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When do switching costs make markets more or less competitive?

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ABSTRACT

In a two-period duopoly setting in which switching costs are the only reason why products may be perceived as differentiated, we provide necessary and sufficient conditions for switching costs to lead to higher prices in the first period as well as to higher overall profitability. We show that this happens if and only if switching costs are not too large. We present the only treatment up to date of how switching costs (and only switching costs) affect competition based on the assumption that switching costs differ across consumers, which allows us to illustrate the undesired byproduct of assuming that products exhibit substantial horizontal differentiation. Not only do we draw implications for the classical literature on competition with switching costs, but also for the more recent one that rests upon such an assumption too.

1. Introduction

It is by now very well documented that many consumption decisions involve making sunk investments that are product-specific (or, more generally, seller-specific). This relationship-specificity creates an inertia towards the continued use of the same product, even if there are sellers of substitute products with very similar functional features.

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Such costs of switching suppliers of a product arise in industries as diverse as computers, telecommunications, paid-TV, ketchup, credit cards, cigarettes, or retail banking, and their extent can range from being relatively small to being quite large, depending on the industry.[1] The most obvious effect of switching costs is that, once a consumer starts consuming a given product, its degree of substitutability with respect to competing products decreases, which has an impact on how firms compete for consumers. In fact, firms that foresee that consumers will get (partly) locked into their products in the future will also factor these elements into their current decision-making.

Given their widespread importance, it should come as no surprise that there has been a lot of theoretical work analyzing the impact of switching costs on competitive outcomes in oligopoly.[2] The most crucial insight from this literature stems from Klemperer's (1987a; 1987b) seminal work. In a two-period symmetric duopoly model, he finds that the second-period profit of a firm is increasing in its first-period market share. This leads to very aggressive competition before consumers are locked-in, but competition greatly relaxes afterwards, with the resulting temporal evolution for prices commonly known as the "bargains-then-ripoffs" pattern. This received wisdom has been very influential in forming the conceptual apparatus for full-fledged dynamic models that greatly extend the workhorse two-period models initially considered.

In the quest for a tractable framework, the switching cost literature has had to address significant technical challenges. Indeed, pure-strategy equilibria easily fail to exist in multiperiod models with switching costs owing to endogenously formed discontinuities and asymmetries in demand functions. This drawback has been amended by making products substantially differentiated from a horizontal point of view.[3] When a consumer is locked-in by a firm, it may be really hard for the competitor to steal it away from the firm (unless tastes are largely uncorrelated over time). As a result, fixing a technical problem creates a fundamental artifact in the economics of the situation to be analyzed: competition once consumers are locked-in ends up being *too* soft, so it should not be surprising that second-period profits are found to be monotone increasing in first-period market share.

The objective of this paper is to provide an analysis of the effects of switching costs, *and only switching costs*, on competitive outcomes. In our two-period symmetric duopoly setting based on Chen (1997), consumers are heterogeneous with respect to their switching costs: the only reason why demand from locked-in consumers is somewhat elastic is because they bear different switching costs. Even though this seems a natural feature of many real world markets in which switching costs are present, heterogeneity in such costs

[1] See Calem and Mester (1995); Dubé et al. (2010); Keane (1997); Larkin (2008); Shcherbakov (2016); Shum (2004); Shy (2002) and Viard (2007) for some relevant empirical studies.

[2] Excellent surveys of earlier literature can be found in Klemperer (1995) and Farrell and Klemperer (2007).

[3] See Klemperer (1987b) for the workhorse model widely used in the literature. A complementary way to deal with equilibrium non-existence is to further assume that that most consumers leave the market before they can possibly bear any switching cost, as also done in Klemperer (1987b).

is unusual in the theoretical literature.[4] It is worth noting that empirical work on the theme of switching costs under competition recognizes that switching costs vary across different individuals (see e.g. Kiss, 2014).

The main contribution of this paper is to analyze the competitive effects of switching costs for any level of such costs in the absence of other factors that interact with them in determining the elasticity of consumers' demands. Under the assumption that switching costs are uniformly distributed, we characterize the full equilibrium set for any value taken by the average switching cost and then use a standard equilibrium refinement (Pareto-dominance) to single out a unique equilibrium on which to perform comparative statics. When switching costs are small in expectation, second-period profits do not vary monotonically with first-period market share. Starting from a low market share in the first period, a firm's second-period profit grows as it sells to more consumers because the firm has a larger base of (partly) captive consumers. However, accumulating market share comes at the expense of the rival. So once the firm has accumulated a large enough market share in the first period, the rival will be led to charge lower prices in the second period in face of its shrinking customer base. Therefore, the firm that develops a larger customer base benefits from having more consumers locked-in, but is harmed by having a more aggressive rival. The latter effect dominates the former one once the firm captures at least half of the market in the first period, so second-period profitability eventually declines as a firm attracts more consumers in the first period. Restricting attention to Pareto-dominant equilibria, we find that an increase in (initially low) switching costs leads to higher prices and profits in both the first and the second period of the game. These results persist when consumers are forward looking: indeed, firms charge higher first-period prices and earn greater profits as consumers care *more* about the future, as first shown by Klemperer (1987b).

As (average) switching costs increase, the nonmonotonic relationship between first-period market share and future profits becomes weaker and eventually disappears. In fact, second-period profits grow with a firm's market share in the first period when switching costs are large enough, so the received wisdom applies in these cases in which second-period competition is quite mild. This results in firms initially competing fiercely for market share in equilibrium, in anticipation of high future prices given the soft competition in the second period. Summarizing our results, first-period prices and payoffs first increase with (average) switching costs and then decrease. Also, second-period profit first increases with a firm's first-period market share and then decreases if switching costs are not too large; otherwise, second-period profits always increase with market share.

In his pioneering contribution to the switching cost literature, Klemperer (1987a) does note as a caveat to his insights on the "bargains-then-ripoffs" pattern that second-period profits may decrease with first-period market share. He does not acknowledge though that such profits may in fact vary nonmonotonically with market share. In fact, he does not even support his claims by giving a single example, let alone by giving conditions on

[4] See Biglaiser et al. (2013) for a recent exception, as well as earlier work by Klemperer (1987a).

primitives that may lead to second-period profits decrease with first-period market share. Indeed, the subsequent literature has treated this aspect as a rarity, if not a pathology (for example, Farrell and Klemperer, 2007, do not even mention it as a possibility in their extensive survey). We show that the decreasingness of future profits on current market share, even if it does not hold everywhere, arises naturally once we focus on settings in which the only source of product differentiation are switching costs, provided such costs are low enough. This aspect was already noticed by Chen (1997), but it did not have any implication for his comparison between the situations in which firms use uniform prices and those in which they price-discriminate (reason why the aspect has probably gone unnoticed). We show however that the nonmonotonicity result is crucial in explaining the comparative statics results regarding the effect of switching costs on the intensity of competition.

It is worth noting that we use the model developed by Chen (1997) to derive results that are unrelated to his findings. His seminal contribution is to prove that behavior-based price discrimination lowers payoffs relative to the case of uniform pricing, but he does note that there are several symmetric equilibria under uniform pricing. Characterizing the set of symmetric equilibria is challenging, so he simply proves that the equilibrium set is nonempty and then provides a lower bound on symmetric equilibrium payoffs for firms, which happens to be sufficient for his purpose. Given our goal of performing comparative statics on the switching cost parameter, one must tackle the equilibrium multiplicity differently. More precisely, we fully characterize the equilibrium set and then refine it based on a reasonable criterion, that of Pareto-dominance, so as to be able to compare our predictions with those of the classical literature on the competitive effects of greater switching costs, an aspect not studied by Chen (1997).

The initial literature on the competitive implications of switching costs worked with simple two-period settings in order to shed light on the main issues at play. Our paper contributes to this classical literature, but it also has implications for more recent work on the topic. Indeed, the early insights were used to build a currently well-developed literature on infinite-horizon games that study competition with switching costs and constant arrival/departure of consumers. In these settings, a firm has to consider incentives to extract rents from locked-in consumers as well as from those not yet locked into any firm. When switching costs are large, the incentive of a firm to milk its customer base pushes price up, but the incentive to grow the future customer base creates a tension to lower price. The common wisdom received from past literature is that the incentive to raise prices dominates, so high switching costs soften dynamic competition (see e.g., Beggs and Klemperer, 1992; Padilla, 1995; To, 1995 and Anderson et al., 2004). Subsequent work by Arie and Grieco (2014); Cabral (2016); Dubé et al. (2009); Fabra and García (2015), and Rhodes (2014) has examined the situations that arise if switching costs are not large, which leads to reversing the previous findings (under some conditions). Many of these models rely on there being enough horizontal differentiation between products. If such differentiation is weak and switching costs are not very high, our results suggest that there might be no conflict between capture of newly arrived consumers and rent extraction from consumers who are locked-in: increasing switching costs might have the effect of unambiguously softening dynamic competition. Technically speaking, our results suggest that the widespread focus on linear Markov Perfect Equilibria may have been somewhat restrictive. Thus, monotonicity of equilibrium strategies should be expected just in the case of high switching costs; if it arises when such costs are not high, they might be driven by other assumptions made.[5]

The remainder of the paper is as follows. Section 2 introduces the game-theoretic model. Under the assumption that consumers are myopic, Section 3 solves the game for the case of small switching costs, whereas Section 4 deals with the other cases. Section 5 shows that consumer foresight does not affect our main findings. Section 6 discusses the results, and Section 7 concludes. All proofs not in the text can be found in the Appendix.

2. The model

We consider a two-period game played by two firms labeled 1 and 2. Such firms produce homogeneous goods at no cost and compete in prices at each period of play.[6] There is also a unit mass of ex-ante homogeneous consumers. Each has unit-demand at each period and is willing to pay at most $v > 0$ for one unit of the good. Upon purchasing firm i's good at the initial period of play, a consumer is supposed to bear a random cost $\widetilde{s}_i$ if she switches to firm i's rival $(i = 1, 2)$. In particular, $\widetilde{s}_i$ is uniformly distributed between 0 and $s > 0$, so the fraction of consumers of firm i who have a cost of switching greater than $s_i \in [0, s]$ equals $\int_{s_i}^{s} \frac{dx}{s} = 1 - \frac{s_i}{s}$ $(i = 1, 2)$.[7] Firms do not observe the switching costs of any consumer, but it is common knowledge how they are distributed across the population of consumers. We assume that consumers have a discount factor equal to zero (see Section 5 for a relaxation of this assumption) and firms have a discount factor equal to one. Our solution concept for this game will be subgame perfect Nash equilibrium (in pure strategies).

Although our model is essentially identical to the one used by Chen (1997) when he assumes that firms cannot price-discriminate, we note that we proceed to fully characterize the set of (pure-strategy) subgame perfect Nash equilibria (unlike him). Despite being somewhat technically challenging for some parameter values, this is necessary because we aim at performing comparative statics on the effect of larger s on equilibrium prices and payoffs, showing at the same time why our results differ from the conventional wisdom (e.g., Farrell and Klemperer, 2007). This contrasts with his distinct approach

[5] Other than a significant extent of horizontal differentiation, another standard assumption is that idiosyncratic tastes towards competing products exhibit no persistence over time.

