Firms' Strategies and Markets
Advertising

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Exercise 1

Assumptions

- Consumers are uniformly distributed along a segment $[0, 1]$. A firm is localized in 0 and another firm in 1.
- A consumer who travels a distance x to buy one unit at price p has a utility $U = v - p - tx$ if he buys and 0 if he does not buy. There is no utility for a second unit.
- A consumer buys only if he receives an ad. Let Φ_i denote the share of consumers who have received an ad from i. The cost to reach this fraction of demand is $A(\phi) = \frac{a\phi^2}{2}$ with $a \geq \frac{t}{2}$.
- If only one firm serves the market, we assume that the market is covered.

Questions

1. What is the demand of consumers who receive only an ad from i?

1. What is the demand of consumers who receive only an ad from i?

- The probability to receive an ad only from firm i is: $\phi_i(1-\phi_j)$.
- Consumers who buy are such that $v - p_i - tx \geq 0$
- $D_i = 1$ if $x_0 = \frac{v-p}{t} > 1$ (covered market)! $\Rightarrow$ We focus on this case for simplicity
- $D_i = \frac{v-p_i}{t}$ otherwise (uncovered market).

2. What is the demand of consumers who receive an ad from i and j?

- The probability to receive an ad from both firms is: $\phi_i \phi_j$.
- Among them the address of the indifferent consumer $\tilde{x}$ is such that $v - p_i - tx = v - p_j - t(1-x)$ or $\tilde{x} = \frac{1}{2} + \frac{(p_j - p_i)}{2t}$.
- $\tilde{x}$ (resp. 1-$\tilde{x}$) is the demand for i (resp. j) when the gap in price is not too high.

3. What is the total demand for firm i? How the price elasticity of demand varies in ϕ in $p_i = p_j = p$ and $\phi_i = \phi_j = \phi$?

- $D_i = \phi_i[(1 - \phi_j) + \phi_j \tilde{x}]$
- At point $p_i = p_j = p$ and $\phi_i = \phi_j = \phi$, the elasticity $\epsilon = \frac{-p_i \partial D_i / \partial p_i}{D_i} = \frac{p\phi}{t(2-\phi)}$ which increases in ϕ.
- A larger ϕ implies a larger the probability that consumers are informed of the existence of both goods: They are thus more sensitive to price.

4. Firms choose simultaneously their price and their ad level. Determine the symmetric Nash equilibrium of this game.

- The profit of firm i is:

$$\Pi_i = (p_i - c)D_i - A(\phi_i)$$

- with $D_i = \phi_i[(1-\phi_j) + \phi_j \frac{p_i - p_j + t}{2t}] = \frac{\phi_i}{2t}[(1-\phi_j)2t + \phi_j(p_i - p_j + t)]$
- The first order conditions are :

$$2p_i = c + t + p_j + \frac{2(1-\phi_j)t}{\phi_j}$$

$$\phi_i = (p_i - c)\frac{(1 - \phi_j + \phi_j \tilde{x})}{a}$$

- At the symmetric equilibrium $p_i = p_j = p^* = c + \sqrt{2at}$ and $\tilde{x} = \frac{1}{2}$ and $\phi_i = \phi_j = \phi^* = \frac{2}{(1+\sqrt{2a/t})}$.

Exercise 2: Advertising as a commitment device (Lal and Matutes, 1994)

Assumption

- Firms A and B are located at the extreme of a segment of lenght 1.
- Consumers are uniformly distributed along the segment and incur linear transport cost tx.
- A and B sell two products 1 and 2.
- Consumers have the same willingness to pay for each good, denoted H.
- Unless they receive an ad (catalog, leaflet,...), consumers are uninformed about prices but make rational expectations about prices.
- Each firm can choose to advertise one or two goods. Advertising costs F and vehicles the information about a product's price to all consumers.
- **We exclude that a consumer visit both stores**. this is a simplifying assumption and in the paper they look at all cases!

Exercise 2

1. What happens if no firm advertise any product?

- If there are no advertising, consumers rationally expect that all prices are equal to H.
  - Once at the store the firm knows that the transportation cost is sunk for the consumer and has an incentive to set a price H.

- Anticipating this, no consumer buy anything and therefore no profit for both firms.

2 What happens if the two firms advertise both products? Is this an equilibrium?

- Assume that the two firms advertise both products at prices (p_{A1}, p_{A2}) and (p_{B1}, p_{B2}) which costs $2F$ to each firm!
- The indifferent consumer is such that the surplus it obtains in visiting A, i.e. $2H - p_{A1} - p_{A2} - t\hat{x}$ is the same as the surplus it obtains in visiting B, i.e. $2H - p_{B1} - p_{B2} - t(1-\hat{x})$

$$\hat{x} = \frac{p_{B1} + p_{B2} - p_{A1} - p_{A2} + t}{2t}$$

- A maximizes its profit $(p_{A1} + p_{A2})\hat{x}$, and B maximizes $(p_{B1} + p_{B2})(1-\hat{x})$!
- This leads to $p_A^* = p_{A1} + p_{A2} = t$ and $p_B = p_{B1} + p_{B2} = t$.

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2 What happens if the two firms advertise both products? **Is this an equilibrium?**

- The first important condition to check is that $t < 2H$. Then, the profit each firm realizes is $\pi_j = \frac{t}{2} - 2F > 0 \rightarrow F < \frac{t}{4}$.
- Another condition to check is that the marginal consumer has a positive surplus, i.e. that $2H - t - \frac{t}{2} > 0 \rightarrow t < \frac{4H}{3}$ (covered market).
- To check whether this is an equilibrium, we check that a firm, say B, has no incentive to deviate unilaterally by only advertising one of its products, say 1.
  - Consumers rationnally expect that a product that is not advertised will be sold at H.
  $$\hat{x} = \frac{p_{B1} + H - p_A^* + t}{2t}$$
  - Maximizing its profit $(p_{B1} + H)(1 - \hat{x})$ with respect to p_{B1}, we obtain $p_{B1} = t - H$.
  - The profit obtained by firm B is therefore $\pi_B = \frac{t}{2} - F > \frac{t}{2} - 2F$: NO.

3. Determine the two types of equilibria of this game. For which conditions on H and F do these equilibria exist?

- There are two symmetric equilibria: (i) one firm advertises 1 and the other 2 or (ii) the two firms advertise the same good.
  - A and B advertise product 1. Consumers expect product 2 to be sold at price H at both stores.
  - The indifferent consumer is:

$$\hat{x} = \frac{p_{B1} + H - p_{A1} - H + t}{2t}.$$

  - A maximizes its profit $(p_{A1} + H)\hat{x}$ whereas B maximizes $(p_{B1} + H)(1 - \hat{x})$.
  - We obtain $p_{A1} = p_{B1} = t - H$ and therefore the profit is $\frac{t}{2} - F > 0$.

3. Determine the two types of equilibria of this game. **For which conditions on H and F do these equilibria exist?**

- There is no incentive for a firm to deviate towards no advertising as it brings no profit.
- There is no incentive to deviate towards advertising both products as it brings a lower profit $\frac{t}{2} - 2F$.
- A firm could deviate by advertising instead the other product. But as everything is symmetric here, it brings the same profit.
- From above it is immediate that there is another symmetric equilibrium in which A advertises 1 and B advertises 2 and conversely. These equilibria exists if $F < t/2$ and if the market is covered, i.e. the marginal consumer has a positive surplus, i.e. that $t < \frac{4H}{3}$. We may have loss leading on product 1 as it is possible to have $H - t < 0$.