Exercise : RPM to eliminate free-riding

Assumptions

- P offers a good produced at a unit cost c to two competing retailers $i = \{1, 2\}$ who compete à la Bertrand.
- Demand for the good is linear $D(p, s) = v + s - p$.
- Total effort service is the sum of the retailer's effort $s_1 + s_2 = s$
- Cost of effort is $c(s_i) = s_i^2$

Questions

1. What are the choices (p^M, s_i^M) of a fully vertically integrated structure?

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1. What are the choices (p^M, s_i^M) of a fully vertically integrated structure?
   - The integrated structure maximizes the profit

     $$\underset{p,s_1,s_2}{Max}(p-c)(v+s_1+s_2-p)-s_1^2-s_2^2$$

     with respect to p, s_1 and s_2.
   - We obtain $p^M = v$, $s_1^M = s_2^M = \frac{v-c}{2}$

2. P and the two retailers are separated. What happens if P offers a simple uniform unit wholesale price contract w?
   - Bertrand competition $p = w$, $s_1 = s_2 = 0$ and, so $s = 0$ and $w = \frac{v+c}{2}$. A shop refrains from providing services that are not appropriable.
   - This leads to a suboptimal level of effort and a suboptimal global demand.

3 P offers a contract (w, F, p) i.e. a contract with two-part tariff and resale price maintenance.

- RPM + two-part tariff can reach the first best!
- The retailer 1 chooses its effort level s_1 to maximize:

$$\underset{s_1}{Max}(p-w)\frac{(v+s_1+s_2-p)}{2}-s_1^2$$

- We obtain $s_i^* = \frac{p-w}{4}$. P controls everything and therefore chooses $p = p^M = v$ and sets $(p^M - w)$ such that $s_i^* = s_i^M$ which implies that

$$\frac{v-w}{4} = s_i^M = \frac{v-c}{2}$$

  Therefore, $w^* = -v + 2c < c$ and $F_i = (p^M - w^*)\frac{(v+s_1^M+s_2^M-p^M)}{2} - (s_i^M)^2 = \frac{3}{4}(v-c)^2$.
- $w = -v + 2c < c$, $p = p^M = v$ and $F_i = \frac{3}{4}(v-c)^2$ to get back the industry profit, $\Pi^M = \frac{(v-c)^2}{2}$.
- $s_1 = s_2 = s^M \Rightarrow$ horizontal externality solved!.