[6] Costless production is without loss of generality if marginal costs are constant (and not too large). Given this normalization, (negative) prices in our model should be interpreted as (negative) markups.

[7] Even though the assumption that some consumers have negligible switching costs seems realistic in quite a few situations, it is worth noting that it is important from a technical point of view: an equilibrium easily fails to exist in the second period if the smallest switching cost borne by consumers is positive but close to zero (see e.g. Klemperer, 1987a, p. 383).

aimed at answering a completely different question ("Does price discrimination result in more intense competition for consumers than uniform pricing?"), so both the current paper and Chen (1997) use basically the same model to derive different insights. Our paper is therefore closer to the classical literature on switching costs, as exemplified by Klemperer (1987b), rather than the literature on behavior-based price discrimination in the presence of switching costs that was pioneered by Chen (1997). We recall that Chen (1997)'s main finding is that firms are worse off when they both engage in such kind of discrimination than when none of them does (see Taylor, 2003, for an extension that shows that more competition when firms price-discriminate lowers prices and profits).

3. Resolution of the model when switching costs are small

We will assume throughout that s/v is so small that all consumers always buy one of the goods at any period of play without being monopolized. In particular, this will simply require that $s/v \leq 1$, so that equilibrium prices are smaller than v. Because we seek to characterize the set of subgame perfect Nash equilibria, we work backwards, starting with the second period of play.

3.1. Second-period competition

We let $\sigma_i \in [0,1]$ denote firm i's customer base, that is, the measure of consumers who purchased firm i's good in the first period, with $\sigma_i \in [0,1]$ for all $i \in \{1,\ 2\}$ and $\sigma_1 + \sigma_2 = 1$. Among the customer base of firm $i \in \{1,\ 2\}$, we can easily compute the measure of consumers who prefer such a firm when it charges p_i over its rival charging $p_{3-i} \leq p_i$: it equals $\Pr(\widetilde{s}_i \geq p_i - p_{3-i}) = (s + p_{3-i} - p_i)/s$. Clearly, all (partially) locked-in consumers prefer firm i over its rival (labeled $3-i$) if $p_{3-i} > p_i$, whereas the converse holds for $p_i \geq p_{3-i} + s$. Therefore, the total demand from consumers in firm i's customer base is:

$$Q_i^{own}(p_i, p_{3-i}) = \begin{cases} \sigma_i & \text{if } p_i - p_{3-i} \leq 0 \\ \sigma_i \left(\dfrac{s + p_{3-i} - p_i}{s} \right) & \text{if } 0 \leq p_i - p_{3-i} \leq s\,. \\ 0 & \text{if } p_i - p_{3-i} \geq s \end{cases}$$

Similarly, the demand from the consumers within the rival's customer base is:[8]

$$Q_i^{rival}(p_i, p_{3-i}) = \begin{cases} \sigma_{3-i} & \text{if } p_i - p_{3-i} \leq -s \\ \sigma_{3-i} \left(\dfrac{p_{3-i} - p_i}{s} \right) & \text{if } -s \leq p_i - p_{3-i} \leq 0\,. \\ 0 & \text{if } p_i - p_{3-i} \geq 0 \end{cases}$$

[8] Note that a firm delivers positive utility to consumers not switching, so a revealed preference argument establishes that consumers who do switch must be attaining a positive utility as well.

As a result, the second-period demand function for firm i is

$$Q_i(p_i, p_{3-i}) = \begin{cases} \sigma_i + \sigma_{3-i} & \text{if } p_i - p_{3-i} \leq -s \\ \sigma_i + \sigma_{3-i}\left(\dfrac{p_{3-i} - p_i}{s}\right) & \text{if } -s \leq p_i - p_{3-i} \leq 0 \\ \sigma_i\left(\dfrac{s + p_{3-i} - p_i}{s}\right) & \text{if } 0 \leq p_i - p_{3-i} \leq s \\ 0 & \text{if } p_i - p_{3-i} \geq s \end{cases},$$

and its profit function is $\pi_i(p_i, p_{3-i}) = p_i Q_i(p_i, p_{3-i})$.

Without loss of generality, let $\sigma_i \geq \sigma_{3-i}$. It is easy to prove that there can exist an equilibrium only in the region in which $0 \leq p_i - p_{3-i} \leq s$, so let us proceed to characterize it. In such a region, $Q_i(p_i, p_{3-i}) = \sigma_i(s + p_{3-i} - p_i)/s$ (i.e., firm i makes fewer sales than in the first period), so $\pi_i(p_i, p_{3-i}) = p_i\sigma_i(s + p_{3-i} - p_i)/s$. Similarly, $Q_{3-i}(p_{3-i}, p_i) = \sigma_{3-i} + \sigma_i(p_i - p_{3-i})/s$ and $\pi_{3-i}(p_{3-i}, p_i) = p_{3-i}[\sigma_{3-i} + \sigma_i(p_i - p_{3-i})/s]$. The first-order (necessary) conditions for an (interior) optimum are:

$$s + p_{3-i} - 2p_i = 0 \tag{1}$$

and

$$\sigma_{3-i} + \sigma_i\left(\frac{p_i - 2p_{3-i}}{s}\right) = 0. \tag{2}$$

Accounting for corner solutions, the respective best response functions for firm i and its rival are

$$b_i(p_{3-i}) = \min\left(v, \frac{s + p_{3-i}}{2}\right) \tag{3}$$

and

$$b_{3-i}(p_i) = \min\left(v, \frac{s(\frac{\sigma_{3-i}}{\sigma_i}) + p_i}{2}\right), \tag{4}$$

since no firm will ever find it optimal to charge a price greater than v. Hence, the game exhibits strategic complementarities, as was to be expected given that firms compete in prices. Also, an increase in firm i's customer base can never make its rival price less aggressively.

We shall seek first for interior equilibria, with the analysis of corner equilibria delayed until Section 4. As we prove in the proposition below, there exists a unique interior equilibrium in which the firm with largest customer base prices higher than its competitor, so the latter steals some consumers in the former's customer base. Further, greater switching costs soften competition, but the firm with largest customer base increases price more than its rival.

Proposition 1. *Let $s \leq v$. Given that $\sigma_i \geq \sigma_{3-i}$, it holds that $p_i^* = \frac{s}{3}(2 + \frac{\sigma_{3-i}}{\sigma_i})$ and $p_{3-i}^* = \frac{s}{3}(1 + 2\frac{\sigma_{3-i}}{\sigma_i})$ in the unique (interior) equilibrium. Profit for firm i is $\pi_i^* =$*

$\frac{s\sigma_i}{9}(2+\frac{\sigma_{3-i}}{\sigma_i})^2$, *whereas profit for firm* $3-i$ *is* $\pi^*_{3-i}=\frac{s\sigma_i}{9}(1+2\frac{\sigma_{3-i}}{\sigma_i})^2$. *It holds that* $p^*_i > p^*_{3-i}$ *and* $\pi^*_i > \pi^*_{3-i}$ *if* $\sigma_i > \sigma_{3-i}$, *whereas* $p^*_i = p^*_{3-i}$ *and* $\pi^*_i = \pi^*_{3-i}$ *if* $\sigma_i = \sigma_{3-i}$.

Proof. See Appendix. □

In order for all consumers to derive a nonnegative utility in an interior equilibrium, we need that $v \geq p^*_i = \frac{s}{3}(2+\frac{\sigma_{3-i}}{\sigma_i})$ regardless of the value taken by $\frac{\sigma_{3-i}}{\sigma_i}$, that is, we need $s \leq v$, which is our working assumption for the moment.[9] Given this assumption, Proposition 1 then applies and it is quite immediate to show the following once one recalls that the market is fully covered (so $\sigma_{3-i} = 1-\sigma_i$).

Corollary 1. *Let* $s \leq v$. *If* $\sigma_i \geq 1/2$, *then both* π^*_i *and* π^*_{3-i} *decrease with* σ_i, *with* π^*_{3-i} *decreasing faster than* π^*_i.

A remarkable feature of Corollary 1 is that π^*_i decreases with σ_i. Even though this aspect was already noticed by Chen (1997), he did not explain this intriguing finding because it did not have any relevant implication for his analysis, but it turns out to be crucial for ours, so let us elaborate on its rationale. Making it explicit that equilibrium prices and profits in the second period are functions of σ_i and σ_{3-i}, so that

$$\pi^*_i(\sigma_i,\sigma_{3-i}) = \sigma_i p^*_i(\sigma_i,\sigma_{3-i})\left(1-\frac{p^*_i(\sigma_i,\sigma_{3-i})-p^*_{3-i}(\sigma_{3-i},\sigma_i)}{s}\right),$$

it follows from the fact that $\sigma_{3-i} = 1-\sigma_i$ and from the envelope theorem that

$$\frac{\partial \pi^*_i}{\partial \sigma_i} = p^*_i\left(1-\frac{p^*_i - p^*_{3-i}}{s}\right) + \frac{\partial \pi_i}{\partial p_{3-i}}\left(\frac{\partial p^*_{3-i}}{\partial \sigma_i} - \frac{\partial p^*_{3-i}}{\partial \sigma_{3-i}}\right).$$

The first term on the right hand side captures the direct effect of increasing firm i's customer base, whereas the second one captures the strategic effect. Because $\sigma_{3-i} = 1-\sigma_i$, it holds that the direct effect equals $(1+\sigma_i)p^*_i/(3\sigma_i) > 0$ and the strategic effect equals $-2p^*_i/(3\sigma_i) < 0$, with $\partial\pi^*_i/\partial\sigma_i < 0$. Therefore, the positive direct effect of increasing σ_i on firm i's second-period profit is more than offset by the negative strategic reaction that increasing σ_i elicits from firm i's rival, since increasing σ_i decreases the size of its customer base and induces it to fight harder to attract firm i's greater customer base given that s is not that large.

Another way to illustrate Corollary 1 is by plotting a firm's second-period profit as a function of its own market share in the first period, given that all consumers transacted

[9] Note that this assumption yields that $v \geq \frac{s+p^*_{3-i}}{2} > \frac{s(\frac{\sigma_{3-i}}{\sigma_i})+p^*_i}{2}$, so best response functions do intersect at interior solutions, as we assumed.

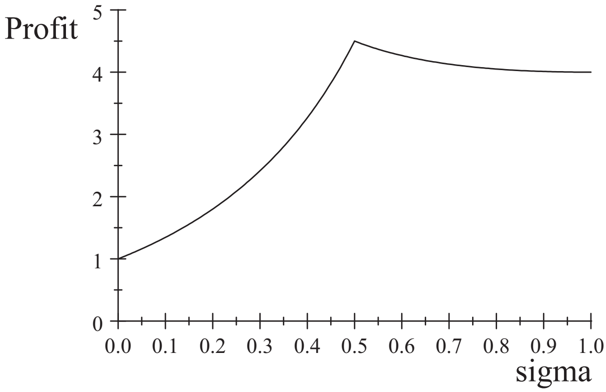

Fig. 1. $\pi^*(\sigma)$ plotted for $s = 9$.

with one of the firms. Denoting this profit function by $\pi^*(\sigma)$, it holds that

$$\pi^*(\sigma) = \begin{cases} \dfrac{s(1-\sigma)}{9}\left(1+\dfrac{2\sigma}{1-\sigma}\right)^2 & \text{if } \sigma \in [0, 1/2] \\ \dfrac{s\sigma}{9}\left(1+\dfrac{1}{\sigma}\right)^2 & \text{if } \sigma \in [1/2, 1] \end{cases},$$

which can be visualized in Fig. 1.

