Firms' Strategies and Markets
Entry

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Exercice 2: Aghion and Bolton (1987)

M sells a good to A who is willing to pay at most $p = 1$ for one unit. The unit cost of M is $c_M = \frac{1}{2}$. An entrant, E can produce the same good at an unknown unit cost c_E uniformly distributed over $[0, 1]$.

- In $t = 0$, A and M sign a contract or not;
- In $t = 1$, E observes the contract, learns its unit cost c_E and chooses to enter or not.
- In $t = 2$, firms set their prices.
- In $t = 3$, A decides where to buy.

1 Without contract, the competition is a la Bertrand.

a. Determine the equilibrium and the probability ϕ of entry. Bertrand $\Rightarrow p^* = max\{c_E, c_M\}$. E enters only if $c_E < c_M$.

The probability of entry is $\phi = P(c < c_M) = c_M = \frac{1}{2}$.

The situation is efficient, the firm who produces is the firm with the lowest unit cost.

b. What are the expected profits? The expected profits are:

$$\Pi_M = \phi 0 + (1 - \phi)(1 - c_M) = \frac{1}{4},$$

$$\Pi_E = \int_0^{c_M} (c_M - c)dc + 0 = \frac{c_M^2}{2} = \frac{1}{8},$$

$$\Pi_A = \phi(1 - c_M) + (1 - \phi)0 = c_M(1 - c_M) = \frac{1}{4}.$$

$$W = \Pi_M + \Pi_E + \Pi_A = \frac{5}{8}$$

2 M offers a take-it-or-leave-it contract (P, P_0) where P is the price that A must pay if he chooses to buy the good from M and P_0 is the penalty A must pay to M if he buys from E.

a. Given (P, P_0), under which conditions does E enter?

$\Pi_A = 1 - P_0 - P_E$ if he buys from E.

$\Pi_A = 1 - P$ if he buys from M.

Therefore A buys from E if $c_E \leq P_E \leq P - P_0$ i.e. $P - P_0 \geq c_E$ and in that case $P_E = P - P_0$.

b. What is the profit of A if he accepts a contract (P, P_0) ?

$\Pi_A = \frac{1}{4}$ without contract.

With the contract,
$\Pi_A(P, P_0) = (P - P_0)(1 - P_E - P_0) + (1 - P + P_0)(1 - P) = 1 - P$ (as $P_E = P - P_0$).

A accepts the contract only if $1 - P \geq \frac{1}{4} \Rightarrow P \leq \frac{3}{4}$.

Solution

c. Determine the optimal contract (P, P_0) for M.

$$\Pi_M(P, P_0) = (P - P_0)P_0 + (1 - P + P_0)(P - C_M)$$

$$\frac{\partial \Pi(P, P_0)}{\partial P_0} = -2P_0 + P + P - c_M = 0$$

Replacing $c_M = \frac{1}{2}$, we obtain:

$$\Rightarrow P_0 = P - \frac{1}{4}.$$

For $P_0 = P - \frac{1}{4}$, the profit of M is $\frac{1}{4}(P - \frac{1}{4}) + \frac{3}{4}(P - \frac{1}{2}) = P - \frac{7}{16}$.

However we know that $P \geq \frac{3}{4}$ to be accepted by A.

The optimal contract is thus $P = \frac{3}{4}$, $P_0 = \frac{1}{2}$.

With the exclusive dealing contract, the probability of entry is reduced to $\frac{1}{4}$.

Solution

d. What are the expected profits under this contract? Comment!
Expected profits are:

$$\Pi_M = (1 - \frac{1}{4})(\frac{3}{4} - c_M) + \frac{1}{4}\frac{1}{2} = \frac{5}{16} > \frac{1}{4},$$

$$\Pi_E = (1 - \frac{1}{4})0 + \int_0^{\frac{1}{4}} (\frac{1}{4} - c)dc = \frac{1}{32} < \frac{1}{8},$$

$$\Pi_A = (1 - \frac{1}{4})(1 - \frac{3}{4}) + \frac{1}{4}(1 - \frac{3}{4}) = \frac{1}{4}.$$

$$W = \frac{19}{32} < \frac{5}{8}$$

The welfare decreases because efficient entries are blockaded.