# Firms' Strategies and Markets Course 4: Dynamic Pricing 

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## Dynamic Pricing

- Repeated interactions among firms may enable collusive strategies (IO class M1)
- High prices over time.
- Reputation or Signaling strategies can occur (Class / Advertising \& Entry)
- Either a low or a high price can signal a high quality to an uninformed consumer in a first period.
- Fighting on one market can create the reputation of being tough.
- We focus here on "consumer inertia" which may come from different sources and imply various firm's dynamic pricing strategies.
- Durable Goods
- Search costs $\rightarrow$ generate temporal price dispersion.
- Switching costs $\rightarrow$ Consumers are locked-in within the same firm

Durable goods: Goods that are not consumed or destroyed in use; Consumers derive the benefit of their purchase for a period of time (several years).

- Cars, Washing Machines, Computers, Smartphones ...

Insights: A durable good monopoly who cannot discriminate in a given period among heterogenous consumers can use intertemporal discrimination to extract more surplus from consumers.

- Some consumers buy in the first period;
- Others delay their purchase expecting a lower price.


## Assumptions

- A durable monopoly with a production cost 0 .
- A continuum of heterogenous consumers live two periods $t=\{1,2\}$. Consumers buy either 0 or 1 unit and their valuation for the good $v$ is uniformly distributed over $[0,1]$.
- $\delta$ is the discount factor.
- The monopoly sets $p_{1}$ in $t=1$ and $p_{2}$ in $t=2$.

Consider first the benchmark case in which the monopoly can sell only in $t=1$ at price $p$.

- A consumer is willing to purchase the good if $(1+\delta) v-p>0$ in $t=1$. The demand is $D(p)=1-\frac{p}{1+\delta}$.
$-\max _{p} p\left(1-\frac{p}{1+\delta}\right) \Leftrightarrow p=\frac{1+\delta}{2}$.
- The corresponding profit $\Pi=\frac{1+\delta}{4}$.


## Consider now the two period pricing strategy

- For a given couple of prices $\left(p_{1}, p_{2}\right)$, we determine the consumer indifferent between purchasing in $t=1$ and in $t=2$.

$$
\underbrace{(1+\delta) \tilde{v}-p_{1}}_{t=1}=\underbrace{\delta\left(\tilde{v}-p_{2}\right)}_{t=2} \Rightarrow \tilde{v}\left(p_{1}, p_{2}\right)=p_{1}-\delta p_{2}
$$

- Suppose that consumers with $v>\tilde{v}$ have purchased the good in $t=1$. The residual demand for the good in $t=2$ is

$$
D_{2}\left(p_{1}, p_{2}\right)=\tilde{v}\left(p_{1}, p_{2}\right)-p_{2} .
$$

In $t=2$, the monopoly chooses $p_{2}$ to maximise $p_{2} D_{2}\left(p_{1}, p_{2}\right)$ and this gives

$$
p_{2}(p 1)=\frac{p_{1}}{2(1+\delta)}
$$

The price in the second period is lower than half of the price in the first period.

- in $t=1$ now, the demand is

$$
D_{1}\left(p_{1}, p_{2}\right)=1-\tilde{v}\left(p_{1}, p_{2}\right)
$$

and the monopoly sets $p_{1}$ to maximise its intertemporal profit

$$
\Pi_{1,2}=p_{1} D_{1}\left(p_{1}, p_{2}\right)+\delta p_{2} D_{2}\left(p_{1}, p_{2}\right)
$$

under the constraint that $p_{2}\left(p_{1}\right)=\frac{p_{1}}{2(1+\delta)}$. This leads to

$$
p_{1}=\frac{2(1+\delta)}{(4+\delta)}<\frac{1+\delta}{2}
$$

and the profit is:

$$
\Pi_{1,2}=\frac{1+\delta}{(4+\delta)}<\Pi
$$

The durable good monopolist
-Obtains lower profit in selling over the two periods than only in the first.
-Cannot prevent from competing with itself.

## Remember

- A durable good monopolist may compete with itself throughout time
- Some business practices may limit this phenomenon
- Renting the good instead of selling it! Here renting at price $p_{1}=p_{2}=\frac{1}{2}$ at each period brings $\Pi$.
- Return policies, money back guarantees or repurchase agreements, ...Contracts that are offered by $M$ to protect the consumers in $t=1$ against any future price cut.
- Reputation
- Technology (capacity constraints, planned obsolescence, new version of the product...)
- If discrete classes of consumers can be identified, intertemporal discrimination can become profitable.