Fig. 1 illustrates the main departure from previous literature (with the exception of Chen, 1997, of course), which has focused on the cases in which second-period profits monotonically increase with first-period market share. As we have shown, the nonmonotonicity of second-period profits may naturally be expected when switching costs are small (recall our maintained assumption that $s \leq v$). Fig. 1 illustrates that there is a tendency towards charging a price that matches that of the rival so that $\sigma = 1/2$ and second-period profits are not too small: when firms have asymmetric customer bases, the small firm is unhappy because it has few consumers in its customer base; even though the large firm has more locked-in customers, the large firm is unhappy too because it faces a very aggressive rival. Note also that a firm that matches the rival's price in the first period faces different incentives depending on whether it is contemplating an upward or a downward deviation: taking into account that products are ex-ante homogeneous, lowering the price by some small amount is preferred over raising it by the very same amount from the point of view of second-period profits. This asymmetry in incentives to lower or raise the price will lead to a multiplicity of symmetric equilibria, as we shall see next.

3.2. First-period competition

We now turn to the first period, keeping in mind that firms anticipate how first-period actions influence second-period competition. We will slightly abuse notation and denote prices with the same notation used for the second period, since second-period prices are not used anymore in this subsection.

Given that products are homogeneous, it is easy to see based on our previous analysis that firm 1's payoff given the price p_1 it charges and the price p_2 charged by its rival is

$$\Pi_1(p_1, p_2) = \begin{cases} p_1 + 4s/9 & \text{if } p_1 < p_2 \\ (p_1 + s)/2 & \text{if } p_1 = p_2 \\ s/9 & \text{if } p_1 > p_2 \end{cases},$$

since $\pi^*(1) = 4s/9$, $\pi^*(1/2) = s/2$ and $\pi^*(0) = s/9$.

We first examine symmetric equilibria in which both firms charge price p^* and each gets half of the consumers. This symmetric pricing will constitute equilibrium behavior if there are no incentives for a firm to slightly decrease or increase its price, that is, if the following two conditions hold at the same time: $(p^* + s)/2 \geq p^* + 4s/9$ and $(p^* + s)/2 \geq s/9$. Therefore, there exists a continuum of symmetric equilibria that can be Pareto-ranked from the firms' points of view: any pair of prices (p^*, p^*) such that $p^* \in [-7s/9, s/9]$ can be charged in equilibrium. Note that our assumption that $v \geq s$ implies that $v \geq p^*$, so we indeed have an equilibrium. Equilibrium profits, denoted by Π^*, range from $s/9$ to $5s/9$, so the equilibrium preferred by firms is the one in which prices are highest, namely, $(p^*, p^*) = (s/9, s/9)$.

To fully characterize the equilibrium set, we now rule out asymmetric equilibria. Suppose now that firm 1 charges p_1^* and firm 2 charges p_2^*, with $p_1^* < p_2^*$, so that firm 1 attracts all consumers. In order for this to constitute an equilibrium, several conditions should be simultaneously satisfied: $p_1^* + 4s/9 \geq (p_2^* + s)/2$, $p_1^* + 4s/9 \geq s/9$, $s/9 \geq (p_1^* + s)/2$ and $s/9 \geq p_1^* + 4s/9$. Thus, firm 1 should have no incentive to either match firm 2's price or price higher than firm 2, whereas firm 2 should have no incentive to either match firm 1's price or slightly undercut firm 1. Because $p_1^* + 4s/9 \geq s/9$ implies $p_1^* \geq -3s/9$ and $s/9 \geq (p_1^* + s)/2$ implies $-7s/9 \geq p_1^*$, it follows that an asymmetric equilibrium cannot exist, and all equilibria must involve market sharing and symmetric pricing, with prices growing over time.

Proposition 2. *Suppose that $s \leq v$. Then (p^*, p^*) constitutes a symmetric equilibrium in the first period if and only if $p^* \in [-7s/9, s/9]$, with profits ranging from $s/9$ to $5s/9$. No other equilibrium exists.*

Underlying any equilibrium there is a strong force that induces firms to behave symmetrically in a way that does not dissipate profits. In fact, a firm that slightly undercuts the competitor discontinuously enhances sales in the short run, but discontinuously intensifies future competitive interaction: when switching costs are not too large, the competitor can still attract some consumers in the second period and will actively try to do so following the undercutting. In some sense, switching costs allow the rival of a firm to credibly commit to punishing the firm if it undercuts the rival's price. It is also worth noting that, despite the multiplicity of symmetric equilibria, the nature of the boundaries of the equilibrium set is very different, each being determined by distinct economic forces that we discussed at the end of the previous subsection. The lower bound on the symmetric equilibrium set arises from ruling out deviations that involve higher prices and hence accumulating a smaller market share, whereas the upper bound arises from ruling out deviations that involve accumulating a larger market share in the first period.

Even though there exist infinitely many equilibria,[10] it seems reasonable from a generic point of view to expect that firms will play the symmetric equilibrium that is Pareto-undominated from their standpoints, a standard equilibrium refinement. Using this refinement to single out a unique subgame perfect Nash equilibrium, we obtain the following result directly from Proposition 2 .[11]

Corollary 2. *Given that $s \leq v$, suppose that firms always coordinate on playing their preferred equilibrium. Then first-period competition is softened as s grows, so payoffs increase.*

Larger s always softens second-period competition, as we showed in Proposition 1. When the Pareto-dominant equilibrium is played, larger s also softens first-period competition. The point is that first-period price cannot be too high in order to dissuade a firm from focusing on short-run profit garnering, even if such short-term focus is associated with lower second-period profitability. Because the harm in second-period profits is greater the higher switching costs are, it follows that higher first-period prices can be sustained as s grows.

Continuing the discussion of Corollary 2, let us point out that the fact that $\pi^*(1/2) > \pi^*(1)$ for $s \in (0, v]$ necessarily implies that first-period price must increase in the switching cost over *some* range of parameter values: when $s = 0$, firms charge an equilibrium price equal to 0, and the fact that $\pi^*(1/2) > \pi^*(1)$ for $s \in (0, v]$ implies that such a price must be positive in a Pareto-dominant equilibrium, so the introduction of switching costs must increase first-period prices. The fact that $\pi^*(1/2) - \pi^*(1)$ always increases with the amount of switching costs shows that this actually happens for the *whole* range of parameter values (provided $s \leq v$ holds).

We conclude this section by noting that Corollary 2 is not driven by the absence of asymmetries in production costs, provided such asymmetries are not excessive. Although we will not show it formally (proof available on request), if firm 2 bears a positive marginal cost of production c in the first period, it holds that several symmetric equilibria exist

[10] We note that the set of symmetric equilibria contains the symmetric equilibrium that would obtain in the absence of switching costs. This holds too when consumers are forward looking.

[11] Harsanyi and Selten (1988) argue in favor of risk-dominance over Pareto-dominance when there is a conflict between both, but this does not happen in our setting. Indeed, firm $i \in \{1, 2\}$ does not care about the level of the symmetric equilibrium price p^* if its rival deviates downwards from such a price; however, when its rival deviates upwards from p^*, firm i has more to gain the greater p^* is. As a result, $(p^*, p^*) = (s/9, s/9)$ both Pareto-dominates and risk-dominates any other symmetric equilibrium, which shows that our approach is quite robust and nothing would change if we replaced Pareto-dominance with risk-dominance as our equilibrium selection criterion.

when $c \le 8s/9$, whereas several asymmetric equilibria in which firm 1 captures all consumers in the first period exist when $c \ge 4s/9$. Symmetric equilibria can be Pareto-ranked in the standard fashion; also, firm 2 is indifferent between all the asymmetric equilibria that may exist, so asymmetric equilibria can be Pareto-ranked as well, since firm 1 prefers higher prices. When the best symmetric equilibrium and the best asymmetric equilibrium coexist (i.e., when $4s/9 \le c \le 8s/9$), both firms prefer the former over the latter, so there always exists a unique Pareto-dominant equilibrium. Because firm 1's price in the best asymmetric equilibrium falls with s, it holds that increasing s lowers first-period prices when $c > 8s/9$, but it increases them when $c \le 8s/9$.

4. Resolution of the model when switching costs are not small

Having examined the equilibrium implications of small switching costs, we turn now to the cases in which $s > v$. Such cases result in corner equilibria in the second stage and were not the object of analysis in Chen (1997). Using the best response functions in (3) and (4), we have the following useful result regarding the second period.

Lemma 1. *Let $s > v$ and suppose that $\sigma_i \ge \sigma_{3-i}$. Then:*

(i) $\frac{v}{s} > \frac{1}{3}(2+\frac{\sigma_{3-i}}{\sigma_i})$ implies that $p_i^* = \frac{s}{3}(2+\frac{\sigma_{3-i}}{\sigma_i})$ and $p_{3-i}^* = \frac{s}{3}(1+2\frac{\sigma_{3-i}}{\sigma_i})$, with $\pi_i^* = \frac{s\sigma_i}{9}(2+\frac{\sigma_{3-i}}{\sigma_i})^2$ and $\pi_{3-i}^* = \frac{s\sigma_i}{9}(1+2\frac{\sigma_{3-i}}{\sigma_i})^2$.

(ii) $\frac{\sigma_{3-i}}{\sigma_i} < \frac{v}{s} \le \frac{1}{3}(2+\frac{\sigma_{3-i}}{\sigma_i})$ implies that $p_i^* = v$ and $p_{3-i}^* = \frac{1}{2}[v+s(\frac{\sigma_{3-i}}{\sigma_i})]$, with $0 < p_i^* - p_{3-i}^* < s$ as well as $\pi_i^* = \frac{\sigma_i v}{2s}[s(2+\frac{\sigma_{3-i}}{\sigma_i})-v]$ and $\pi_{3-i}^* = \frac{\sigma_i}{4s}[v+s(\frac{\sigma_{3-i}}{\sigma_i})]^2$.

(iii) $\frac{v}{s} \le \frac{\sigma_{3-i}}{\sigma_i}$ implies that $p_i^* = v$ and $p_{3-i}^* = v$, with $0 = p_i^* - p_{3-i}^* \le s$, as well as $\pi_i^* = \sigma_i v$ and $\pi_{3-i}^* = \sigma_{3-i} v$.

Proof. See Appendix. □

Letting $\sigma_{3-i} = 1-\sigma_i$ and $\sigma_i = \sigma$ in the previous lemma, and rewriting the conditions that give rise to each of the three cases in terms of σ, we can distinguish two situations, depending on whether switching costs are "large" or "intermediate."

