
Result 1

There exists a separating sequential equilibrium, such that a high quality firm chooses (P, A) and a low quality firm P_L^L, if and only if for some (P, A):

$$\pi(P, H, H) - \pi(P_L^H, H, L) \geq A \geq \pi(P, L, H) - \pi(P_L^L, L, L) \quad (1)$$

- $\pi(P, H, H) - A \geq \pi(P_L^H, H, L)$: a firm of quality H earns a larger profit in selecting (P, A) which conveys the signal H to consumers than her best profit when consumers believe it is of quality L.
- $\pi(P, L, H) - A \leq \pi(P_L^L, L, L)$: a firm of quality L earns a smaller profit in selecting (P, A) rather than its best profit when consumers believe its quality is L.

- Isoprofit curves:
  - $A(P) = \pi(P, H, H) - \pi(P_L^H, H, L)$ (Above)
  - $A(P) = \pi(P, L, H) - \pi(P_L^L, L, L)$ (Below)



- Elimination of equilibria with dominated strategies.

Result 2

There exists a separating equilibrium if and only if there is some (P, A) such that eq(1) holds. At any separating equilibrium, the choice (P, A) of the high-quality firm must be a solution to the following programme (2):

$$\underset{P,A}{max} \pi(P, H, H) - A$$

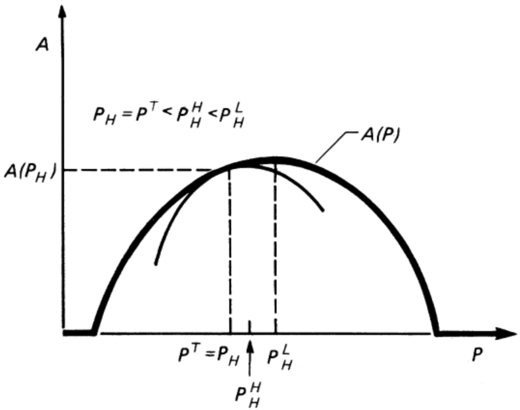
$$\texttt{s.t} \ \ \pi(P, L, H) - A = \pi(P_L^L, L, L).$$

If the solution (P^*, A^*) to (2) is such that $A^* > 0$, then P^* solves

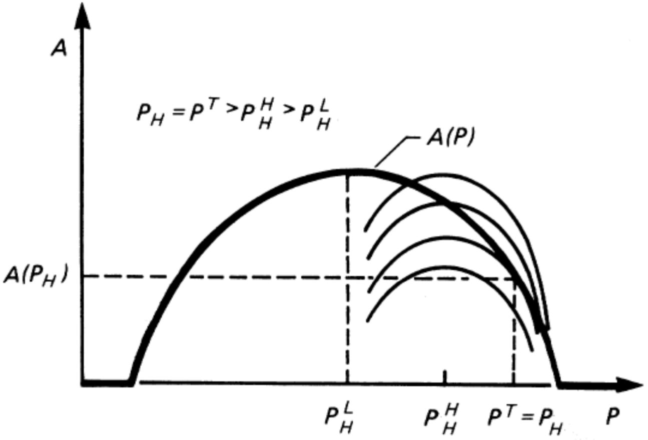
$$\underset{P}{max} \pi(P, H, H) - \pi(P, L, H)$$

$$\Rightarrow \frac{\partial \pi(P, H, H)}{\partial P} = \frac{\partial \pi(P, L, H)}{\partial P}$$

- Assume $\pi(P, H, H) - \pi(P, L, H)$ has a maximum in P.
- $A(P) = \pi(P, L, H) - \pi(P_L^L, L, L)$
- The other curve is $\pi(P, H, H) - A$
- The separating equilibrium is at the tangency point (P^T, A^T).



- The separating equilibrium is at the tangency point (P^T, A^T).
- In the case below there is an upward distortion in price $P^T > P_H^H$



Assume that $\pi(P, L, H)$ is strictly concave in P and that $A(P)$ is positive on an interval $(\underline{P}, \overline{P})$ with $P > 0$.

- A necessary condition for advertising to occur at equilibrium is $P_H^H \in (\underline{P}, \overline{P})$ or, equivalently,

$$\pi(P_H^H, L, H) > \pi(P_L^L, L, L)$$

This condition says that an L would willingly set its price at P_H^H if

doing it could change its perceived quality from L to H.

  - **Case in which** $P_H^H > \overline{P}$: If a new high-quality product is very expensive to produce and is aimed at a limited market.
  - **Case in which** $P_H^H < \underline{P}$: If the new high-quality product is very cheap to produce the introducing firm may set a low initial price or give away free samples in launching the product.