## Durable Goods with discrete class of consumers

## Assumptions

- A durable good monopoly, M , with a production cost $c$.
- Two consumers who live two periods $t=\{1 ; 2\}$. Two consumers buy either 0 or 1 unit. C1 has a valuation 1 and $\mathrm{C} 2 v_{l}$ with $c<v_{l}<1$.
- $\delta$ is the discount factor.
- M sets $p_{1}$ in $t=1$ and $p_{2}$ in $t=2$.
- We proceed in two steps:
- We determine a benchmark if M only sells in $t=1$.
- We then determine the two period equilibrium and make the comparison.


## Benchmark if if M only sells in $t=1$

In a one period game, the problem boils down to a usual discrimination issue: M can choose either to sell only to C 1 or to serve both consumers C1 and C2.

- If M sells only to $\mathrm{C} 1, p=1+\delta$ and its profit is $\Pi=1+\delta-c$.
- If $M$ sells to $C 1$ and $C 2 p=v_{l}(1+\delta)$ and its profit is $\Pi=2\left(v_{l}(1+\delta)-c\right)$.
- The first option is chosen if $c<v_{l}<\frac{1}{2}\left(1+\frac{c}{1+\delta}\right)$, i.e. a when the two types of consumers are sufficiently different.


## The two period equilibrium

- Prices are $\left(p_{1}, p_{2}\right)$ and profit $\Pi_{1,2}$ of $M$.
- M is willing to serve C 1 in $t=1$ and C 2 in $t=2$.

To make sure C 1 buys in $t=1: 1+\delta-p_{1}>\delta\left(1-p_{2}\right) \Rightarrow$

$$
\begin{equation*}
p_{1}<1+\delta p_{2} \tag{1}
\end{equation*}
$$

- Now, $p_{2}$ depends on the behavior of C 1 in $t=1$. If C 1 has not purchased the good in $t=1$,
1). If $v_{l}<\frac{1}{2}(1+c), M$ sets $p_{2}=1$ given the result of the benchmark.

Therefore, given (1) M sets $p_{1}=1+\delta$ and sells to C 1 . Then, M sets $p_{2}=v_{1}$ and sells to C2.
$M$ obtains $\Pi_{1,2}=1+\delta-c+\delta\left(v_{l}-c\right)$.
2). If $v_{1}>\frac{1}{2}(1+c), M$ sets $p_{2}=v_{1}$.

Thus, given (1), $M$ sets $p_{1}=1+\delta v_{l}$ and sells to $C 1$. Then $M$ sets $p_{2}=v_{l}$ and sells to C2.
$M$ obtains $\Pi_{1,2}=1+\delta v_{l}-c+\delta\left(v_{l}-c\right)$.

## Comparison

- If $v_{l}<\frac{1}{2}\left(1+\frac{c}{1+\delta}\right)<\frac{1+c}{2}$,

$$
\Pi=1+\delta-c<\Pi_{1,2}=1+\delta-c+\delta\left(v_{l}-c\right) .
$$

Intertemporal discrimination is profitable!

- The reverse is true when $v_{l}>\frac{1}{2}(1+c)$ !

$$
\Pi=2\left(v_{l}(1+\delta)-c\right)>\Pi_{1,2}=1+\delta v_{l}-c+\delta\left(v_{l}-c\right)
$$

## Search Costs \& The Diamond Paradox

Search costs: Consumers might be imperfectly informed about prices

- If getting information is costly, $p_{1}=p_{2}>c$ can be an equilibrium.
- Diamond Paradox: in a duopoly $p_{1}=p_{2}=p^{M}$ might be an equilibrium
- All consumers are uninformed about prices
- They have no cost to learn one price and a cost $\epsilon$ to learn the second price!
- For any $p_{1}=p_{2}=p<p^{M}$, a firm has an incentive to deviate towards $p+\frac{\epsilon}{2}$ !