4.1. Large switching costs

Let $s \ge 3v/2$. Then the cases that should be dealt with based on Lemma 1 are $0 \le \sigma \le 1-\frac{s}{v+s}$, $\frac{v}{v+s} \le \sigma \le \frac{s}{v+s}$ and $\frac{s}{v+s} < \sigma \le 1$, so second-period profit for a firm as a function of σ is as follows:

$$\pi^*(\sigma) = \begin{cases} \dfrac{(1-\sigma)}{4s}\left[v + s\left(\dfrac{\sigma}{1-\sigma}\right)\right]^2 & \text{if } 0 \le \sigma \le \dfrac{v}{v+s} \\ \sigma v & \text{if } \dfrac{v}{v+s} \le \sigma \le \dfrac{s}{v+s} \\ \dfrac{\sigma v}{2s}\left[s\left(1+\dfrac{1}{\sigma}\right) - v\right] & \text{if } \dfrac{s}{v+s} < \sigma \le 1 \end{cases}.$$

By contrast with our initial analysis, second-period profit can be easily shown to be monotonic in first-period market share when switching costs are large. The point now is that a small rival that is very aggressive will not matter that much for a large firm with a customer base that is "sticky" and hard to steal away.

Solving for an equilibrium in the first period of the game then leads to the following result.

Proposition 3. *Suppose that $s \ge 3v/2$. Then (p^*, p^*) constitutes a symmetric equilibrium in the first period if and only if $p^* \in [-v(s - v/2)/s, -v(s - v)/s]$, with profits ranging from $v^2/(4s)$ to $v^2/(2s)$. No other equilibrium exists.*

Proof. See Appendix. □

If firms do not play an equilibrium that is Pareto-dominated by another one, it holds that the unique equilibrium played when switching costs are large exhibits the following properties.

Corollary 3. *Suppose that firms always coordinate on playing their preferred equilibrium. Given that $s \ge 3v/2$, first-period competition toughens as s grows, with overall payoffs decreasing.*

As switching costs increase, both price and profits decrease in the Pareto-dominant equilibrium, and indeed profits vanish as $s \to \infty$, since $p^* \to -v$ in such a case, as is well-known.

4.2. Intermediate switching costs

We conclude our analysis by studying the cases in which $s \in (v, 3v/2)$. Since $\sigma_{3-i} = 1 - \sigma_i$ and $\sigma_i = \sigma$, Lemma 1 yields the following subintervals for market share: $0 \le \sigma \le 1 - \dfrac{s}{3v-s}$, $\dfrac{3v-2s}{3v-s} \le \sigma \le 1 - \dfrac{s}{v+s}$, $\dfrac{v}{v+s} \le \sigma \le \dfrac{s}{v+s}$, $\dfrac{s}{v+s} < \sigma \le \dfrac{s}{3v-s}$ and $\dfrac{s}{3v-s} < \sigma \le 1$. Accounting for the profit made by a firm in each of these subintervals, we can easily construct its second-period profit as a function of first-period market share:

$$\pi^*(\sigma) = \begin{cases} \dfrac{s(1-\sigma)}{9}\left(1+\dfrac{2\sigma}{1-\sigma}\right)^2 & \text{if } 0 \leq \sigma \leq \dfrac{3v-2s}{3v-s} \\ \dfrac{(1-\sigma)}{4s}\left[v+s\left(\dfrac{\sigma}{1-\sigma}\right)\right]^2 & \text{if } \dfrac{3v-2s}{3v-s} \leq \sigma \leq \dfrac{v}{v+s} \\ \sigma v & \text{if } \dfrac{v}{v+s} \leq \sigma \leq \dfrac{s}{v+s} \\ \dfrac{\sigma v}{2s}\left[s\left(1+\dfrac{1}{\sigma}\right)-v\right] & \text{if } \dfrac{s}{v+s} < \sigma \leq \dfrac{s}{3v-s} \\ \dfrac{s\sigma}{9}\left(1+\dfrac{1}{\sigma}\right)^2 & \text{if} \dfrac{s}{3v-s} < \sigma \leq 1 \end{cases}.$$

As in the small switching cost case, this function is again nonmonotonic: it first increases as σ grows, peaks at $\frac{s}{3v-s} \in (\frac{1}{2}, 1)$ and then decreases as σ is further increased. Solving the first period of the game leads to the following equilibrium characterization for the cases in which $s \in (v, 3v/2)$.

Proposition 4. *Suppose that $s \in (v, 3v/2)$. Then (p^*, p^*) constitutes a symmetric equilibrium in the first period if and only if $p^* \in [2s/9 - v, v - 8s/9]$, with profits ranging from $s/9$ to $v - 4s/9$. No other equilibrium exists.*

Proof. See Appendix. □

Assuming that firms do not play Pareto-dominated equilibria singles out a unique equilibrium with the following properties.

Corollary 4. *Suppose that firms always coordinate on playing their preferred equilibrium. Given that $s \in (v, 3v/2)$, first-period competition toughens as s grows, with overall payoffs decreasing.*

As in Corollary 2, first-period prices are positive if s is slightly above v (in particular, if $s \in (v, 9v/8)$) because $\pi^*(1/2) > \pi^*(1)$ in such cases; however, prices/markups in the first period are nonpositive if $s \in [9v/8, 3v/2)$ because equally sharing the market creates a strong incentive to become a large firm in the second period, and the only way to deter deviations is to make consumer attraction costly in the first period. Unlike Corollary 2, it is worth noting that first-period prices fall with switching costs when s is at an intermediate level. This happens because the second-period profit of a firm that sells to all consumers grows with s, so the payoff to deviating grows with s, and the only manner to make such deviations less tempting is to reduce the price/markup charged in the first period.

5. Extension to forward looking consumers

The purpose of this section is to analyze how equilibrium pricing is affected by dropping the assumption that consumers are myopic. We will work separately for the cases in which $s \leq v$ and $s > v$ because a (symmetric) equilibrium fails to exist in the latter case.

Assuming for now that $s \leq v$ (switching costs are "small"), the first-period demand of consumers becomes more inelastic as they become more forward looking, so the prices that can be sustained in a symmetric equilibrium should be greater in principle. We proceed to show that this is partly true: the highest price that can be sustained in a symmetric equilibrium increases, but the lowest one does not vary. As first pointed out by Klemperer (1987b), firms might perfectly obtain greater profits as consumers become better foresighted. Indeed, this will be the case if firms coordinate on playing the Pareto-dominant symmetric equilibrium.

To formally demonstrate all this, suppose from now on that consumers discount future utility at rate $\delta \in [0, 1]$.[12] In order to avoid problems of equilibrium non-existence in the second period,[13] suppose also that consumers do not learn their switching costs until they consume one product. As Chen (1997, p. 895) discusses, this assumption can be quite realistic in several situations, since consuming a product for the first time can allow a consumer to learn several factors that are unknown at the time of first purchase but are nevertheless relevant for the decision of whether or not to switch to another firm. For example, someone who purchases a computer program for the first time need not know how costly it is for him/her to learn how to operate it, how the program interacts or complements other programs (s)he has, how many computing resources are required for the program to function, how much (s)he cares about bugs whose extent is not known in advance, etc. All these uncertainties are likely to be resolved right upon consuming the product, but not before.

In order to solve the model with forward looking consumers who do not know in advance their realized switching cost and hence are ex-ante identical, note that none of them can individually affect second-period competition. Hence, a consumer's expected future net utility when dealing with firm $i \in \{1, 2\}$ if its market share equals $\sigma \geq 1/2$ can be computed as follows:

$$u_i(\sigma) = \int_0^{p_i^* - p_{3-i}^*} (v - p_{3-i}^* - x)\frac{dx}{s} + \int_{p_i^* - p_{3-i}^*}^{s} (v - p_i^*)\frac{dx}{s}$$
$$= v - p_i^* + \frac{(p_i^* - p_{3-i}^*)^2}{2s}$$

[12] The case we analyzed earlier corresponds to consumers being myopic (i.e., $\delta = 0$), a case not studied by Chen (1997).

[13] The proof of non-existence of second-period Nash equilibria when each consumer knows her switching cost at the outset of the game can be made available upon request.

$$= v - \frac{s}{3}\left(1 + \frac{1}{\sigma}\right) + \frac{s}{18}\left(2 - \frac{1}{\sigma}\right)^2.$$

In turn, dealing with firm $3-i$ if its market share equals $1-\sigma \leq 1/2$ delivers the following future utility to a consumer:

$$\begin{aligned} u_{3-i}(1-\sigma) &= v - p^*_{3-i} \\ &= v - \frac{s}{3}\left(\frac{2}{\sigma} - 1\right). \end{aligned}$$

So the total utility of a consumer who buys from firm $i \in \{1, 2\}$ when it charges p_i and attracts $\sigma \geq 1/2$ consumers equals:

$$U_i(\sigma) = v - p_i + \delta u_i(\sigma).$$

Its rival delivers the following utility:

$$U_{3-i}(1-\sigma) = v - p_{3-i} + \delta u_{3-i}(1-\sigma).$$

When consumers are forward looking, a firm's demand function becomes continuous, since consumers as a whole do not respond to lower prices by massively switching consumption, in anticipation of the competitive imbalance that would otherwise result in the second period.

Lemma 2. *When $s \leq v$, it holds that firm 1's demand is as follows:*

$$\sigma(p_1, p_2) = \begin{cases} 1 & \text{if } p_1 < p_2 - 5\delta s/18 \\ \dfrac{1+\sqrt{9-\frac{18(p_2-p_1)}{\delta s}}}{8-\frac{18(p_2-p_1)}{\delta s}} & \text{if } p_2 - 5\delta s/18 < p_1 \leq p_2 \\ 1 - \dfrac{1+\sqrt{9-\frac{18(p_1-p_2)}{\delta s}}}{8-\frac{18(p_1-p_2)}{\delta s}} & \text{if } p_2 \leq p_1 < p_2 + 5\delta s/18 \\ 0 & \text{if } p_1 > p_2 + 5\delta s/18 \end{cases},$$

with firm 2's demand equal to $1 - \sigma(p_1, p_2)$.

Proof. See Appendix. □

If one restricts attention to symmetric equilibria, it can be readily shown that forward looking behavior by consumers simply increases the largest price that can be sustained in a symmetric equilibrium, as the following proposition shows.

Proposition 5. *Suppose that $s \leq v$. Then it holds that (p^*, p^*) constitutes a symmetric equilibrium in the first period if and only if $p^* \in [-\frac{7s}{9}, \frac{(1+5\delta)s}{9}]$, with profits ranging from $\frac{s}{9}$ to $\frac{5(2+\delta)s}{18}$.*

Proof. See Appendix. □

If we restrict attention to symmetric equilibria that are not Pareto-dominated by others, then we obtain the following corollary.

Corollary 5. *Suppose that firms always coordinate on playing their preferred equilibrium. Given that $s \leq v$, first-period competition is softened as δ or s grow, so payoffs increase.*

When $s \leq v$, consumer foresight simply adds an extra force that reinforces the one that gives rise to Corollary 2, namely, the inelasticity of first-period demand that is generated by consumers realizing that price cuts in the first period are partly intended to lock them in later on.

When switching costs are not "small" ($s > v$), first-period demands are not continuous and there exist no symmetric equilibria, as the following proposition demonstrates.

Proposition 6. *Suppose that $s > v$. Then the set of symmetric equilibria is empty if and only if $\delta > 0$.*

No symmetric equilibrium exists when consumers are not completely myopic for $s > v$, and the complexity of demand functions makes the task of characterizing mixed-strategy equilibria quite formidable.[14] As emphasized by Rhodes (2014), though, average switching costs found in the empirical literature rarely exceed 50% of the price paid by consumers, so the fact that market prices cannot exceed valuations yields that $s \leq v$ is a remarkably weak assumption. This lends support to our finding that increasing switching costs softens competition to lock possibly foresighted consumers into a firm when such costs are "small" (Corollary 5), which is at odds with common wisdom. The next section elaborates on how the assumption of substantial product differentiation drives the results derived by past literature.

