Firms' Strategies and Markets
Advertising

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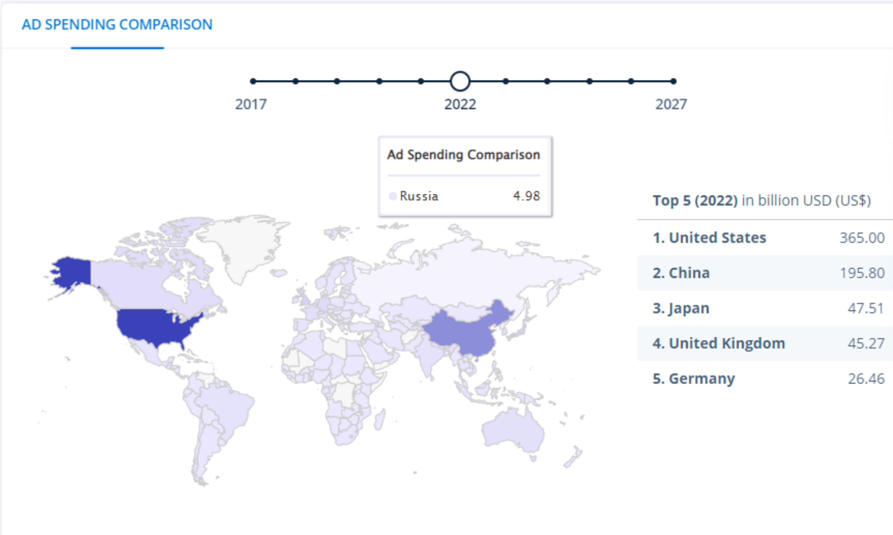
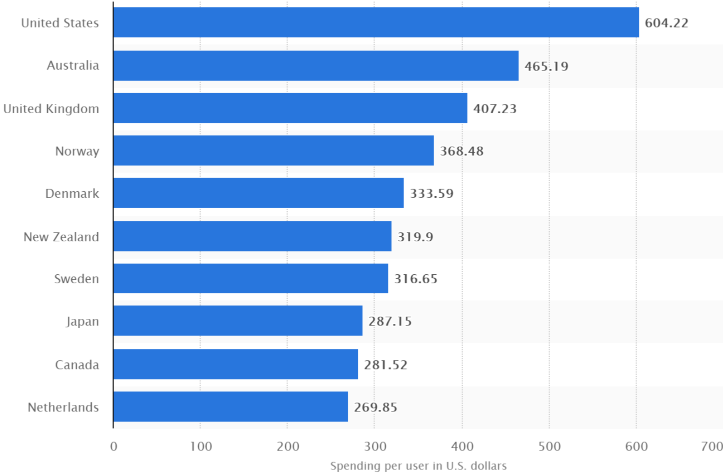
Introduction

- Worlwide amount of ad spending in 2022 is about 781 billlion \$;
- More than 60% of this amount are digital advertising and mobile phone (growing)–the rest are mainly TV and radio ($\approx$ 30%) or print medias (newspapers and magazine $<$5%);
  - Google is the largest digital ad seller in the world in 2019;
  - Google and Facebook have a 60% market share of online advertising.
  - CMA report in 2020 / role of consumer data in digital market ads.
- The largest advertisers in 2017 are Samsung and Procter & Gamble ($>$10 billions US \$ in 2017 for P&G)

Countries with highest advertising spending in 2022



Countries with highest advertising spending per person in 2016 (US $)



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$$\max_{Q,A} \Pi(Q, A) = pQ(p, A) - C(Q(p, A)) - A$$

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The First Order Conditions are:

$$\Pi_p = (p - C_Q)Q_p + Q = 0 \Rightarrow \frac{p - C_Q}{p} = \frac{-1}{\epsilon_{Q/p}}$$

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Result

The advertising intensity is equal to the ratio of the advertising elasticity of demand and the price elasticity of demand: $\frac{A}{pQ} = \frac{\epsilon_{Q/A}}{-\epsilon_{Q/p}}$:
Dorfman-Steiner condition !

Typology of advertising

- **Persuasive Advertising** enhances consumers' tastes for a given product
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  - Advertising changes the distribution of consumers' tastes.
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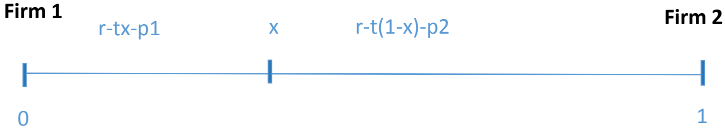
- **Signaling Quality**: the amount of ads spent or the price indirectly convey information about the quality of the products to consumers.