## Search Costs and Temporal Price Dispersion Varian (1980): A model of "sales".

## Assumptions

- Monopolistic competition among $n$ symmetric firms with free entry.
- I informed consumers and $U=\frac{M}{n}$ uninformed consumers per store.
- $r$ is the reservation price of consumers.
- $C(q)$ is a firm cost function with strictly decreasing average cost (ex: $c q+f$ ).
- If a firm sets the lowest price, it obtains $I+U$ consumers.
- If the firm does not set the lowest price, it obtains $U$ consumers.
- If several firms have the identical lowest price, there is a tie, and they share equally I consumers among them.


## There exists no symmetric pure strategy Nash equilibrium

- First, the relevant range of prices is $\left[p^{*}, r\right]$. If $p>r$, there is no demand and if $p<p^{*}=\frac{C(I+U)}{l+U}$ the firm obtains a negative profit even in the best case, i.e. when serving all consumers.
- If all firms set $p=p^{*}$, there is a tie and then profits are negative: $p^{*} x\left(U+\frac{l}{n}\right)-C\left(U+\frac{l}{n}\right)<0$.
- If all firms set $\left.p \in] p^{*}, r\right]$, a slight price cut by one of the firms enables to capture all informed customers and realize a positive profit.
There is a symmetric equilibrium in mixed strategy.
- Each firm randomly chooses a price according to the same density of probability $f(p)(F(p)$ is the distribution function $) \Rightarrow$ Temporal price dispersion arises!

Assume that all firms have the same distribution $F(p)$.
We build the expected profit function for a firm for any price $p$

- With probability $(1-F(p))^{n-1}, p$ is the lowest price and then the firm earns $\pi_{s}(p)=p(U+I)-C(U+I)$ (Success).
- With probability $1-(1-F(p))^{n-1}, p$ is not the lowest price and it obtains $\pi_{f}(p)=p U-C(U)$.

- The expected profit of the firm therefore is:

$$
\int_{p^{*}}^{r}\left[\pi_{s}(p)(1-F(p))^{n-1}+\pi_{f}(p)\left(1-(1-F(p))^{n-1}\right)\right] f(p) d p
$$

- Maximizing the above profit with respect to $p$, the FOC is:

$$
\pi_{s}(p)(1-F(p))^{n-1}+\pi_{f}(p)\left(1-(1-F(p))^{n-1}\right)=0
$$

Rearranging, we obtain:

$$
F(p)=\left\{\begin{array}{cc}
0 & p<p^{*} \\
1-\left(\frac{\pi_{f}(p)}{\pi_{f}(p)-\pi_{s}(p)}\right)^{\frac{1}{n-1}} & p \in\left[p^{*}, r\right] \\
1 & p>r
\end{array}\right.
$$

- If firms compete in a market with both informed and uninformed consumers, temporal price dispersion may arise in equilibrium. In equilibrium firms alternate (ramdomly) relatively high prices and periods of sales.


## An example with $c(q)=f$

- $\pi_{f}(r)=r U-f=0 \Rightarrow U=\frac{f}{r}$
- $\pi_{s}\left(p^{*}\right)=p^{*}(I+U)-f=0 \Rightarrow p^{*}=\frac{f}{l+\frac{f}{r}}$
- The corresponding $f(p)$ has the following shape:

- Firms tend to charge extreme prices with higher probability.
- Prices are lower as $I$ increases and $f$ is low (more competitive) but high prices are always charged with positive probability.
- This model also applies to competition among stores that have a base of loyal consumers and other consumers that tend to switch among stores when the store cannot distinguish among these consumers (see Narasimhan, 1988).
- There is a tension between an incentive to set the monopoly price to loyal consumers and a competitive price for those who may go to the rival $\rightarrow$ Mixed strategy equilibrium
- These results on temporal price dispersion are robust if consumers can endogenously decide whether they want to acquire additional information through costly search.
- Empirical evidence for search costs - online vs offline.


## Switching costs

Definition: The presence of switching costs give consumers an incentive to purchase repeatedly from the same supplier.

- Transaction costs: Time and effort to change supplier (e.g. changing bank accounts, insurances, telephone company, etc...)
- Contractual costs : Mobile phone company that offers a contract with a phone at low price for a 24 month lock-in contract.
- Shopping costs : Purchasing several goods from one supplier rather than shopping around for different products.
- Search costs
- ...