6. Discussion of the results

The model we have just solved allows us to derive results on how switching costs, and only switching costs, affect competition and payoffs (in fact, notice that there is no loss of generality in letting $v = 1$). We have seen that the competitive implications of switching costs critically depend on how the relationship between second-period profitability and

[14] It is also hard to know whether asymmetric equilibria exist, even though it is not very hard to show that equilibria in which one of the firms corners the market do not exist.

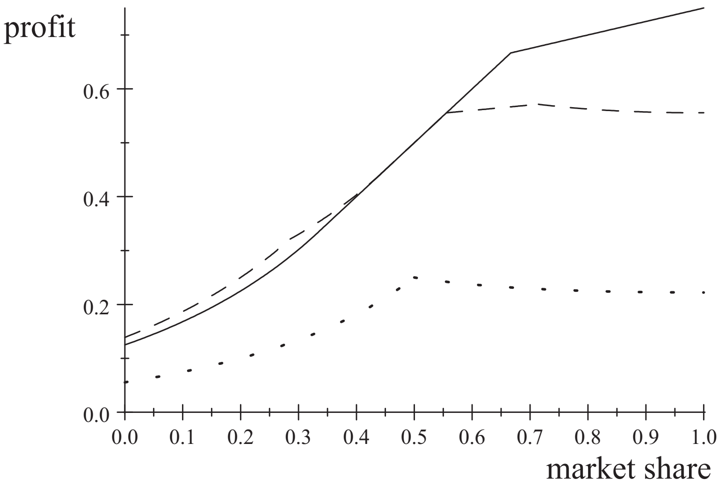

Fig. 2. Second-period profits as a function of first-period market share for $v = 1$ as well as $s = 1/2$ (dotted curve), $s = 5/4$ (dashed curve) and $s = 2$ (solid curve).

first-period market share is affected by s. In the case of myopic consumers, we can readily see in Fig. 2 that its nonmonotonic relationship for low switching costs is mitigated as s grows, eventually becoming monotone increasing.

The change in this relationship between second-period profit and market share in the first period is due to the change in the demand elasticity of firms' customer bases: as s increases, such demand functions become more inelastic. Previous literature has made technical assumptions that have the undesired byproduct of making second-period demands become very inelastic, so they correspond to the large switching cost case we have analyzed when consumers are myopic.[15] Such literature has remained silent on the standalone effects of switching costs on competition when they are not too large. As we have shown, demands can naturally be elastic enough so that a large firm will care negatively about accumulating more market share because of the negative strategic reaction elicited on the rival. It is not innocuous to assume that customer bases are quite "sticky" for reasons other than switching costs. For small switching costs, we have shown (c.f. Proposition 1) that the small firm charges a higher price as it gathers more market share, whereas the converse happens for a large firm that gathers more market share. As a result, prices are not monotone in market shares, which may have substantive implications for the current literature on infinite-horizon models of competition with switching costs and arrival of new consumers over time. In particular, the standard focus on prices that monotonically vary with market share probably holds because products are significantly

[15] In fact, when $t = 0$ in Klemperer's (1987b) classical paper, the sufficient condition that he mentions is that the switching cost be large enough relative to the valuation of the product (see top of p. 143 in Klemperer, 1987b). When $t > 0$, he can examine the case of small switching costs, but these results are driven by the existence of horizontal differentiation, as we will argue shortly.

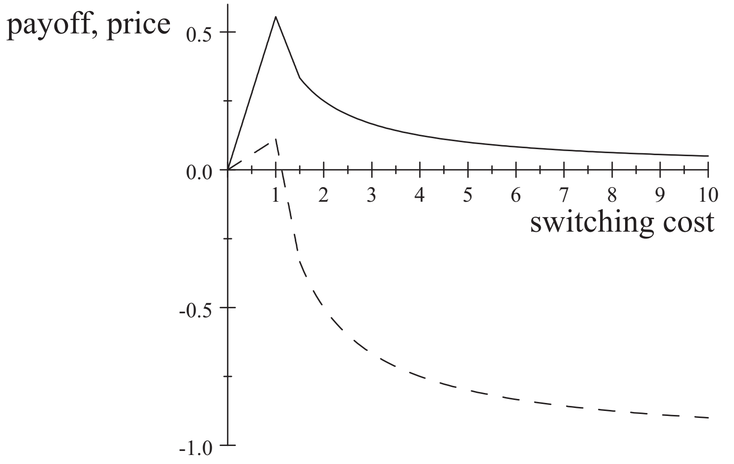

Fig. 3. First-period price (dashed curve) and overall payoffs (solid curve) as a function of s for $v = 1$.

differentiated from a horizontal standpoint: our findings suggest that low switching costs might have different effects from the ones unveiled so far.

Having a framework for examining how switching costs, and only switching costs, affect competitive outcomes is valuable in that we can study whether firms benefit from competing for consumers who bear costs of switching across different products. Our results, based on selecting out the Pareto-undominated equilibrium as the unique one, unambiguously indicate that greater switching costs increase first-period prices and overall profits when switching costs are small and reduce them otherwise. This is represented in Fig. 3.

The traditional message from the literature that switching costs intensify first-period competition is therefore driven by the assumption that customer bases are very sticky because switching costs are very large or for reasons other than the switching costs themselves. When customer bases are easy to steal away, switching costs need not intensify competition in the first period and need not be detrimental for overall profitability. Our results are robust to introducing horizontal differentiation provided it is not too strong (see supplementary material), so an empirical finding that introductory prices decrease with switching costs would point to either switching costs being sufficiently high or products being horizontally differentiated enough.

7. Conclusion

In a two-period game played by two symmetric firms and by consumers who differ in their switching costs, we have analyzed how increasing (average) switching costs affects first-period competition and firms' payoffs. When switching costs are low, a firm that increases its market share in the first period need not enhance future profitability, a situation overlooked by past literature, and the unique Pareto-undominated equilibrium is such that greater (average) switching costs lead to greater prices and profits in both periods. When switching costs are high, though, greater (average) switching costs lead to a lower price in the first period and a lower overall profitability.

We have analyzed how switching costs affect competition without making any assumption on the degree of horizontal differentiation between products. Such an assumption is a key feature of previous literature on the topic, including the more recent one using infinite-horizon models. Indeed, there is a currently ongoing debate on whether higher switching costs always lead to higher prices and profits in dynamic settings with continuous arrival of new consumers over time. Our results seem to indicate that a positive relationship between prices and switching costs may be expected for small switching costs,[16] but the result for larger switching costs is in principle ambiguous. We hope our model can be used as a building block for future analyses that clear this issue.

Another relevant extension that would be worthwhile pursuing is to somehow allow for market expansion effects. In the equilibrium of our model, no consumer switches or is left out of the market, so a change in switching costs simply redistributes overall surplus created between firms and consumers. Our point was to analyze how changes in switching costs shape competition and firms' payoffs. Examining their effect on social welfare is an important issue that is very challenging and is left for future research on the topic.

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Appendix

Proof of Proposition 1. Solving the system that consists of Eqs. (1) and (2) yields $p_i^* = \frac{s}{3}(2 + \frac{\sigma_{3-i}}{\sigma_i})$ and $p_{3-i}^* = \frac{s}{3}(1 + 2\frac{\sigma_{3-i}}{\sigma_i})$, with $0 \leq p_i^* - p_{3-i}^* \leq s$ because of our working assumption that $\sigma_i \geq \sigma_{3-i}$. Also, $\pi_i(p_i^*, p_{3-i}^*) = \frac{s\sigma_i}{9}(2 + \frac{\sigma_{3-i}}{\sigma_i})^2$ and $\pi_{3-i}(p_{3-i}^*, p_i^*) = \frac{s\sigma_i}{9}(1 + 2\frac{\sigma_{3-i}}{\sigma_i})^2$, so $\pi_{3-i}^* \leq \pi_i^*$, with equality if and only if $\sigma_i = \sigma_{3-i}$. It only remains to show that, keeping the rival's pricing strategy fixed, neither firm has an incentive to charge a substantially different price from the one it is supposed to.

[16] It can be shown (see supplementary material) that our results also hold in a simple infinite-horizon model in which firms compete for a single consumer whose switching cost at each period of play is an independent random variable uniformly distributed between 0 and $s > 0$.

We start by ruling out profitable deviations by firm $3-i$, taking into account that

$$\pi_{3-i}(p_{3-i},p_i^*)=\begin{cases} p_{3-i}(\sigma_{3-i}+\sigma_i) & \text{if } p_{3-i}-p_i^*\le -s \\ p_{3-i}\left[\sigma_{3-i}+\sigma_i\left(\dfrac{p_i^*-p_{3-i}}{s}\right)\right] & \text{if } -s\le p_{3-i}-p_i^*\le 0 \\ p_{3-i}\sigma_{3-i}\left(\dfrac{s+p_i^*-p_{3-i}}{s}\right) & \text{if } 0\le p_{3-i}-p_i^*\le s \\ 0 & \text{if } p_{3-i}-p_i^*\ge s \end{cases}$$

is a continuous function (even if it is not differentiable at three points). We have just shown that the optimal price to be charged by firm $3-i$ within $[p_i^*-s,p_i^*]$ is p_{3-i}^*. Also, it is clear that charging a price below p_i^*-s cannot be profitable.[17] So the only deviations to be considered by firm $3-i$ would involve (substantial) increases in the price so as to focus on milking its customer base, but not above p_i^*+s, since they would yield a zero payoff. Note that $p_{3-i}\sigma_{3-i}(s+p_i^*-p_{3-i})/s$ is maximized at $\widehat{p}_{3-i}\equiv(s+p_i^*)/2$, which is always smaller than p_i^*+s. Also, $\widehat{p}_{3-i}\ge p_i^*$, so $\pi_{3-i}(p_{3-i},p_i^*)$ has a local maximum at $\widehat{p}_{3-i}\in[p_i^*,p_i^*+s)$, and we need to show that

$$\pi_{3-i}(p_{3-i}^*,p_i^*)=\frac{s\sigma_i}{9}\left(1+2\frac{\sigma_{3-i}}{\sigma_i}\right)^2\ge\pi_{3-i}(\widehat{p}_{3-i},p_i^*)=\frac{s\sigma_{3-i}}{36}\left(5+\frac{\sigma_{3-i}}{\sigma_i}\right)^2$$

in order to rule out a profitable deviation for firm $3-i$. This inequality holds if and only if

$$\left(2+4\frac{\sigma_{3-i}}{\sigma_i}\right)^2\ge\frac{\sigma_{3-i}}{\sigma_i}\left(5+\frac{\sigma_{3-i}}{\sigma_i}\right)^2,$$

which is always satisfied for any $\dfrac{\sigma_{3-i}}{\sigma_i}\in[0,1]$.[18]

We conclude by ruling out profitable deviations by firm i, taking into account that

$$\pi_i(p_i,p_{3-i}^*)=\begin{cases} p_i(\sigma_i+\sigma_{3-i}) & \text{if } p_i-p_{3-i}^*\le -s \\ p_i[\sigma_i+\sigma_{3-i}(\dfrac{p_{3-i}^*-p_i}{s})] & \text{if } -s\le p_i-p_{3-i}^*\le 0 \\ p_i\sigma_i(\dfrac{s+p_{3-i}^*-p_i}{s}) & \text{if } 0\le p_i-p_{3-i}^*\le s \\ 0 & \text{if } p_i-p_{3-i}^*\ge s \end{cases}$$

is a continuous function (even if it is not differentiable at three points). We have shown that the optimal price to be charged by firm i within $[p_{3-i}^*,p_i^*+s]$ is p_i^*. Also, it is clear that charging a price above p_{3-i}^*+s cannot be profitable. So the only deviations to be considered by firm i would involve (substantial) drops in the price so as to focus on capturing some of the rival's customer base. Note that $p_i[\sigma_i+\sigma_{3-i}(\dfrac{p_{3-i}-p_i}{s})]$ is

[17] Given the continuity of the payoff function, this follows from the facts that $\pi_{3-i}(p_{3-i}^*,p_i^*)>\pi_{3-i}(p_i^*-s,p_i^*)$ and that $\pi_{3-i}(p_{3-i},p_i^*)$ is everywhere increasing on the set $[0,p_i^*-s]$.

