Firms' Strategies and Markets
Dynamic pricing

Claire Chambolle

October 05, 2022

1/8

Exercice 1

Assumptions

- A durable good monopoly, M, with a production cost c.
- Two consumers who live two periods $t = \{1; 2\}$. Two consumers buy either 0 or 1 unit. C1 has a valuation 1 and C2 v_l with $c < v_l < 1$.
- δ is the discount factor.
- M sets p_1 in $t = 1$ and p_2 in $t = 2$.

Questions

1. Determine the price equilibrium p and profit Π if M only sells in $t = 1$.
2. Determine the two period equilibrium (p_1, p_2) and profit $\Pi_{1,2}$ of M.
3. Compare the two profits when $c < v_l < \frac{1}{2}(1 + \frac{c}{1+\delta})$. What happens if $v_l > \frac{1}{2}(1 + c)$?

Solution-Exercice 1

1. Determine the price equilibrium p and profit Π if M only sells in $t = 1$.

- If M sells only to C1, $p = 1 + \delta$ and its profit is $\Pi = 1 + \delta - c$.
- If M sells to C1 and C2 $p = v_l(1 + \delta)$ and its profit is $\Pi = 2(v_l(1 + \delta) - c)$.
- The first option is chosen if $c < v_l < \frac{1}{2}(1 + \frac{c}{1+\delta})$.

2. Determine the two period equilibrium (p_1, p_2) and profit $\Pi_{1,2}$ of M.

- M is willing to serve C1 in $t = 1$ and C2 in $t = 2$.
  To make sure C1 buys in $t = 1$: $1 + \delta - p_1 > \delta(1 - p_2) \Rightarrow$

$$p_1 < 1 + \delta p_2 \tag{1}$$

- Now, p_2 depends on the behavior of C1 in $t = 1$. If C1 has not purchased the good in $t = 1$,

  - If $v_l < \frac{1}{2}(1 + c)$, M sets $p_2 = 1$.
    Therefore, given (1) M sets $p_1 = 1 + \delta$ and sells to C1. Then, M sets $p_2 = v_l$ and sells to C2.

    M obtains $\Pi_{1,2} = 1 + \delta - c + \delta(v_l - c)$.

  - If $v_l > \frac{1}{2}(1 + c)$, M sets $p_2 = v_l$.
    Thus, given (1), M sets $p_1 = 1 + \delta v_l$ and sell to C1. Then M sets $p_2 = v_l$ and sells to C2.

    M obtains $\Pi_{1,2} = 1 + \delta v_l - c + \delta(v_l - c)$.

Solution-Exercice 1

3. Compare the two profits when $v_l < \frac{1}{2}(1 + \frac{c}{1+\delta})$. What happens if $v_l > \frac{1}{2}(1 + c)$?

- If $v_l < \frac{1}{2}(1 + \frac{c}{1+\delta}) < \frac{1+c}{2}$,

$$\Pi = 1 + \delta - c < \Pi_{1,2} = 1 + \delta - c + \delta(v_l - c).$$

Intertemporal discrimination is profitable!

- The reverse is true when $v_l > \frac{1}{2}(1 + c)$!

$$\Pi = 2(v_l(1 + \delta) - c) > \Pi_{1,2} = 1 + \delta v_l - c + \delta(v_l - c)$$

Solutions: Exercice 2

1. Determine the equilibrium of the two period game.
   - The one shot game is repeated twice: no dynamic effect here!
   - $p_A^t = p_B^t = 1$ in both periods and each firm gets a market share $\frac{1}{2}$, the equilibrium profit is 1.

   Firms now observe consumer's identities and can set personalized prices p_{kA}^2 and p_{kB}^2 for consumers who bought from A or B in $t = 1$.
2. If α is the market share of firm A in $t = 1$, determine the second period equilibrium.
   - The indifferent consumers addresses are $\hat{x}_A = \frac{1}{2} + \frac{(p_{BA}^2 - p_{AA}^2)}{2}$ and $\hat{x}_B = \frac{1}{2} + \frac{(p_{BB}^2 - p_{AB}^2)}{2}$. BOUTON
     Firms A and B's maximization problems are:

$$\max_{p_{AA}^2, p_{AB}^2} \quad p_{AA}^2 \hat{x}_A + p_{AB}^2 (\hat{x}_B - \alpha)$$
$$\max_{p_{BA}^2, p_{BB}^2} \quad p_{BA}^2 (\alpha - \hat{x}_A) + p_{BB}^2 (1 - \hat{x}_B)$$

Solutions: Exercice 2

- The solution is
  $p_{AA}^2 = \frac{1}{3}(1+2\alpha), p_{AB}^2 = \frac{1}{3}(3-4\alpha), p_{BA}^2 = \frac{1}{3}(4\alpha-1), p_{BB}^2 = \frac{1}{3}(3-2\alpha)$

- $\hat{x}_A = \frac{1}{6} + \frac{1}{3}\alpha, \hat{x}_B = \frac{1}{2} + \frac{1}{3}\alpha$

3. Consumers are forward looking. Determine the address of the indifferent consumer α in $t = 1$.

   - In $t = 1$, anticipating a symmetric equilibrium the indifferent consumer anticipates that it will switch in $t = 2$ and therefore its address is such that:

$$r - \alpha - p_A^1 + (r - (1 - \alpha) - p_{BA}^2) = r - (1 - \alpha) - p_B^1 + (r - \alpha - p_{AB}^2)$$

$$\alpha = \frac{4 + 3(p_B^1 - p_A^1)}{8}$$

4. Determine the first period equilibrium prices.
   - Firm A's intertemporal profit is:

$$\max_{p_A^1} p_A^1 \alpha + p_{AA}^2 \hat{x}_A + p_{AB}^2 (\hat{x}_B - \alpha)$$

   - $p_A^1 = p_B^1 = \frac{4}{3}$, $\alpha = \frac{1}{2}$, $p_{AA}^2 = p_{BB}^2 = \frac{2}{3}$, $p_{BA}^2 = p_{AB}^2 = \frac{1}{3}, \hat{x}_A = \frac{1}{3}, \hat{x}_B = \frac{2}{3}$.
   - Poaching relaxes competition in period 1 and intensifies it in the second. Total profit is $\Pi_A = \Pi_B = \frac{5}{6}$.
   - Firms would be better off if they could refrain from poaching.

Initial Condition

back

- We check here that, in equilibrium, the initial condition is met $\hat{x}_A < \alpha < \hat{x}_B$