## Imperfect competition and switching costs

## Assumptions

- Two-period model with imperfect competition.
- Consumers are uniformly distributed along a Hotelling line $[0,1]$ with a linear transportation cost $-x$ for a distance $x$. Two firms $A$ and $B$ are located at the extremes.
- Switching costs
- After $t=1$, a share $\lambda$ of consumers leaves the market and is replaced by new consumers.
- The remaining share of consumers $(1-\lambda)$ who has bought from firm $K=A, B$ in $t=1$ incurs a cost $z$ to switch to the other firm in $t=2$.
- Old consumers keep their preference from one period to the next.
- Consumers have a reservation price $r$ such that the market is fully covered.
- Consumers are myopic.


## Benchmark without switching cost

- Both periods are identical and independent.
- Old and new consumers behave in the same way:
- A consumer $x$ buys from $A$ in $t=1,2$ if:

$$
r-x-p_{A}^{t} \geq r-(1-x)-p_{B}^{t} \Rightarrow x \geq \tilde{x}=\frac{1}{2}\left(1+p_{B}^{t}-p_{A}^{t}\right)
$$

- In each $t=1,2$ firm $A$ (resp. firm $B$ ) maximizes:

$$
p_{A}^{t} \tilde{x} \Rightarrow p_{A}^{t}=p_{B}^{t}=1
$$

- Equilibrium profits are $\Pi_{K}^{t}=\frac{1}{2}$ for each firm.


## Competition in $t=2$

- Assume that in $t=1$, each firm $A$ and $B$ has obtained respectively a share $\alpha$ and $1-\alpha$ of the market.
- A fraction $(1-\lambda)$ of consumers remain
- A consumer $x$ who bought from $A$ in $t=1$ buys again from $A$ if:

$$
r-x-p_{A}^{2} \geq r-(1-x)-p_{B}^{2}-z \Rightarrow x \leq \hat{x}_{A}=\frac{1}{2}\left(1+p_{B}^{2}-p_{A}^{2}+z\right)
$$

- A fraction $\lambda$ are new consumers
- A new consumer $x$ buys from $A$ in $t=2$ if:

$$
r-x-p_{A}^{2} \geq r-(1-x)-p_{B}^{2} \Rightarrow x \leq \hat{x}=\frac{1}{2}\left(1+p_{B}^{2}-p_{A}^{2}\right)
$$

- Assume $\hat{x}_{A}>\alpha$ (we check ex post this condition), the demand is:

$$
q_{A}^{2}\left(p_{A}^{1}, p_{B}^{1}, p_{A}^{2}, p_{B}^{2}\right)=(1-\lambda) \alpha\left(p_{A}^{1}, p_{B}^{1}\right)+\lambda \frac{1}{2}\left(1+p_{B}^{2}-p_{A}^{2}\right)
$$

- The same reasoning applies for $B$.


## Competition in $t=2$

The FOC writes as:

$$
\frac{\partial \pi_{A}^{2}}{\partial p_{A}^{2}}=q_{A}^{2}+p_{A}^{2} \frac{\partial q_{A}^{2}}{\partial p_{A}^{2}}=0
$$

We obtain :

$$
p_{A}^{2}\left(p_{B}^{2}\right)=\frac{1-\lambda}{\lambda} \alpha+\frac{1}{2}\left(1+p_{B}^{2}\right)
$$

- Firms compete more aggressively to gain new costumers:

$$
\frac{\partial p_{A}^{2}\left(p_{B}^{2}\right)}{\partial \lambda}<0
$$

- Firms compete less aggressively as the share of "captive consumer" increases: $\frac{\partial p_{A}^{2}\left(p_{B}^{2}\right)}{\partial \alpha}>0$
- In $t=2$ equilibrium, $\pi_{A}^{2}\left(\alpha\left(p_{A}^{1}, p_{B}^{1}\right)\right)=\frac{1}{2 \lambda}\left(1+\frac{1}{3}(2 \alpha-1)(1-\lambda)\right)^{2}$ with $\alpha\left(p_{A}^{1}, p_{B}^{1}\right)=\frac{1}{2}\left(1+p_{B}^{1}-p_{A}^{1}\right)$.