Persuasive Advertising

Assumptions

- Game: Stage 1- Advertising & Stage 2- price competition;
- Consumers are distributed according to $F(x)$ over $[0, 1]$
- The cost of advertising intensity λ_i is $a\lambda_i^2/2$.



- Advertising increases consumers' willingness to pay: $r_i(\lambda_i)$
- Advertising changes the distribution of consumers' tastes: $F(x, \lambda_i, \lambda_j)$
- Advertising increases perceived product difference : $t(\lambda_i, \lambda_j)$

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The indifferent consumer address $\hat{x}$ is such that:

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Firms maximize their profit with respect to p_i and the reaction functions are symmetric and increasing : Prices are strategic complement!

$$\underset{p_i}{Max}\Pi_i \Rightarrow p_i(p_j) = \frac{1}{2}(c + t + p_j)$$

Results

There is a symmetric equilibrium: $p_1^* = p_2^* = c + t$ and $\Pi_1^* = \Pi_2^* = \frac{t}{2}$.

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$$\begin{aligned} \Pi_1 &= (p_1 - c)\hat{x}(p_1, p_2, \lambda_1, \lambda_2) - a\lambda_1^2/2 \\ \Pi_2 &= (p_2 - c)(1 - \hat{x}(p_1, p_2, \lambda_1, \lambda_2)) - a\lambda_2^2/2 \end{aligned}$$

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In stage 1, each firm i maximizes its profit with respect to λ_i anticipating the stage 2 competition in prices:

$$\underset{\lambda_i}{Max}\Pi_i(\lambda_i, \lambda_j) \Rightarrow \lambda_i(\lambda_j) = \frac{\beta(3t - \beta\lambda_j)}{9at - \beta^2}$$

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Results

$\lambda_1^* = \lambda_1^* = \frac{\beta}{3a}$, $p_1^* = p_2^* = c + t$ and $\Pi_1^* = \Pi_2^* = \frac{t}{2} - \frac{\beta^2}{18a} < \frac{t}{2}$. Firms are worse-off with advertising. If they could coordinate, they would refrain from investing.

Advertising changes the distribution of consumers' tastes

Assumptions

- We denote $F(x, \lambda_1, \lambda_2) = (1 + \lambda_1 - \lambda_2)x - (\lambda_1 - \lambda_2)x^2$ with a continuous density $f(x, \lambda_1, \lambda_2) = (1 + \lambda_1 - \lambda_2) - 2x(\lambda_1 - \lambda_2)$.
- If $\lambda_1 = \lambda_2$ we find a uniform distribution, $\lambda_1 = 1$ and $\lambda_2 = 0$ a distribution that favors firm 1. Distribution Function

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Maximizing their profit **simultaneously** with respect to p_i and λ_i, and focusing on the symmetric equilibrium:

Results

$p_1^* = p_2^* = c + t$ and $\lambda_1^* = \lambda_2^* = \frac{t}{4a}$. $\Pi_1^* = \Pi_2^* = \frac{t}{2} - \frac{t^2}{32a} < \frac{t}{2}$. Firms are worse-off with advertising. If they could coordinate, they would refrain from investing.

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Result

Advertising that increases perceived product difference relaxes competition and therefore firms' investment is profitable.

Public good: coordination raises investment.

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  - Advertising characteristics of the products may increase the perceived differentiation among products and soften competition !
- Heavy regulation of ads – in France:
  - Comparative ads are regulated (not authorized to depreciate/lie the product of a rival)!!
  - Law “Evin" (1991) forbids any ads on tobacco or alcool. greenhouse gaz emissions
  - Since January 2022, are ads on some products that are bad for environment (high GHG emissions- SUV) or for health (food products listed by PNNS).

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With advertising : Competition is Bertrand like, because the product is

homogenous: $p = c$.

Result

Informative advertising on prices may intensify competition by reducing consumers' search costs.

- Argument often put forward in favor of “online" sales.