Remember

- Burning money, i.e. a high level of advertising may signal a high quality
- Together with advertising, a high price (ie. higher than the high quality monopoly) may signal a high quality: it claims that the producer is confident enough in its product quality
- Together with advertising, a low price may signal a high quality (i.e lower than the high quality monopoly price): it claims that consumers that will taste it won't be disappointed.

Exercise 1

Assumptions

- Consumers are uniformly distributed along a segment $[0, 1]$. A firm is localized in 0 and another firm in 1.
- A consumer who travels a distance x to buy one unit at price p has a utility $U = v - p - tx$ if he buys and 0 if he does not buy. There is no utility for a second unit.
- A consumer buys only if he receives an ad. Let Φ_i denote the share of consumers who have received an ad from i. The cost to reach this fraction of demand is $A(\phi) = \frac{a\phi^2}{2}$ with $a \geq \frac{t}{2}$.

Questions

1. What is the demand of consumers who receive only an ad from i?
2. What is the demand of consumers who receive an ad from i and j?
3. What is the total demand for firm i? How the price elasticity of demand varies in ϕ in $p_i = p_j = p$ and $\phi_i = \phi_j = \phi$?
4. Firms choose simultaneously their price and their ad level. Determine the symmetric Nash equilibrium of this game.

Exercise 2

Advertising as a commitment device (Lal and Matutes, 1994)

Assumption

- Firms A and B are located at the extreme of a segment of lenght 1.
- Consumers are uniformly distributed along the segment and incur linear transport cost tx.
- A and B sell two products 1 and 2.
- Consumers have the same willingness to pay for each good, denoted H.
- Unless they receive an ad (catalog, leaflet,...), consumers are uninformed about prices but make rational expectations about prices.
- Each firm can choose to advertise one or two goods. Advertising costs F and vehicles the information about a product's price to all consumers.
- **We exclude that a consumer visit both stores**. this is a symplifying assumption and in the paper they look at all cases!

Exercise 2

Questions

1. What happens if no firm advertise any product?
2. What happens if the two firms advertise both products? Is this an equilibrium?
3. Determine the two types of equilibria of this game. For which conditions on H and F do these equilibria exist?

References

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Signaling Game

- Player 1 has a private information about his type $\theta \in \Theta$ and chooses a signal $s \in S$.
- Player 2 observes s and chooses an action $b \in B$.
- Player 2 has prior belief about Player 1's type p(.). After observing s, Player 2 revises its beliefs according to the Baye's rule and has a posterior belief $\mu(./s)$ over Θ.
- Player 1 determines $\sigma_1(s/\theta)$, the probability to send a signal s when being of type θ.
- Player 2 determines $\sigma_2(b/s)$, the probability to choose the action b given the signal s and posterior belief $\mu(./s)$.

Definition . A perfect Bayesian equilibrium of a signaling game is a strategy profile (σ_1^*, σ_2^*) in which each player's strategy is the best reaction to the other's strategy according to the posterior beliefs $\mu(./s)$.

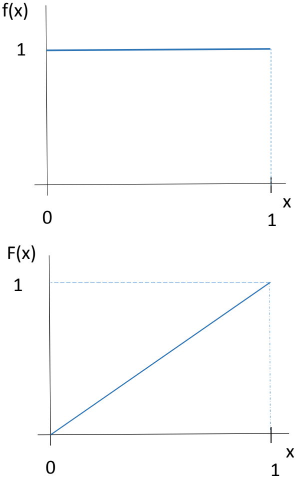
A perfect Bayesian equilibrium is such that given a set of receiver's beliefs about the sender's type, the receiver chooses the action that is a best reaction to the message received and the sender chooses a message that is a best reaction to the action of the receiver. back

Types of equilibria

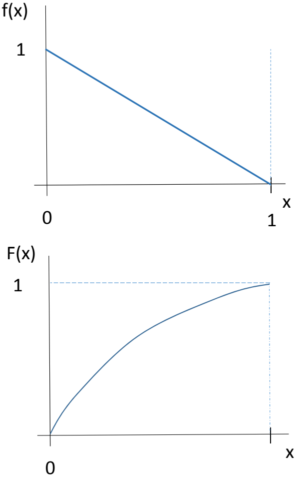
A **separating equilibrium** is an equilibrium where Players 1 of different types always choose different messages and therefore fully reveal their type to Player 2.

A **pooling equilibrium** is an equilibrium where Players 1 of different types always choose the same message and no information is revealed to Player 2.

Uniform distribution: $\lambda_1 = \lambda_2$



Distribution in favor of 1: $\lambda_1 = \lambda_2$



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	Colgate	P&G CREST
Help reduce Cavities	★★★	★★★
Help brush away Plaque	★★	★
Prevent Gingivitis	★	★★
White teeth	★★	★
Fresh feeling breath	★	★★