[18] To show this, let $y\equiv\sigma_{3-i}/\sigma_i$ and define $\Gamma(y)\equiv(2+4y)^2-y(5+y)^2$. Because $\Gamma(0)>0=\Gamma(1)$, $\Gamma'(1)=0$ and $\Gamma''(y)=6(2-y)>0$ for $y\in[0,1]$, it holds that $\Gamma(y)\ge0$ for all $y\in[0,1]$.

maximized at $\widehat{p}_i \equiv (\frac{\sigma_i}{\sigma_{3-i}} s + p^*_{3-i})/2$, where $\widehat{p}_i \geq p^*_{3-i}$.[19] Because $\widehat{p}_i > p^*_{3-i} - s$, charging a price below $p^*_{3-i} - s$ cannot be profitable. Also, $\widehat{p}_i \geq p^*_{3-i}$ implies that $\pi_i(p_i, p^*_{3-i})$ is everywhere increasing on the set $[p^*_{3-i} - s, p^*_{3-i}]$, and hence $\pi_i(p_i, p^*_{3-i})$ is increasing for $p_i < p^*_i$, which shows that no profitable deviation exists for firm i either. □

Proof of Lemma 1. We have already characterized interior equilibria in Proposition 1 (case (i)), so we now seek for equilibria in which $p^*_i = v$ and $p^*_{3-i} = \min(v, \frac{1}{2}(p_i + s(\dfrac{\sigma_{3-i}}{\sigma_i})))$ (i.e., cases (ii) and (iii)). In order for firm i to be indeed best responding, we need that $v \leq \dfrac{s + p^*_{3-i}}{2}$ holds, so we distinguish two cases:

(a) Suppose that $p^*_{3-i} = \min(v, \dfrac{1}{2}(v + s(\dfrac{\sigma_{3-i}}{\sigma_i}))) = \dfrac{1}{2}(v + s(\dfrac{\sigma_{3-i}}{\sigma_i})) < v$. The latter inequality implies that $v \leq \dfrac{s + p^*_{3-i}}{2} = \dfrac{2s + s(\dfrac{\sigma_{3-i}}{\sigma_i}) + v}{4}$ can be equivalently written as $\dfrac{v}{s} \leq \dfrac{1}{3}(2 + \dfrac{\sigma_{3-i}}{\sigma_i})$. Because $\dfrac{1}{2}(v + s(\dfrac{\sigma_{3-i}}{\sigma_i})) < v$ is equivalent to $\dfrac{\sigma_{3-i}}{\sigma_i} < \dfrac{v}{s}$, this case therefore arises when $\dfrac{\sigma_{3-i}}{\sigma_i} < \dfrac{v}{s} \leq \dfrac{1}{3}(2 + \dfrac{\sigma_{3-i}}{\sigma_i})$. It remains to rule out incentives to perform large price deviations. Clearly, firm i has no incentive to increase the price, but it might be tempted to lower it a lot. To rule this possibility out, note that $p_i[\sigma_i + \sigma_{3-i}(\dfrac{p_{3-i} - p_i}{s})]$ is maximized at $\widehat{p}_i \equiv (\frac{\sigma_i}{\sigma_{3-i}} s + p^*_{3-i})/2$, which can never be smaller than $p^*_{3-i} > p^*_{3-i} - s$.[20] This shows that $\pi_i(p_i, p^*_{3-i})$ is increasing for $p_i < p^*_i$, which proves that no profitable deviation exists for firm i. Regarding incentives to deviate by firm $3 - i$, the only deviations that firm $3 - i$ would contemplate would involve (substantial) increases in the price so as to focus on milking its customer base, but not above v, since they would yield a zero payoff. Note that $p_{3-i}\sigma_{3-i}(s + p^*_i - p_{3-i})/s$ is maximized at $\widehat{p}_{3-i} \equiv (s + p^*_i)/2$, which is always greater than v, so the optimal deviation by firm $3 - i$ would involve charging v, which was ruled out as being preferred over p^*_{3-i}.

(b) Suppose now that $p^*_{3-i} = \min(v, \dfrac{1}{2}(v + s(\frac{\sigma_{3-i}}{\sigma_i}))) = v \leq \dfrac{1}{2}(v + s(\dfrac{\sigma_{3-i}}{\sigma_i}))$. The latter inequality implies that $v \leq \dfrac{s + p^*_{3-i}}{2} = \dfrac{s + v}{2}$ can be equivalently written as $\dfrac{v}{s} \leq 1$. Because $v < \dfrac{1}{2}(v + s(\dfrac{\sigma_{3-i}}{\sigma_i}))$ is equivalent to $\dfrac{v}{s} \leq \dfrac{\sigma_{3-i}}{\sigma_i}$, the working assumption that $\dfrac{\sigma_{3-i}}{\sigma_i} \leq 1$ yields that this case arises when $\dfrac{v}{s} \leq \dfrac{\sigma_{3-i}}{\sigma_i}$. It remains to rule out incentives to undertake large price drops. Clearly, firm i has no incentive to increase the

[19] To demonstrate this, note that $\widehat{p}_i \geq p^*_{3-i}$ is equivalent to $2(\dfrac{\sigma_{3-i}}{\sigma_i})^2 + \dfrac{\sigma_{3-i}}{\sigma_i} - 3 \leq 0$. Let $y \equiv \sigma_{3-i}/\sigma_i \in [0, 1]$ and define $\Theta(y) \equiv 2y^2 + y - 3$. Because $\Theta(0) < 0 = \Theta(1)$ and $\Theta''(y) > 0$, it holds that $\Theta(y) \leq 0$ for all $y \in [0, 1]$, as desired.

[20] To demonstrate this, note that $\widehat{p}_i \geq p^*_{3-i}$ is equivalent to $\dfrac{v}{s} \leq 2\dfrac{\sigma_i}{\sigma_{3-i}} - \dfrac{\sigma_{3-i}}{\sigma_i}$. Since we are studying the cases in which $\dfrac{v}{s} \leq \dfrac{1}{3}(2 + \dfrac{\sigma_{3-i}}{\sigma_i})$ and $\dfrac{\sigma_i}{\sigma_{3-i}} \geq 1$ implies that $\dfrac{1}{3}(2 + \dfrac{\sigma_{3-i}}{\sigma_i}) \leq 2\dfrac{\sigma_i}{\sigma_{3-i}} - \dfrac{\sigma_{3-i}}{\sigma_i}$, it follows that $\widehat{p}_i \geq p^*_{3-i}$ is always fulfilled.

price, but it might be tempted to lower it a lot. To rule this possibility out, note that $p_i[\sigma_i + \sigma_{3-i}(\frac{p_{3-i} - p_i}{s})]$ is maximized at $\widehat{p}_i \equiv (\frac{\sigma_i}{\sigma_{3-i}} s + p^*_{3-i})/2$, which can never be smaller than $p^*_{3-i} > p^*_{3-i} - s$.[21] This shows that $\pi_i(p_i, p^*_{3-i})$ is increasing for $p_i < p^*_i$, which shows that no profitable deviation exists for firm i. Regarding incentives to deviate by firm $3-i$, $p_{3-i}[\sigma_{3-i} + \sigma_i(\frac{p^*_i - p_{3-i}}{s})]$ is maximized at $\widehat{p}_{3-i} \equiv (\frac{\sigma_{3-i}}{\sigma_i} s + p^*_i)/2$, which can never be smaller than $p^*_i = v > p^*_i - s$ because we are studying the cases in which $\frac{v}{s} \leq \frac{\sigma_{3-i}}{\sigma_i}$. This shows that $\pi_{3-i}(p_{3-i}, p^*_i)$ is increasing for $p_{3-i} < p^*_{3-i}$, which shows that no profitable deviation exists for firm $3-i$ either.

Therefore, $\frac{v}{s} \leq \frac{\sigma_{3-i}}{\sigma_i}$ implies that $p^*_i = v$ and $p^*_{3-i} = v$, with $0 = p^*_i - p^*_{3-i} \leq s$, as well as $\pi^*_i = \sigma_i v$ and $\pi^*_{3-i} = \sigma_{3-i} v$. In turn, $\frac{\sigma_{3-i}}{\sigma_i} < \frac{v}{s} \leq \frac{1}{3}(2 + \frac{\sigma_{3-i}}{\sigma_i})$ implies that $p^*_i = v$ and $p^*_{3-i} = \frac{1}{2}[v + s(\frac{\sigma_{3-i}}{\sigma_i})]$, with $0 < p^*_i - p^*_{3-i} < s$ as well as $\pi^*_i = \frac{\sigma_i v}{2s}[s(2 + \frac{\sigma_{3-i}}{\sigma_i}) - v]$ and $\pi^*_{3-i} = \frac{\sigma_i}{4s}[v + s(\frac{\sigma_{3-i}}{\sigma_i})]^2$. □

Proof of Proposition 3. Because firm 1's payoff as a function of first-period prices p_1 and p_2 is

$$\Pi_1(p_1, p_2) = \begin{cases} p_1 + v(2s - v)/(2s) & \text{if } p_1 < p_2 \\ (p_1 + v)/2 & \text{if } p_1 = p_2 \\ v^2/(4s) & \text{if } p_1 > p_2 \end{cases},$$

any symmetric equilibrium price p^* must satisfy both $(p^* + v)/2 \geq p^* + v(2s - v)/(2s)$ and $(p^* + v)/2 \geq v^2/(4s)$, that is, $-v(s - v/2)/s \leq p^* \leq -v(s - v)/s$, with profits ranging from $v^2/(4s)$ to $v^2/(2s)$. Regarding asymmetric equilibria, the following conditions must be satisfied by an equilibrium (p^*_1, p^*_2) in which firm 1 captures all consumers: $p^*_1 + v(2s - v)/(2s) \geq (p^*_2 + v)/2$, $p^*_1 + v(2s - v)/(2s) \geq v^2/(4s)$, $v^2/(4s) \geq (p^*_1 + v)/2$ and $v^2/(4s) \geq p^*_1 + v(2s - v)/(2s)$. Because $p^*_1 + v(2s - v)/(2s) \geq v^2/(4s) \geq p^*_1 + v(2s - v)/(2s)$ implies that $p^*_1 = v(3v - 4s)/(4s)$, so we cannot possibly have $v^2/(4s) \geq (p^*_1 + v)/2 = 3v^2/(8s)$. As in the case in which $s \leq v$, no asymmetric equilibrium exists, which concludes the proof. □