Informative advertising on product's existence

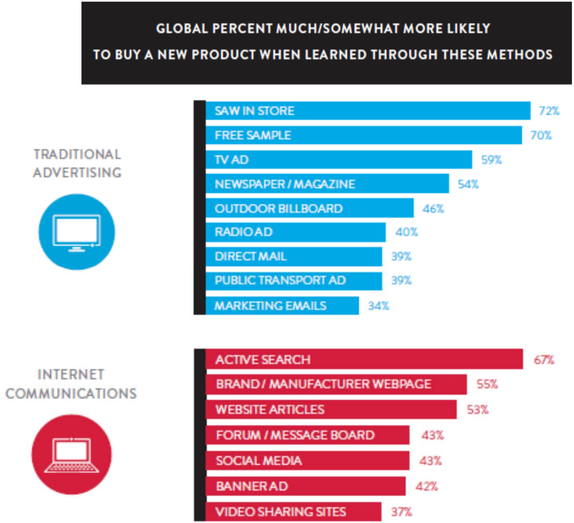
Grossman & Shapiro (1984)

- Consumers unaware of a new product's existence: no utility and no demand.
- Consumers aware of a new product's existence
  - $u(q) > 0$ with $u'(q) > 0$ and $u''(q) < 0$.
  - Maximising $u(q) - pq$ where p is the price, we derive a demand $q(p) > 0$, with $q'(p) < 0$.

Information about the existence of a product

Advertising can inform consumers about the very existence of a product!

Advertising is key to launch a new product



Source: Nielsen Global Survey of New Product Purchase Sentiment, Q3 2012

Remember

- In a competition framework: different types of informative advertising lead to different outcomes
  - It might increase competition when it vehicles information on prices.
  - Informative advertising is profitable when it reveals the product's existence (See Exercice 1).

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- A consumer buys only if he receives an ad. Let Φ_i denote the share of consumers who have received an ad from i. The cost to reach this fraction of demand is $A(\phi) = \frac{a\phi^2}{2}$ with $a \geq \frac{t}{2}$.

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- The probability to receive an ad only from firm i is: $\phi_i(1-\phi_j)$.
- Consumers who buy are such that $v - p_i - tx \geq 0$
- $D_i = 1$ if $x_0 = \frac{v-p}{t} > 1$ (covered market)! $\Rightarrow$ We focus on this case for simplicity
- $D_i = \frac{v-p_i}{t}$ otherwise (uncovered market).

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- $\tilde{x}$ (resp. 1-$\tilde{x}$) is the demand for i (resp. j) when the gap in price is not too high.

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- $D_i = \phi_i[(1 - \phi_j) + \phi_j \tilde{x}]$
- At point $p_i = p_j = p$ and $\phi_i = \phi_j = \phi$, the elasticity $\epsilon = \frac{-p_i \partial D_i / \partial p_i}{D_i} = \frac{p\phi}{t(2-\phi)}$ which increases in ϕ.

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- A larger ϕ implies a larger the probability that consumers are informed of the existence of both goods: They are thus more sensitive to price.

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- At the symmetric equilibrium $p_i = p_j = p^* = c + \sqrt{2at}$ and $\tilde{x} = \frac{1}{2}$ and $\phi_i = \phi_j = \phi^* = \frac{2}{(1+\sqrt{2a/t})}$.

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Assumptions

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Full Information

Consumers know the quality and thus firms do not advertise.
A high quality firm sets $p_H = v_H$ and gets $\Pi_H = 2(v_H - c)$);
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Asymmetric Information

We look for a separating equilibrium BOUTON. We assume that only advertising amounts (not price) can convey a signal about quality.

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Result

Burning money through advertising can be a credible means for a firm to signal a high quality in particular in the case of experience good with repeated purchases.

Price and Advertising signals

Milgrom and Roberts (1986)

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Result 1

There exists a separating sequential equilibrium, such that a high quality firm chooses (P, A) and a low quality firm P_L^L, if and only if for some (P, A):

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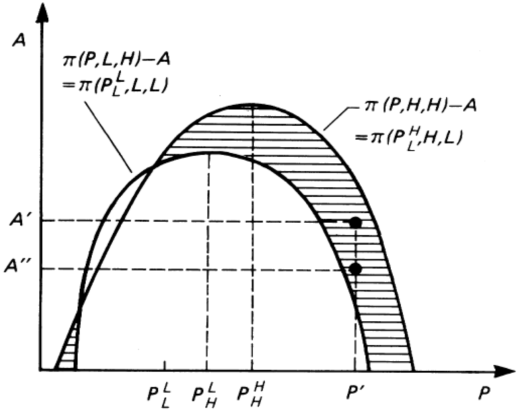
- $\pi(P, L, H) - A \leq \pi(P^L_L, L, L)$: a firm of quality L earns a smaller profit in selecting (P, A) rather than its best profit when consumers believe its quality is L.