## Competition in $t=1$

In $t=1$ firms take into account their intertemporal profit over the two periods.

$$
\pi_{A}\left(p_{A}^{1}, p_{B}^{1}\right)=\pi_{A}^{1}\left(p_{A}^{1}, p_{B}^{1}\right)+\pi_{A}^{2}\left(\alpha\left(p_{A}^{1}, p_{B}^{1}\right)\right)
$$

The FOC is:

$$
\frac{\partial \pi_{A}\left(p_{A}^{1}, p_{B}^{1}\right)}{\partial p_{A}^{1}}=\frac{\partial \pi_{A}^{1}\left(p_{A}^{1}, p_{B}^{1}\right)}{\partial p_{A}^{1}}+\underbrace{\frac{\partial \pi_{A}^{2}\left(\alpha\left(p_{A}^{1}, p_{B}^{1}\right)\right)}{\partial \alpha}}_{+} \underbrace{\frac{\partial \alpha\left(p_{A}^{1}, p_{A}^{2}\right)}{\partial p_{A}^{1}}}_{-}=0
$$

- For $\lambda>\frac{2}{5}$, in equilibrium $\alpha=\frac{1}{2}$, and $p_{K}^{1}=\frac{5 \lambda-2}{3}$ and $p_{K}^{2}=\frac{1}{\lambda}$. For $\lambda \leq \frac{2}{5}$, in equilibrium $\alpha=\frac{1}{2}$, and $p_{K}^{1}=0$ and $p_{K}^{2}=\frac{1}{\lambda}$.
- In the benchmark case without switching costs: $p_{K}^{1}=p_{K}^{2}=1$.
- In the first period $p_{K}^{1}<1$ is lower to lock in as much consumers as possible ( second period profit effect).
- In the second period though, $p_{K}^{2}>1$ the equilibrium price is higher because firms compete only for new consumers.
- In terms of profit, each firm loses in $t=1$ but earns more in $t=2$ than absent switching costs.
- In equilibrium the intertemporal profit with switching costs is:

$$
\pi_{A}= \begin{cases}\frac{1}{6}\left(\frac{1}{\lambda}+5\right) & \text { for } \lambda>\frac{2}{5}, \\ \frac{1}{2 \lambda} & \text { for } \lambda<\frac{2}{5}\end{cases}
$$

- In equilibrium, the intertemporal profit without switching cost is 1 .
- Here firms are always better off when they can lock-in consumers and the effect on consumers surplus is negative.


## Endogenous switching cost: Coupons

- Coupons are discount offered on the price of the product at the next purchase.
- The oldest "Coupon" by TheCCC



## Assumptions

- Consumers redraw their types in $t=2$.
- In $t=1$ firms can offer coupons $c_{K} \geq 0$ to their loyal consumers. In $t=2$ the consumer will pay $p_{A}^{2}-c_{A}$ if he buys again from $A$.
- Consumers are forward looking.

Competition in period 2

- A consumer who purchased from $A$ in $t=1$, buys from $A$ again if its new address $x$ is such that
$r-x-\left(p_{A}^{2}-c_{A}\right)>r-(1-x)-p_{B}^{2} \Rightarrow x<\hat{x}_{A}=\frac{1}{2}\left(1+p_{B}^{2}-p_{A}^{2}+c_{A}\right)$
- Similarly, consumers who purchased from $B$ in $t=1$ buys from $B$ again if $x>\hat{X}_{B}=\frac{1}{2}\left(1+p_{B}^{2}-p_{A}^{2}-c_{B}\right)$
- We assume that $0<\hat{x}_{B} \leq \hat{x}_{A}<1$ i.e., that there is switching in equilibrium. (We check ex post this condition)
- In $t=2, A$ sells to consumers who had bought from $A$ in $t=1(\alpha)$ and do not switch $\left(x<\hat{x}_{A}\right)$, and those who bought from $B(1-\alpha)$ and switch $\left(x<\hat{x}_{B}\right)$.
- The maximization program is:

$$
\max _{p_{A}^{2}} \alpha \hat{x}_{A}\left(p_{A}^{2}-c_{A}\right)+(1-\alpha) \hat{x}_{B} p_{A}^{2}
$$

- The best reaction function is:

$$
p_{A}^{2}\left(p_{B}^{2}\right)=\frac{1}{2}\left(1+p_{B}^{2}+2 \alpha c_{A}-(1-\alpha) c_{B}\right)
$$

- Conversely, we obtain: $p_{B}^{2}\left(p_{A}^{2}\right)=\frac{1}{2}\left(1+p_{A}^{2}-\alpha c_{A}+2(1-\alpha) c_{B}\right)$
- In equilibrium,

$$
p_{A}^{2}=1+\alpha c_{A}, p_{B}^{2}=1+(1-\alpha) c_{B}
$$

Prices paid by switching (resp. loyal) consumers are higher (resp. lower).

