## ECO 650: Final Exam 2022

December, 7th 2022

## 1 Exercice 1: Innovation - 7 pts

Suppose that two firms $i=1,2$ each consider incuring a fixed cost $f$ to establish a research division, in the hope of finding a new product. If created, a research division has a probability $\lambda$ to succeed. If one firm succeeds, it obtains the monopoly profit $\Pi^{m}$ on the product market. If both firms find a new product, they will obtain the duopoly profit $\Pi^{d}$.

1. Assume that only one research division is created. What is its expected gain?

$$
\lambda \Pi^{m}-f
$$

2. Assume now that two research divisions were created. What is their expected gains?

$$
\lambda(1-\lambda) \Pi^{m}+\lambda^{2} \Pi^{d}-f
$$

3. Determine the level $\hat{f}$ below which there exists a Nash equilibrium in which both research divisions are created.

$$
f \leq \hat{f}=\lambda(1-\lambda) \Pi^{m}+\lambda^{2} \Pi^{d}
$$

is a sufficient condition to ensure that the unique Nash equilibrium is such that the two research divisions are created.
4. Determine the level $f^{*}$ below which it is optimal for the industry to have both research divisions created.

$$
f \leq f^{*}=\lambda \Pi^{m}-2 \lambda^{2}\left(\Pi^{m}-\Pi^{d}\right)
$$

5. Compare $\hat{f}$ and $f^{*}$ and comment.
$f^{*} \leq \hat{f}$ which implies that in the interval $\left.] f^{*}, \hat{f}\right]$, it would be socially optimal to have only one research labs created but two are created. Each firm does not take into account the negative externality it generates on the other.

## 2 Bundling (6 pts)

Two consumers A and B have the following valuations for Sport tickets:
On an annual basis, SPORT 24 offer annual supscription for basketball and Tennis games. Each game costs 5 euros to the Company. Sport 24 cannot discriminate among consumers. To simplify, consider that there is 1 consumer of each type (A and B).

## Questions:

1. Determine the best pricing strategy for SPORT 24 if it offers an annual card fee per sport type? $p_{B}=70 \pi_{B}=(70-25) \cdot 2=90 p_{T}=40 \pi_{B}=(40-25) \cdot 2=30$ Total profit is 120 .
2. Determine the optimal price for SPORT 24 if it offers only a Gold card membership (Full access to all games- pure bundling) $? p_{b}=110$ and $\Pi_{b}=2 .(110-50)=120$. bundling is not profitable.
3. Consumers now have the following valuations:
4. Answer to the same questions (1) and (2). $p_{B}=90, \pi_{B}=(90-25)=55$ and $P_{T}=50$ with $\Pi_{T}=(50-25) \cdot 2=50$. Total profit is 105 and $p_{b}=(110-50)=120$.
5. In which case bundling is the most profitable? Explain. Bundling is profitable in the second case because consumer's valuation are negatively correlated.

## 3 Vertical Relations (7 pts.)

Assume there is one upstream firm $U$ that relies on one downstream firm $D$ to sell its product to consumers. The unit cost of the product is normalized to 0 . Consumers' demand is given by $q=a-p$, where $a>0$ is a parameter, $q$ is the quantity demanded, and $p$ is the final
price charged to consumers. Assume that $D$ can also buy the product at cost $c \in[0, a]$ from a competitive fringe.

## Questions:

1. Assume that $U$ and $D$ have signed a two-part tariff contract $(w, F)$. Determine the equilibrium profits of firms $U$ and $D . \Pi_{D}=\frac{(a-w)^{2}}{4}-F, \Pi_{U}=\frac{w(a-w)}{2}+F$.
2. Assume now that, anticipating the profit fonctions determined in 1$), U$ and $D$ bargain (with equal power) over the contract $(w, F)$. Determine the equilibrium contract, price and profits. (3 pts.)

$$
\begin{gathered}
\operatorname{Max}\left(\frac{(a-w)^{2}}{4}-F-\frac{(a-c)^{2}}{4}\right)\left(\frac{w(a-w)}{2}+F\right) \\
w=0 \\
F=\frac{a^{2}}{8}-\frac{(a-c)^{2}}{8} \\
\Pi_{U}=\frac{a^{2}}{8}-\frac{(a-c)^{2}}{8}
\end{gathered}
$$

and

$$
\Pi_{D}=\frac{a^{2}}{8}+\frac{(a-c)^{2}}{8}
$$

3. What is the impact of $c \in[0, a]$ on the profit sharing. Comment. When $c=0$, the retailer gets all the profit, because the producer has no value compared to the competitive fringe and thus no bargaining power. When $c$ increases, the profit of the retailer is reduces and that of the producer increases. When $c=a$, all the profit is in the hand of the manufacturer.