- Isoprofit curves:
    - $A(P) = \pi(P, H, H) - \pi(P_L^H, H, L)$ (Above)
    - $A(P) = \pi(P, L, H) - \pi(P_L^L, L, L)$ (Below)



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Result 2

There exists a separating equilibrium if and only if there is some (P, A) such that eq(1) holds. At any separating equilibrium, the choice (P, A) of the high-quality firm must be a solution to the following programme (2):

$$\max_{P,A} \pi(P, H, H) - A$$

$$\texttt{subject to} \pi(P, L, H) - A \leq \pi(p_L^L, L, L)$$

. If the solution (P^*, A^*) to (2) is such that $A^* > 0$, then P^* solves

$$\max_{P} \pi(P, H, H) - \pi(P, L, H)$$

$$\Rightarrow \frac{\partial \pi(P, H, H)}{\partial P} = \frac{\partial \pi(P, L, H)}{\partial P}$$

- Assume $\pi(P,H,H) - \pi(P,L,H)$ has a maximum in P.
- $A(P) = \pi(P,L,H) - \pi(P^L_L, L, L)$
- The other curve is $\pi(P,H,H) - A$
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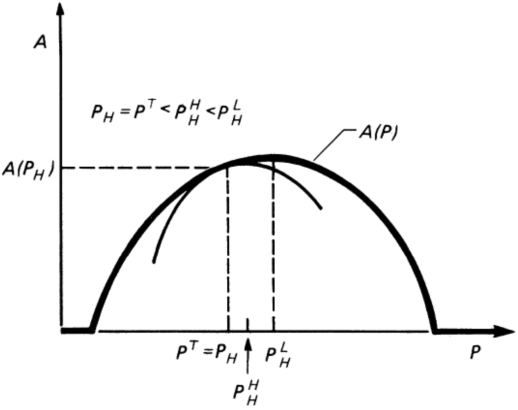
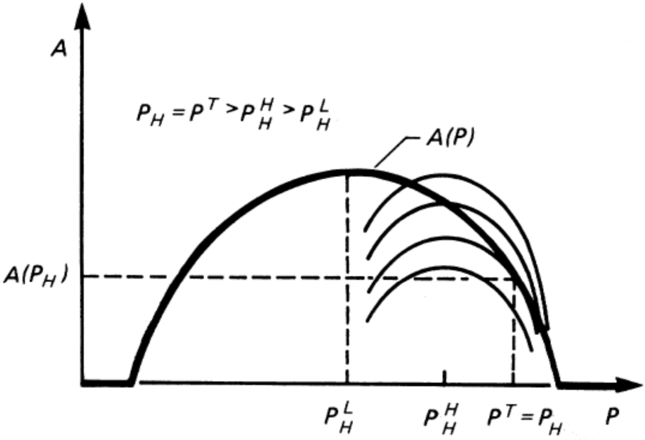


- The separating equilibrium is at the tangency point (P^T, A^T).
- In the case below there is an upward distortion in price $P^T > P_H^H$



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This condition says that an L would willingly set its price at P_H^H if doing it could change its perceived quality from L to H.

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  - **Case in which** $P_H^H > \overline{P}$: If a new high-quality product is very expensive to produce and is aimed at a limited market.
  - **Case in which** $P_H^H < \underline{P}$: If the new high-quality product is very cheap to produce the introducing firm may set a low initial price or give away free samples in launching the product.

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- Together with advertising, a low price may signal a high quality (i.e lower than the high quality monopoly price): it claims that consumers that will taste it won't be disappointed.

Exercise 2

Advertising as a commitment device (Lal and Matutes, 1994)

Assumption

- Firms A and B are located at the extreme of a segment of lenght 1.
- Consumers are uniformly distributed along the segment and incur linear transport cost tx.
- A and B sell two products 1 and 2.
- Consumers have the same willingness to pay for each good, denoted H.
- Unless they receive an ad (catalog, leaflet,...), consumers are uninformed about prices but make rational expectations about prices.
- Each firm can choose to advertise one or two goods. Advertising costs F and vehicles the information about a product's price to all consumers.
- **We exclude that a consumer visit both stores**. this is a symplifying assumption and in the paper they look at all cases!