Proof of Proposition 4. One can easily construct firm 1's payoff in the first period with the aid of $\pi^*(\sigma)$:

$$\Pi_1(p_1, p_2) = \begin{cases} p_1 + 4s/9 & \text{if } p_1 < p_2 \\ (p_1 + v)/2 & \text{if } p_1 = p_2 \\ s/9 & \text{if } p_1 > p_2 \end{cases}.$$

Also, it is easy to see that a symmetric equilibrium in which firms charge p^* must satisfy the following two conditions: $(p^* + v)/2 \geq p^* + 4s/9$ and $(p^* + v)/2 \geq s/9$. As a result,

[21] To demonstrate this, note that $\widehat{p}_i \geq p^*_{3-i}$ is equivalent to $\frac{v}{s} \leq \frac{\sigma_i}{\sigma_{3-i}}$. Since we are studying the cases in which $\frac{v}{s} \leq \frac{\sigma_{3-i}}{\sigma_i}$, and $\frac{\sigma_i}{\sigma_{3-i}} \geq 1$ implies that $\frac{\sigma_{3-i}}{\sigma_i} \leq \frac{\sigma_i}{\sigma_{3-i}}$, it follows that $\widehat{p}_i \geq p^*_{3-i}$ is always fulfilled.

any (p^*, p^*) such that $p^* \in [2s/9 - v, v - 8s/9]$ constitutes a symmetric equilibrium for the first stage, with profits ranging from $s/9$ to $v - 4s/9$. With respect to asymmetric equilibria in which firm 1 charges some p_1^* and firm 2 charges some $p_2^* > p_1^*$, they should satisfy the following set of conditions: $p_1^* + 4s/9 \geq (p_2^* + v)/2$, $p_1^* + 4s/9 \geq s/9$, $s/9 \geq (p_1^* + v)/2$ and $s/9 \geq p_1^* + 4s/9$. Because $p_1^* + 4s/9 \geq s/9 \geq p_1^* + 4s/9$ implies that $p_1^* = -3s/9$, it is impossible to have $s/9 \geq (p_1^* + v)/2 = (v - 3s/9)/2$ (since the case we are examining requires that $s < 3v/2$). Consequently, only symmetric equilibria exist, and the desired result follows. □

Proof of Lemma 2. Note that $U_1(1) > U_2(0)$ if and only if $p_1 < p_2 - 5\delta s/18$, so firm 1's demand is 1 if $p_1 < p_2 - 5\delta s/18$, and it is 0 if $p_1 > p_2 + 5\delta s/18$. For $p_1 \in [p_2 - 5\delta s/18, p_2]$, firm 1's demand solves $U_1(\sigma) = U_2(1 - \sigma)$, that is,

$$(4 + \frac{1}{\sigma})(2 - \frac{1}{\sigma}) = \frac{18(p_2 - p_1)}{\delta s}. \tag{5}$$

Therefore,

$$\sigma = \frac{1 + \sqrt{9 - \dfrac{18(p_2 - p_1)}{\delta s}}}{8 - \dfrac{18(p_2 - p_1)}{\delta s}}.$$

If $p_1 \in [p_2, p_2 + 5s/18]$, then

$$\sigma = 1 - \frac{1 + \sqrt{9 - \dfrac{18(p_1 - p_2)}{\delta s}}}{8 - \dfrac{18(p_1 - p_2)}{\delta s}}.$$

□

Proof of Proposition 5. Suppose that firm 1 chooses p_1 to maximize

$$\Pi_1(p_1, p_2) = \begin{cases} p_1\sigma + \dfrac{s\sigma}{9}\left(1 + \dfrac{1}{\sigma}\right)^2 & \text{if } p_1 - p_2 \leq 0 \\ p_1\sigma + \dfrac{s(1-\sigma)}{9}\left(1 + \dfrac{2\sigma}{1-\sigma}\right)^2 & \text{if } p_1 - p_2 \geq 0 \end{cases},$$

where σ is a shortcut for the demand function in Lemma 2. Suppose that firm 2 charges some price p^*. Note that $\Pi_1(\,\cdot, p^*)$ is continuous, but it is not differentiable at $p_1 = p^*$, so the following should hold in order for firm 1 to have incentives to charge price p^* as well:

$$\left.\frac{\partial \Pi_1(p_1, p^*)}{\partial p_1}\right|_{p_1 \uparrow p^*} \geq 0 \geq \left.\frac{\partial \Pi_1(p_1, p^*)}{\partial p_1}\right|_{p_1 \downarrow p^*}.$$

Taking into account that

$$\left.\frac{\partial\sigma(p_1,p^*)}{\partial p_1}\right|_{p_1\uparrow p^*}=\left.\frac{\partial\sigma(p_1,p^*)}{\partial p_1}\right|_{p_1\downarrow p^*}=-\frac{3}{4s\delta}$$

and

$$\lim_{p_1\uparrow p^*}\sigma(p_1,p^*)=\lim_{p_1\downarrow p^*}\sigma(p_1,p^*)=1/2,$$

this condition can be rewritten as:

$$\frac{1}{2}-\frac{3}{4s\delta}\left(p^*-\frac{3s}{9}\right)\geq 0\geq\frac{1}{2}-\frac{3}{4s\delta}\left(p^*+\frac{15s}{9}\right).$$

Hence, any symmetric equilibrium price p^* must be such that $(2\delta-5)s/3\leq p^*\leq(1+2\delta)s/3$.

Up to now, our analysis was local around the nondifferentiability point displayed by profit functions, so we did not yet account for large price deviations. This is important because ruling out incentives to perform such price deviations may discard some prices $p^*\in[(2\delta-5)s/3,(1+2\delta)s/3]$, perhaps all. So let us consider the incentives by firm 1 to deviate from charging price $p^*\in[\frac{(2\delta-5)s}{3},\frac{(1+2\delta)s}{3}]$ given that its rival is indeed charging p^*. Suppose first that firm 1 contemplates charging $p_1<p^*$, so that its profit becomes $\Pi_1^l(p_1,p^*)=p_1\sigma+\frac{s\sigma}{9}(1+\frac{1}{\sigma})^2$. We will work on the quantity space, so using expression (5) yields that $p_1=p^*-\frac{\delta s}{18}(4+\frac{1}{\sigma})(2-\frac{1}{\sigma})<p^*$ is the price that implements sales equal to $\sigma>1/2$ given that the rival is charging p^*. Then firm 1 chooses $\sigma>1/2$ so as to maximize

$$\mathring{\Pi}_1(\sigma)\equiv\left[p^*-\frac{\delta s}{18}\left(4+\frac{1}{\sigma}\right)\left(2-\frac{1}{\sigma}\right)\right]\sigma+\frac{s\sigma}{9}\left(1+\frac{1}{\sigma}\right)^2.$$

Because $\frac{d^2\mathring{\Pi}_1(\sigma)}{d\sigma^2}=\frac{s(2+\delta)}{9\sigma^3}>0$, it is clear that the only deviation that firm 1 would consider if it lowered the price would be accumulating the maximum possible market share.

Suppose now that firm 1 contemplates charging $p_1>p^*$, so that its profit becomes $\Pi_1^r(p_1,p^*)=p_1\sigma+\frac{s(1-\sigma)}{9}(1+\frac{2\sigma}{1-\sigma})^2$. Again, we will work on the quantity space, so let $p_1=p^*+\frac{\delta s}{18}(4+\frac{1}{1-\sigma})(2-\frac{1}{1-\sigma})>p^*$ be the price that implements sales equal to $\sigma<1/2$.[22] Then firm 1 chooses $\sigma<1/2$ so as to maximize

$$\widehat{\Pi}_1(\sigma)\equiv\left[p^*+\frac{\delta s}{18}\left(4+\frac{1}{1-\sigma}\right)\left(2-\frac{1}{1-\sigma}\right)\right]\sigma+\frac{s(1-\sigma)}{9}\left(1+\frac{2\sigma}{1-\sigma}\right)^2.$$

[22] This follows from replacing σ with $1-\sigma$ and exchanging firms' indices in expression (5) given that $p_1=p^*$, since such an expression was derived under the presumption that $\sigma>1/2$ and that firm 1 is the one with bigger customer base in the second period.

Because $\sigma < \frac{1}{2}$ implies that $\frac{d^2\widehat{\Pi}_1(\sigma)}{d\sigma^2} = \frac{s[8(1-\sigma)-\delta(4-\sigma)]}{9(1-\sigma)^4} > 0$, it is clear that the only deviation that firm 1 would consider if it raised the price would be not accumulating any market share.

We have shown that the only deviations worthwhile taking involve either not capturing any market share or capturing all of it. Therefore, we need both

$$\mathring{\Pi}_1(1/2) = \frac{p^* + s}{2} \geq \mathring{\Pi}_1(1) = p^* + \frac{(8-5\delta)s}{18}$$

and

$$\mathring{\Pi}_1(1/2) = \frac{p^* + s}{2} \geq \widehat{\Pi}_1(0) = \frac{s}{9},$$

that is, $-\frac{7s}{9} \leq p^* \leq \frac{(1+5\delta)s}{9}$. Because $(2\delta-5)s/9 < -7s/9$ and $(1+5\delta)s/9 < (1+2\delta)s/3$, only prices p^* such that $p^* \in [-\frac{7s}{9}, \frac{(1+5\delta)s}{9}]$ are (symmetric) equilibrium prices.

Note that we have implicitly assumed throughout that all consumers would be willing to trade with firm 2 when firm 1 deviates from p^* and increases its price so as to accumulate no market share. This requires that $U_2(1) = v - p^* + \delta u_2(1) = v - p^* + \delta(v - \frac{11s}{18})$ be nonnegative, so we must have

$$v > \frac{p^* + 11\delta s/18}{1+\delta},$$

which of course ensures that $U_1(1/2) = U_2(1/2) \geq 0$. Because we have assumed that $v \geq s$ and

$$s > \frac{(1+5\delta)s/9 + 11\delta s/18}{1+\delta} \geq \frac{p^* + 11\delta s/18}{1+\delta},$$

there is no need to include any further constraint for the equilibrium candidates we exhibited to indeed be equilibria. □

Proof of Proposition 6. We have already shown that the set of symmetric equilibria is not empty when $\delta = 0$, so let $\delta > 0$ in what follows. We consider first case in which $s/v \geq 3/2$ and we conclude by analyzing the case in which $s/v \in (1, 3/2)$.