- Equilibrium profit in $t=2$ is: $\pi_{A}^{2}=\frac{1}{2}-\frac{1}{2} \alpha(1-\alpha) c_{A}\left(c_{A}+c_{B}\right)<\frac{1}{2}$

Competition in $t=1$
$-\operatorname{In} t=1, A$ maximizes its intertemporal profit:

$$
\max _{p_{A}^{1}, c_{A}} p_{A}^{1} \alpha+\pi_{A}^{2}\left(\alpha, c_{A}\right)
$$

- To determine $\alpha$ we need to find the address of the indifferent consumer. Assuming consumers are forward looking, we compute the difference in consumer's surplus in $t=1$ :

$$
\Delta_{s}^{1}=\left(r-\alpha-p_{A}^{1}\right)-\left(r-(1-\alpha)-p_{B}^{1}\right)=1-2 \alpha+p_{B}^{1}-p_{A}^{1}
$$

and the difference in consumer's surplus in $t=2$ :

$$
\begin{aligned}
\Delta_{s}^{2} & =\int_{0}^{\hat{x}_{A}}\left(r-\left(p_{A}^{2}-c_{A}\right)-x\right) d x+\int_{\hat{x}_{A}}^{1}\left(r-p_{B}^{2}-(1-x)\right) d x \\
& -\int_{0}^{\hat{x}_{B}}\left(r-p_{A}^{2}-x\right) d x+\int_{\hat{x}_{B}}^{1}\left(r-\left(p_{B}^{2}-c_{B}\right)-(1-x)\right) d x \\
& =\frac{1}{4}\left(\left(c_{A}+c_{B}\right)^{2}+2\left(c_{A}-c_{B}\right)\right)-\frac{1}{2}\left(c_{A}+c_{B}\right)^{2} \alpha
\end{aligned}
$$

Competition in $t=1$

- $\Delta_{s}^{1}+\Delta_{s}^{2}=0$ gives:

$$
\alpha=\frac{4\left(1+p_{B}^{1}-p_{A}^{1}\right)+\left(c_{A}+c_{B}\right)^{2}+2\left(c_{A}-c_{B}\right)}{2\left(4+\left(c_{A}+c_{B}\right)^{2}\right)}
$$

- Deriving the intertemporal profit $\max _{p_{A}^{1}, c_{A}} p_{A}^{1} \alpha+\pi_{A}^{2}\left(\alpha, c_{A}\right)$ for $A$ and $B$ and focusing on a symetric equilibrium, we find:

$$
\begin{gathered}
c_{A}=c_{B}=\frac{2}{3}, p_{A}^{1}=p_{B}^{1}=\frac{13}{9}>1, p_{A}^{2}=p_{B}^{2}=\frac{4}{3}>1, \pi_{A}=\pi_{B}=\frac{10}{9}>1 . \\
\alpha=\frac{1}{2}, \hat{x}_{A}=\frac{5}{6}, \hat{x}_{B}=\frac{1}{6}
\end{gathered}
$$

## BOUTON

- Without coupons, prices would be equal to 1 in both periods and the intertemporal profit would be 1.
- Prices with coupon are $p_{A}^{2}-c_{A}=\frac{2}{3}<1$
- Firms are better off with coupons, it enables them to relax competition and all consumers (except loyal costumers in $t=2$ who pay $\frac{2}{3}$ ) pay a higher price.


## Exercice : Poaching

## Assumptions

- Two firms $k \in\{A, B\}$ are located at the extremes of a Hotelling line and compete during two periods, $t \in\{1,2\}$. Prices are denoted $p_{k}^{t}$.
- Consumers with a reservation price $r$ uniformly distributed along the line, incur a linear transportation cost $-x$ to travel distance $x$
- No production cost.


## Questions

1. Determine the equilibrium of the two period game.

## Solutions: Exercice

1. Determine the equilibrium of the two period game.

- The one shot game is repeated twice: no dynamic effect here!
$-p_{A}^{t}=p_{B}^{t}=1$ in both periods and each firm gets a market share $\frac{1}{2}$, the equilibrium profit is 1 .
Firms now observe consumer's identities and can set personalized prices $p_{k A}^{2}$ and $p_{k B}^{2}$ for consumers who bought from $A$ or $B$ in $t=1$.