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  - Once at the store the firm knows that the transportation cost is sunk for the consumer and has an incentive to set a price H.
- Anticipating this, no consumer buy anything and therefore no profit for both firms.

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- This leads to $p_A^* = p_{A1} + p_{A2} = t$ and $p_B = p_{B1} + p_{B2} = t$.

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- The first important condition to check is that $t < 2H$. Then, the profit each firm realizes is $\pi_j = \frac{t}{2} - 2F > 0 \rightarrow F < \frac{t}{4}$.
- Another condition to check is that the marginal consumer has a positive surplus, i.e. that $2H - t - \frac{t}{2} > 0 \rightarrow t < \frac{4H}{3}$ (covered market).
- To check whether this is an equilibrium, we check that a firm, say B, has no incentive to deviate unilaterally by only advertising one of its products, say 1.
  - Consumers rationally expect that a product that is not advertised will be sold at H.

$$\hat{x} = \frac{p_{B1} + H - p_A^* + t}{2t}$$

  - Maximizing its profit $(p_{B1} + H)\hat{x}$ with respect to p_{B1}, we obtain $p_{B1} = t - H$.
  - The profit obtained by firm B is therefore $\pi_B = \frac{t}{2} - F > \frac{t}{2} - 2F$: NO.

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  - We obtain $p_{A1} = p_{B1} = t - H$ and therefore the profit is $\frac{t}{2} - F > 0$.

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- From above it is immediate that there is another symmetric equilibrium in which A advertises 1 and B advertises 2 and conversely.

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Signaling Game

- Player 1 has a private information about his type $\theta \in \Theta$ and chooses a signal $s \in S$.
- Player 2 observes s and chooses an action $b \in B$.
- Player 2 has prior belief about Player 1's type p(.). After observing s, Player 2 revises its beliefs according to the Baye's rule and has a posterior belief $\mu(./s)$ over Θ.
- Player 1 determines $\sigma_1(s/\theta)$, the probability to send a signal s when being of type θ.
- Player 2 determines $\sigma_2(b/s)$, the probability to choose the action b given the signal s and posterior belief $\mu(./s)$.

Definition . A perfect Bayesian equilibrium of a signaling game is a strategy profile (σ_1^*, σ_2^*) in which each player's strategy is the best reaction to the other's strategy according to the posterior beliefs $\mu(./s)$.

A perfect Bayesian equilibrium is such that given a set of receiver's beliefs about the sender's type, the receiver chooses the action that is a best reaction to the message received and the sender chooses a message that is a best reaction to the action of the receiver. back

Types of equilibria

A **separating equilibrium** is an equilibrium where Players 1 of different types always choose different messages and therefore fully reveal their type to Player 2.

A **pooling equilibrium** is an equilibrium where Players 1 of different types always choose the same message and no information is revealed to Player 2.

back

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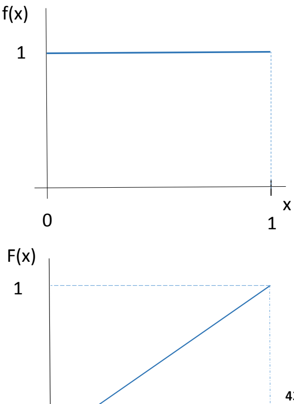
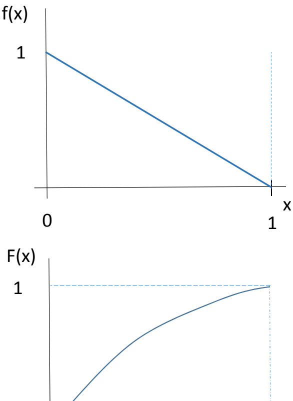
Uniform distribution: $\lambda_1 = \lambda_2$



Distribution in favor of 1: $\lambda_1 = \lambda_2$



back

A perfect Bayesian equilibrium is such that given a set of receiver's beliefs about the sender's type, the receiver chooses the action that is a

	Colgate	P&G CREST
Help reduce Cavities	★ ★ ★	★ ★ ★
Help brush away Plaque	★ ★	★
Prevent Gingivitis	★	★ ★
White teeth	★ ★	★
Fresh feeling	★	★ ★