Firms' Strategies and Markets
Advertising

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1/12

Exercise 1

Assumptions

- Consumers are uniformly distributed along a segment $[0, 1]$. A firm is localized in 0 and another firm in 1.
- A consumer who travels a distance x to buy one unit at price p has a utility $U = v - p - tx$ if he buys and 0 if he does not buy. There is no utility for a second unit.
- A consumer buys only if he receives an ad. Let Φ_i denote the share of consumers who have received an ad from i. The cost to reach this fraction of demand is $A(\phi) = \frac{a\phi^2}{2}$ with $a \geq \frac{t}{2}$.

Questions

1. What is the demand of consumers who receive only an ad from i?

2/12

1. What is the demand of consumers who receive only an ad from i?

- The probability to receive an ad only from firm i is: $\phi_i(1-\phi_j)$.
- Consumers who buy are such that $v - p_i - tx \geq 0$
- $D_i = 1$ if $x_0 = \frac{v-p}{t} > 1$ (covered market)! $\Rightarrow$ We focus on this case for simplicity
- $D_i = \frac{v-p_i}{t}$ otherwise (uncovered market).

3/12

2. What is the demand of consumers who receive an ad from i and j?

- The probability to receive an ad from both firms is: $\phi_i \phi_j$.
- Among them the address of the indifferent consumer $\tilde{x}$ is such that $v - p_i - tx = v - p_j - t(1-x)$ or $\tilde{x} = \frac{1}{2} + \frac{(p_j - p_i)}{2t}$.
- $\tilde{x}$ (resp. 1-$\tilde{x}$) is the demand for i (resp. j) when the gap in price is not too high.

4/12

3. What is the total demand for firm i? How the price elasticity of demand varies in ϕ in $p_i = p_j = p$ and $\phi_i = \phi_j = \phi$?

- $D_i = \phi_i[(1 - \phi_j) + \phi_j \tilde{x}]$
- At point $p_i = p_j = p$ and $\phi_i = \phi_j = \phi$, the elasticity $\epsilon = \frac{-p_i \partial D_i / \partial p_i}{D_i} = \frac{p\phi}{t(2-\phi)}$ which increases in ϕ.
- A larger ϕ implies a larger the probability that consumers are informed of the existence of both goods: They are thus more sensitive to price.

4. Firms choose simultaneously their price and their ad level. Determine the symmetric Nash equilibrium of this game.

- The profit of firm i is:

$$\Pi_i = (p_i - c)D_i - A(\phi_i)$$

- with $D_i = \phi_i[(1-\phi_j) + \phi_j \frac{p_i - p_j + t}{2t}] = \frac{\phi_i}{2t}[(1-\phi_j)2t + \phi_j(p_i - p_j + t)]$
- The first order conditions are :

$$2p_i = c + t + p_j + \frac{2(1-\phi_j)t}{\phi_j}$$

$$\phi_i = (p_i - c)\frac{(1 - \phi_j + \phi_j \tilde{x})}{a}$$

- At the symmetric equilibrium $p_i = p_j = p^* = c + \sqrt{2at}$ and $\tilde{x} = \frac{1}{2}$ and $\phi_i = \phi_j = \phi^* = \frac{2}{(1+\sqrt{2a/t})}$.

6/12

Exercise 2: Advertising as a commitment device (Lal and Matutes, 1994)

Assumption

- Firms A and B are located at the extreme of a segment of lenght 1.
- Consumers are uniformly distributed along the segment and incur linear transport cost tx.
- A and B sell two products 1 and 2.
- Consumers have the same willingness to pay for each good, denoted H.
- Unless they receive an ad (catalog, leaflet,...), consumers are uninformed about prices but make rational expectations about prices.
- Each firm can choose to advertise one or two goods. Advertising costs F and vehicles the information about a product's price to all consumers.
- **We exclude that a consumer visit both stores**. this is a simplifying assumption and in the paper they look at all cases!

7/12

Exercise 2

1. What happens if no firm advertise any product?

- If there are no advertising, consumers rationally expect that all prices are equal to H.
  - Once at the store the firm knows that the transportation cost is sunk for the consumer and has an incentive to set a price H.
- Anticipating this, no consumer buy anything and therefore no profit for both firms.

8/12

2 What happens if the two firms advertise both products? Is this an equilibrium?

- Assume that the two firms advertise both products at prices (p_{A1},p_{A2}) and (p_{B1},p_{B2}) which costs $2F$ to each firm!
- The indifferent consumer is such that the surplus it obtains in visiting A, i.e. $2H - p_{A1} - p_{A2} - t\hat{x}$ is the same as the surplus it obtains in visiting B, i.e. $2H - p_{B1} - p_{B2} - t(1-\hat{x})$

$$\hat{x} = \frac{p_{B1} + p_{B2} - p_{A1} - p_{A2} + t}{2t}$$

- A maximizes its profit $(p_{A1} + p_{A2})\hat{x}$, and B maximizes $(p_{B1} + p_{B2})(1-\hat{x})$!
- This leads to $p_A^* = p_{A1} + p_{A2} = t$ and $p_B = p_{B1} + p_{B2} = t$.

9/12

2 What happens if the two firms advertise both products? **Is this an equilibrium?**

- The first important condition to check is that $t < 2H$. Then, the profit each firm realizes is $\pi_j = \frac{t}{2} - 2F > 0 \rightarrow F < \frac{t}{4}$.
- Another condition to check is that the marginal consumer has a positive surplus, i.e. that $2H - t - \frac{t}{2} > 0 \rightarrow t < \frac{4H}{3}$ (covered market).
- To check whether this is an equilibrium, we check that a firm, say B, has no incentive to deviate unilaterally by only advertising one of its products, say 1.
  - Consumers rationnally expect that a product that is not advertised will be sold at H.

$$\hat{x} = \frac{p_{B1} + H - p_A^* + t}{2t}$$

  - Maximizing its profit $(p_{B1} + H)\hat{x}$ with respect to p_{B1}, we obtain $p_{B1} = t - H$.
  - The profit obtained by firm B is therefore $\pi_B = \frac{t}{2} - F > \frac{t}{2} - 2F$: NO.

10/12

3. Determine the two types of equilibria of this game. For which conditions on H and F do these equilibria exist?

- There are two symmetric equilibria: (i) one firm advertises 1 and the other 2 or (ii) the two firms advertise the same good.
  - A and B advertise product 1. Consumers expect product 2 to be sold at price H at both stores.
  - The indifferent consumer is:

$$\hat{x} = \frac{p_{B1} + H - p_{A1} - H + t}{2t}.$$

  - A maximizes its profit $(p_{A1} + H)\hat{x}$ whereas B maximizes $(p_{B1} + H)(1 - \hat{x})$.
  - We obtain $p_{A1} = p_{B1} = t - H$ and therefore the profit is $\frac{t}{2} - F > 0$.

11/12

3. Determine the two types of equilibria of this game. **For which conditions on H and F do these equilibria exist?**

- There is no incentive for a firm to deviate towards no advertising as it brings no profit.
- There is no incentive to deviate towards advertising both products as it brings a lower profit $\frac{t}{2} - 2F$.
- A firm could deviate by advertising instead the other product. But as everything is symmetric here, it brings the same profit.
- From above it is immediate that there is another symmetric equilibrium in which A advertises 1 and B advertises 2 and conversely. These equilibria exists if $F < t/2$ and if the market is covered, i.e. the marginal consumer has a positive surplus, i.e. that $t < \frac{4H}{3}$. We may have loss leading on product 1 as it is possible to have $H - t < 0$.

12/12