# ECO 650: Firms' Strategies and Markets
# Vertical Relationships and Bargaining(II)

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# Exercise 1

**Assumptions:**

- A manufacturer produces a good at a unit cost $c$.
- A retailer faces a demand $D(p) = 1 - p$.
- The game:
  1. The manufacturer and the retailer bargain over a two-part tariff contract $(w, F)$;
  2. The retailer sets a final price $p$ to consumers.

**Questions:**

1. Given the contract $(w, F)$, determine the optimal price set by the retailer in stage 2. Determine the stage-2 equilibrium profits of firms $\pi_U(w) + F$ and $\pi_D(w) - F$.
2. Write down the Nash program and determine the optimal contract $(w, F)$. Is it efficient?

# Exercise 1: Solution

1. In stage 2, the retailer maximizes $\max_p (p-w)(1-p) - F$; The FOC is: $1 - 2p + w = 0 \Rightarrow p = \frac{1+w}{2}$; $\pi_U(w) = (w-c)(\frac{1-w}{2})$ and $\pi_D(w) = (\frac{1-w}{2})^2$.

2. The Nash program in stage 1 is

$$\max_{(w,F)} (\pi_U(w) + F)(\pi_D(w) - F)$$

FOCS are:

$$\begin{aligned} -(\pi_U(w) + F) + (\pi_D(w) - F) &= 0 \qquad (1) \\ \frac{\partial \pi_U(w)}{\partial w}(\pi_D(w) - F) + \frac{\partial \pi_D(w)}{\partial w}(\pi_U(w) + F) &= 0 \qquad (2) \end{aligned}$$

(1) is the split the difference rule, $F$ is used to share profits equally. Plugging (1) into (2): $\underbrace{(\frac{\partial \pi_U(w)}{\partial w} + \frac{\partial \pi_D(w)}{\partial w})}_{0} \underbrace{(\pi_D(w) - F)}_{>0} = 0$. $w$ is set to maximize joint profits $w^* = c$: Efficiency!

# Exercise 1: Solutions

$\pi_U(w) + \pi_D(w) = (\frac{1-w}{2})(\frac{1+w-2c}{2})$.
Deriving this joint profit w.r.t $w$ gives:
$-(1+w-2c) + (1-w) = 0 \Rightarrow w^* = c$. $\pi_U(c) = 0, \pi_D(c) = \frac{(1-c)^2}{4}$
$F^* = \frac{\pi_D(c) - \pi_U(c)}{2} = \frac{(1-c)^2}{8}$.
In equilibrium both firms obtain a profit $\frac{(1-c)^2}{8}$.

3. The outside option profit $\bar{\Pi}$ is such that $\max_p (p-\bar{c})(1-p)$.
$\bar{\Pi} = (\frac{1-\bar{c}}{2})^2$. The Nash program in stage 1 is

$$\max_{(w,F)} (\pi_U(w) + F)(\pi_D(w) - F - \bar{\Pi})$$

FOCS are:

$$-(\pi_U(w) + F) + (\pi_D(w) - F - \bar{\Pi}) = 0 \quad (3)$$

$$\frac{\partial \pi_U(w)}{\partial w}(\pi_D(w) - F - \bar{\Pi}) + \frac{\partial \pi_D(w)}{\partial w}(\pi_U(w) + F) = 0 \quad (4)$$

Plugging (3) into (4), again $w$ maximizes the joint profit $w^* = c$: unchanged!