2. If $\alpha$ is the market share of firm $A$ in $t=1$, determine the second period equilibrium.

- The indifferent consumers addresses are $\hat{x}_{A}=\frac{1}{2}+\frac{\left(\rho_{B A}^{2}-\rho_{A A}^{2}\right)}{2}$ and

$$
\hat{x}_{B}=\frac{1}{2}+\frac{\left(p_{B B}^{2}-p_{A B}^{2}\right)}{2} \text {. BOUTON }
$$

Firms $A$ and $B$ 's maximization problems are:

$$
\begin{array}{ll}
\max _{p_{A A}^{2}, p_{A B}^{2}} & p_{A A}^{2} \hat{x}_{A}+p_{A B}^{2}\left(\hat{x}_{B}-\alpha\right) \\
\max _{p_{B A}^{2}, p_{B B}^{2}} & p_{B A}^{2}\left(\alpha-\hat{x}_{A}\right)+p_{B B}^{2}\left(1-\hat{x}_{B}\right)
\end{array}
$$

## Solutions: Exercice 2

- The solution is
$p_{A A}^{2}=\frac{1}{3}(1+2 \alpha), p_{A B}^{2}=\frac{1}{3}(3-4 \alpha), p_{B A}^{2}=\frac{1}{3}(4 \alpha-1), p_{B B}^{2}=\frac{1}{3}(3-2 \alpha)$
- $\hat{x}_{A}=\frac{1}{6}+\frac{1}{3} \alpha, \hat{x}_{B}=\frac{1}{2}+\frac{1}{3} \alpha$

3. Consumers are forward looking. Determine the address of the indifferent consumer $\alpha$ in $t=1$.

- In $t=1$, anticipating a symmetric equilibrium the indifferent consumer anticipates that it will switch in $t=2$ and therefore its address is such that:

$$
\begin{gathered}
r-\alpha-p_{A}^{1}+\left(r-(1-\alpha)-p_{B A}^{2}\right)=r-(1-\alpha)-p_{B}^{1}+\left(r-\alpha-p_{A B}^{2}\right) \\
\alpha=\frac{4+3\left(p_{B}^{1}-p_{A}^{1}\right)}{8}
\end{gathered}
$$

4. Determine the first period equilibrium prices.

- Firm A's intertemporal profit is:

$$
\max _{p_{A}^{1}} p_{A}^{1} \alpha+p_{A A}^{2} \hat{x}_{A}+p_{A B}^{2}\left(\hat{x}_{B}-\alpha\right)
$$

- $p_{A}^{1}=p_{B}^{1}=\frac{4}{3}, \alpha=\frac{1}{2}, p_{A A}^{2}=p_{B B}^{2}=\frac{2}{3}$,
$p_{B A}^{2}=p_{A B}^{2}=\frac{1}{3}, \hat{x}_{A}=\frac{1}{3}, \hat{x}_{B}=\frac{2}{3}$.
- Poaching relaxes competition in period 1 and intensifies it in the second. Total profit is $\Pi_{A}=\Pi_{B}=\frac{5}{6}$.
- Firms would be better off if they could refrain from poaching.


## Références

Bulow, J., 1982, "Durable-Goods Monopolists", Journal of Political Economy, Vol. 90, No. 2 , pp. 314-332.

Caminal, R., and C. Matutes, (1990), "Endogenous Switching Costs in a duopoly Model", International Journal of Industrial Organization, 8, pp 353-373.

Narasimhan, C., 1988, "Competitive Promotional Strategies", The Journal of Business, Vol. 61, pp.427-450.

Varian, H., 1980, "A Model of Sales", The American Economic Review, Vol. 70, No. 4, pp. 651-659.

## Initial Condition

## back

- We check here that, in equilibrium, the initial condition is met, i.e. that all old consumers who have bought from $A$ buy again from $A$ in $t=2$.
- Formally we had assume that $\hat{x}_{A}=\frac{1}{2}(1+z)>\alpha=\frac{1}{2}$.


Consumers do not switch.

## Initial Condition

## back

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Consumers that do not switch.

## Initial Condition

## back

- We check here that, in equilibrium, the initial condition is met $\hat{x}_{A}<\alpha<\hat{x}_{B}$


$$
t=2
$$

