ECO 650: Firms' Strategies and Markets
Vertical Relationships and Bargaining(II)

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Exercise 1

Assumptions:

- A manufacturer produces a good at a unit cost c.
- A retailer faces a demand $D(p) = 1 - p$.
- The game:
  1. The manufacturer and the retailer bargain over a two-part tariff contract (w, F);
  2. The retailer sets a final price p to consumers.

Questions:

1. Given the contract (w, F), determine the optimal price set by the retailer in stage 2. Determine the stage-2 equilibrium profits of firms $\pi_U(w) + F$ and $\pi_D(w) - F$.
2. Write down the Nash program and determine the optimal contract (w, F). Is it efficient?
3. Assume now that the retailer can access another supply source at marginal cost $\bar{c} > c$ (competitive fringe). What is the outside option profit of the retailer $\bar{\Pi}$? How does it affect the bargaining?

Exercise 1: Solution

1. In stage 2, the retailer maximizes $\max_p (p-w)(1-p) - F$; The FOC is: $1 - 2p + w = 0 \Rightarrow p = \frac{1+w}{2}$; $\pi_U(w) = (w-c)(\frac{1-w}{2})$ and $\pi_D(w) = (\frac{1-w}{2})^2$.

2. The Nash program in stage 1 is

$$\max_{(w,F)} (\pi_U(w) + F)(\pi_D(w) - F)$$

FOCS are:

$$
\begin{aligned}
-(\pi_U(w) + F) + (\pi_D(w) - F) &= 0 \quad (1)\\
\frac{\partial \pi_U(w)}{\partial w}(\pi_D(w) - F) + \frac{\partial \pi_D(w)}{\partial w}(\pi_U(w) + F) &= 0 \quad (2)
\end{aligned}
$$

(1) is the split the difference rule, F is used to share profits equally. Plugging (1) into (2): $\underbrace{(\frac{\partial \pi_U(w)}{\partial w} + \frac{\partial \pi_D(w)}{\partial w})}_{0} \underbrace{(\pi_D(w) - F)}_{>0} = 0$. w is set to maximize joint profits $w^* = c$: Efficiency!

Exercise 1: Solutions

$\pi_U(w) + \pi_D(w) = (\frac{1-w}{2})(\frac{1+w-2c}{2})$.
Deriving this joint profit w.r.t w gives:
$-(1+w-2c) + (1-w) = 0 \Rightarrow w^* = c$. $\pi_U(c) = 0, \pi_D(c) = \frac{(1-c)^2}{4}$
$F^* = \frac{\pi_D(c) - \pi_U(c)}{2} = \frac{(1-c)^2}{8}$.
In equilibrium both firms obtain a profit $\frac{(1-c)^2}{8}$.

3. The outside option profit $\bar{\Pi}$ is such that $\max_p (p-\bar{c})(1-p)$.

   $\bar{\Pi} = (\frac{1-\bar{c}}{2})^2$. The Nash program in stage 1 is

$$\max_{(w,F)} (\pi_U(w) + F)(\pi_D(w) - F - \bar{\Pi})$$

   FOCS are:

$$-(\pi_U(w) + F) + (\pi_D(w) - F - \bar{\Pi}) = 0 \quad (3)$$

$$\frac{\partial \pi_U(w)}{\partial w}(\pi_D(w) - F - \bar{\Pi}) + \frac{\partial \pi_D(w)}{\partial w}(\pi_U(w) + F) = 0 \quad (4)$$

   Plugging (3) into (4), again w maximizes the joint profit $w^* = c$: unchanged!

Exercise 1: Solution

$F^* = \frac{\pi_D(c) - \pi_U(c) - \bar{\Pi}}{2} = \frac{(1-c)^2}{8} - \frac{\bar{\Pi}}{2} > 0$. Note that $F^* > 0$ for any $\bar{c} > c$. Now the retailer earns $\frac{(1-c)^2}{8} + \frac{\bar{\Pi}}{2}$ which is higher than the profit of the producer $\frac{(1-c)^2}{8} - \frac{\bar{\Pi}}{2}$. This is because of the retailer's outside option. It only impacts the sharing of joint profits but efficiency remains!