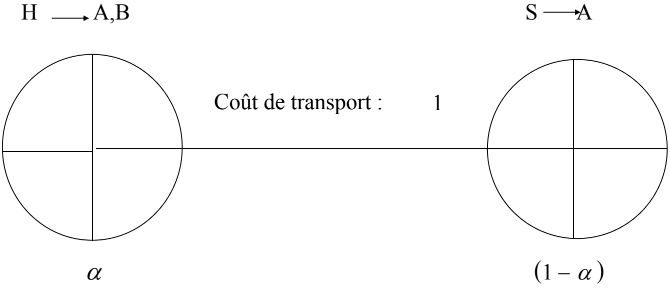
ECO 650: Firms' Strategies and Markets
Course 1: Multiproduct firms' pricing strategies

Claire Chambolle

September 20, 2017

Exercice 1

- Two stores H (Hypermarket) and S (Supermarket)
- H sells A and B – S sells A
- $\alpha \in [0, \frac{1}{2}]$ consumers are located at H and $1 - \alpha$ in S.
- Transportation cost among the stores is normalized to 1.
- $u_A = 1$; u_B uniformly distributed over $[0, 1]$ around each store.
- $b \in [0, 1]$ is the unit cost for B. No cost for A.



1. Which consumers may travel from one store to the other?

2. We note $p^H = p_A^H + p_B^H$ the sum of prices for the two goods at store H; p^S the price of B at store S.
   Determine the demand at each store.

3. Determine the two candidates Nash equilibria in pure strategy.

4. Assume that $\alpha = \frac{1}{9}$ and show that the loss-leading equilibrium is the unique Nash equilibrium in pure strategy.

5. How do you explain the emergence of this loss-leading equilibrium?

Solution–Exercice 1

1. No consumer in H will travel to S as $u_A = 1$. In contrast, when $1 + u_B - p^H - 1 > 1 - p^S$ (i.e. $u_B > 1 + p^H - p^S$) consumers located in S may choose to travel to H to buy the two goods A and B instead of A alone.

2.
   - If $p^H < p^S$, some consumers travel from S to H to buy the two goods and $D^H_A = \alpha + (1-\alpha)(p^S - p^H)$ (with $p^H_A \leq 1$) whereas $D^H_B = \alpha(1 - p^H_B) + (1-\alpha)(p^S - p^H)$. Demand for A at S is $D^S = (1-\alpha)(1 + p^H - p^S)$.
   - If $p^H > p^S$, $D^H_A = \alpha$ whereas $D^H_B = \alpha(1 - p^H_B)$ and $D^S = 1 - \alpha$.

3.
   - If $p^H < p^S$, the profit of H can be written as:

$$\Pi^H = (p^H - b)[\alpha + (1-\alpha)(p^S - p^H)] - \alpha p^H_B(p^H_B - b)$$

$$\Pi^S = (1-\alpha)p^S(1 + p^H - p^S)$$

   Maximizing Π^H with respect to p^H and p^H_B, and Π^S with respect to p^S, we obtain the first loss-leading equilibrium candidate:

$$p^{H*} = \frac{1+\alpha}{3(1-\alpha)} + \frac{2b}{3}, p^{H*}_B = \frac{b}{2}, p^{S*} = \frac{2-\alpha}{3(1-\alpha)} + \frac{b}{3}$$

3. ▸ If $p^H > p^S$, the profit of H and S can be respectively written as:

$$\Pi^H = p_A^H \alpha + \alpha(1 - p_B^H)(p_B^H - b),\ \Pi^S = (1-\alpha)p^S$$

Maximizing Π^H with respect to p_A^H and p_B^H, and Π^S with respect to p^S, we obtain the second local monopoly equilibrium candidate:

$$\hat{p}_A^H = 1, \hat{p}_B^H = \frac{1+b}{2}, \hat{p}^S = 1$$

4. The equilibrium profit in the loss-leading case is:

$$\Pi^{H*} = \frac{(1+\alpha)^2}{9(1-\alpha)} + \frac{b^2(4+5\alpha)}{36}, \Pi^{S*} = \frac{(2-\alpha)^2}{9(1-\alpha)} + \frac{b^2(1-\alpha)}{9}$$

In the local monopoly case:

$$\hat{\Pi}^H = \frac{(5-2b+b^2)\alpha}{4}, \hat{\Pi}^S = 1 - \alpha$$

Assume $b \to 0$, when $\alpha = \frac{1}{9}$: H has no incentive to deviate unilaterally from the loss leading strategy by raising its price to the local monopoly level as $\Pi^{H*} > \hat{\Pi}^H$. Again, S cannot unilaterally deviate by raising her price as it would remain in the competition situation. It is not profitable for S to lower its price to p^H (no corner solution!).

5. The logic under the result here is complementarity.
    - A complementarity between the two independent products arises through the transportation cost.
    - H has an incentive to sell B below cost because this is the product which has an elastic demand, and therefore lowering this price below cost can attract consumers from S.
    - If instead $\alpha = \frac{1}{3}$ there is a local monopoly equilibrium. H has no incentive to compete to attract consumers from S.