Unless otherwise noted, let us assume that $s/v \geq 3/2$. Suppose on the one hand that a consumer deals with firm 1 when its first-period market share is $\sigma \in (\frac{s}{v+s}, 1]$ (where $\frac{1}{2} < \frac{s}{v+s} < 1$ because $s/v > 1$). Since $p_1^* = v$ and $p_2^* = \frac{1}{2}[v + s(\frac{1-\sigma}{\sigma})]$ in the second period, it holds that the consumer expects the following second period utility:

$$u_1(\sigma) = \int_0^{p_1^*-p_2^*} (v - p_2^* - x)\frac{dx}{s} + \int_{p_1^*-p_2^*}^{s} (v - p_1^*)\frac{dx}{s}$$
$$= \frac{1}{2s}\left(\frac{v+s}{2} - \frac{s}{2\sigma}\right)^2.$$

On the other hand, she expects the following utility if she trades with firm 2:

$$u_2(1-\sigma) = \frac{v+s}{2} - \frac{s}{2\sigma}.$$

Therefore, $v - p_1 + \delta u_1(\sigma) = v - p_2 + \delta u_2(1-\sigma)$ yields that firm 1's market share σ is given by the solution to the following equation:

$$p_2 - p_1 - \frac{\delta}{2s}\left(\frac{v+s}{2} - \frac{s}{2\sigma}\right)\left(\frac{s}{2\sigma} + \frac{3s-v}{2}\right) = 0.$$

We have that the demand function for firm 1 equals

$$\sigma(p_1, p_2) = \frac{s\delta}{2\sqrt{s\delta[s\delta - 2(p_2 - p_1)]} - (s-v)\delta}$$

for $p_2 - \dfrac{\delta v(4s-v)}{8s} \leq p_1 < p_2$ (since $\sigma(p_1, p_2) \in (\dfrac{s}{v+s}, 1]$ must hold). Paralleling this argument when $\sigma \in [0, \dfrac{v}{v+s})$ yields that the demand function for firm 1 equals

$$\sigma(p_1, p_2) = 1 - \frac{s\delta}{2\sqrt{s\delta[s\delta - 2(p_1 - p_2)]} - (s-v)\delta}$$

when prices p_1 and p_2 are such that $p_2 < p_1 \leq p_2 + \dfrac{\delta v(4s-v)}{8s}$. Finally, suppose that a consumer deals with firm 1 when its market share is expected to equal $\sigma \in [\dfrac{v}{v+s}, \dfrac{s}{v+s}]$, so that she foresees having to pay $p_1^* = p_2^* = v$ in the second period. Because the consumer foresees the same second-period utility regardless of which firm she trades with, she will be inclined to purchase from the cheapest firm in the first period, if there is such a firm. If $p_1 = p_2$, then $\sigma = K$ is consistent with $\sigma \in [\dfrac{v}{v+s}, \dfrac{s}{v+s}]$ whenever $\dfrac{v}{v+s} \leq K \leq \dfrac{s}{v+s}$. Suppose now that $p_1 < p_2$, so that all consumers should purchase from firm 1. Since consumers form rational expectations, all consumers buying from the same firm cannot be consistent with their expectation that $\sigma \in [\dfrac{v}{v+s}, \dfrac{s}{v+s}]$, so the only case in which $\sigma \in [\dfrac{v}{v+s}, \dfrac{s}{v+s}]$ must arise is because $p_1 = p_2$. We have therefore shown the following for $K \in [\dfrac{v}{v+s}, \dfrac{s}{v+s}]$:

$$\sigma(p_1, p_2) = \begin{cases} 1 & \text{if } p_2 < p_2 - \delta v(4s-v)/(8s) \\ \frac{s\delta}{2\sqrt{s\delta[s\delta - 2(p_2 - p_1)]} - (s-v)\delta} & \text{if } p_2 - \dfrac{\delta v(4s-v)}{8s} \leq p_1 < p_2 \\ K & \text{if } p_1 = p_2 \\ 1 - \frac{s\delta}{2\sqrt{s\delta[s\delta - 2(p_1 - p_2)]} - (s-v)\delta} & \text{if } p_2 < p_1 < p_2 + \delta v(4s-v)/(8s) \\ 0 & \text{if } p_2 + \delta v(4s-v)/(8s) \leq p_1 \end{cases}.$$

Importantly, note that $\sigma(p_1, p_2)$ exhibits a discontinuity at $p_1 = p_2$, since

$$\lim_{p_1 \uparrow p_2} \sigma(p_1, p_2) = \frac{s}{s+v} \geq K \geq \frac{v}{s+v} = \lim_{p_1 \downarrow p_2} \sigma(p_1, p_2),$$

where one of the weak inequalities must be strict because $s/v \geq 3/2 > 1$. This feature of firms' demands that does not appear when $s/v \leq 1$ will be critical in showing that a symmetric equilibrium fails to exist when $s/v \geq 3/2$.

Based on our analysis so far, the profit function can be easily shown to be as follows:

$$\Pi_1(p_1, p_2) = \begin{cases} p_1\sigma + \dfrac{\sigma v}{2s}\left[s\left(1+\dfrac{1}{\sigma}\right) - v\right] & \text{if } p_1 - p_2 < 0 \\ K(p_1 + v) & \text{if } p_1 - p_2 = 0. \\ p_1\sigma + \dfrac{(1-\sigma)}{4s}\left[v + s\left(\dfrac{\sigma}{1-\sigma}\right)\right]^2 & \text{if } p_1 - p_2 > 0 \end{cases}$$

Whenever $K \in [\frac{v}{v+s}, \frac{s}{v+s})$,[23] we prove by contradiction that there can exist no symmetric equilibrium in which firms charge price p^*. First, we need to rule out unilateral incentives to slightly lower the price. Hence, we need that

$$K(p^* + v) \geq \frac{(p^* + v)s}{v+s},$$

that is, $p^* \leq -v$ (since $K < \frac{s}{v+s}$). Given that the rival is charging $p^* \leq -v$, consider any deviation by firm 1 such that it loses all market share. Such a deviation yields $v^2/(4s)$, which clearly exceeds $K(p^* + v)$ whenever $p^* \leq -v$. As a result, firm 1 can profitably deviate in a unilateral fashion, and it is therefore impossible that a symmetric equilibrium exists when $\delta > 0$ if $s/v \geq 3/2$.

To conclude the proof, let us assume that $s/v \in (1, 3/2)$ given that $\delta > 0$. When firm 1's first-period market share is such that $\sigma \in (\frac{s}{3v-s}, 1]$, the fact that $\frac{s}{3v-s} > \frac{1}{2}$ implies that $p_2^* = \frac{s(2-\sigma)}{3\sigma} < p_1^* = \frac{s(1+\sigma)}{3\sigma} < s$. Thus, if a consumer deals with firm 1 given that its market share in the first period is $\sigma \in (\frac{s}{3v-s}, 1]$, it holds that the consumer obtains an expected utility in the second period equal to

$$\begin{aligned} u_1(\sigma) &= \int_0^{p_1^* - p_2^*} (v - p_2^* - x)\frac{dx}{s} + \int_{p_1^* - p_2^*}^{s} (v - p_1^*)\frac{dx}{s} \\ &= v - \frac{s}{9} + \frac{s(1 - 10\sigma)}{18\sigma^2}. \end{aligned}$$

[23] The case in which $K = \frac{s}{v+s}$ is identical by replacing the indices of firms, so we will ignore it henceforth.

Dealing with firm 1's competitor delivers a utility equal to

$$\begin{aligned} u_2(\sigma) &= v - p^*_{3-i} \\ &= v - \frac{s(2-\sigma)}{3\sigma}. \end{aligned}$$

From the equality that $v - p_1 + \delta u_1(\sigma) = v - p_2 + \delta u_2(1-\sigma)$, one gets after rearranging that

$$p_2 - p_1 + \frac{s\delta(1+2\sigma)}{18\sigma^2} - \frac{4s\delta}{9} = 0,$$

so

$$\sigma = \frac{s\delta}{3\sqrt{s\delta[s\delta - 2(p_2-p_1)]} - s\delta}.$$

Since $\sigma \in (\frac{s}{3v-s}, 1]$ must hold, we must have that $\sigma = \frac{s\delta}{3\sqrt{s\delta[s\delta - 2(p_2-p_1)]} - s\delta}$ for $p_2 - \frac{5s\delta}{18} \leq p_1 < p_2 - \frac{\delta(s^2-v^2)}{2s}$, whereas $\sigma = 1$ for $p_1 < p_2 - \frac{5s\delta}{18}$. Based on this and on our previous results derived for the case in which $s/v \geq 3/2$ (which apply because they simply required that $s > v$), one can easily obtain firm 1's demand function in the first period after some straightforward changes of variables:

$$\sigma(p_1,p_2) = \begin{cases} 1 & \text{if } p_1 < p_2 - \frac{5s\delta}{18} \\ \frac{s\delta}{3\sqrt{s\delta[s\delta-2(p_2-p_1)]}-s\delta} & \text{if } p_2 - \frac{5s\delta}{18} \leq p_1 < p_2 - \frac{\delta(s^2-v^2)}{2s} \\ \frac{s\delta}{2\sqrt{s\delta[s\delta-2(p_2-p_1)]}-(s-v)\delta} & \text{if } p_2 - \frac{\delta(s^2-v^2)}{2s} \leq p_1 < p_2 \\ K & \text{if } p_1 = p_2 \\ 1 - \frac{s\delta}{2\sqrt{s\delta[s\delta-2(p_1-p_2)]}-(s-v)\delta} & \text{if } p_2 < p_1 < p_2 + \frac{\delta(s^2-v^2)}{2s} \\ 1 - \frac{s\delta}{3\sqrt{s\delta[s\delta-2(p_1-p_2)]}-s\delta} & \text{if } p_2 + \frac{\delta(s^2-v^2)}{2s} < p_1 < p_2 + \frac{5s\delta}{18} \\ 0 & \text{if } p_2 + \frac{5s\delta}{18} \leq p_1 \end{cases}.$$

Given firm 1's demand function, it is straightforward to construct its profit function:

$$\Pi_1(p_1,p_2) = \begin{cases} p_1\sigma + \frac{s\sigma}{9}(1+\frac{1}{\sigma})^2 & \text{if } p_1 - p_2 < -\frac{\delta(s^2-v^2)}{2s} \\ p_1\sigma + \frac{\sigma v}{2s}[s(1+\frac{1}{\sigma}) - v] & \text{if } -\frac{\delta(s^2-v^2)}{2s} < p_1 - p_2 < 0 \\ K(p_1+v) & \text{if } p_1 - p_2 = 0 \\ p_1\sigma + \frac{(1-\sigma)}{4s}[v + s(\frac{\sigma}{1-\sigma})]^2 & \text{if } \frac{\delta(s^2-v^2)}{2s} > p_1 - p_2 > 0 \\ p_1\sigma + \frac{s(1-\sigma)}{9}[1+2(\frac{\sigma}{1-\sigma})]^2 & \text{if } p_1 - p_2 > \frac{\delta(s^2-v^2)}{2s} \end{cases}.$$

As in the case in which $s/v \geq 3/2$, this profit function is not continuous at $p_1 = p_2$ when $K \in [\frac{v}{v+s}, \frac{s}{v+s}]$ consumers purchase firm 1's product when its first-period price coincides with that of firm 2. Again, this discontinuity results in non-existence of a symmetric equilibrium when $\delta > 0$ because it is impossible to rule out at the same time incentives to slightly undercut the rival and incentives to lose all the demand by raising the price. □

Supplementary material

Supplementary material associated with this article can be found, in the online version, at 10.1016/j.ijindorg.2016.04.002.

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