Exercice 1

Assumptions

- Consumers are uniformly distributed along a segment $[0, 1]$. A firm is localized in 0 and another firm in 1.
- A consumer who travels a distance x to buy one unit at price p has a utility $U = v - p - tx$ if he buys and 0 if he does not buy. There is no utility for a second unit.
- A consumer buys only if he receives an ad. Let Φ_i denote the share of consumers who have received an ad from i. The cost to reach this fraction of demand is $A(\phi) = \frac{a\phi^2}{2}$ with $a \geq \frac{t}{2}$.

Questions

1. What is the demand of consumers who receive only an ad from i?
2. What is the demand of consumers who receive an ad from i and j?
3. What is the total demand for firm i? How the price elasticity of demand varies in ϕ in $p_i = p_j = p$ and $\phi_i = \phi_j = \phi$?
4. Firms choose simultaneously their price and their ad level. Determine the symmetric Nash equilibrium of this game.

1. $\phi_i(1-\phi_j)$ if $p_i \leq v-t$, they all buy to firm i.
2. $\phi_i\phi_j$. Among them the indifferent consumer $\tilde{x}$ is such that $v-p_i-tx = v-p_j-t(1-x)$ or $\tilde{x} = \frac{1}{2}+\frac{(p_j-p_i)}{2t}$. This is the demand for i when the gap in price is not too high.
3. $D_i = \phi_i[(1-\phi_j)+\phi_j\tilde{x}]$. At point $p_i = p_j = p$ and $\phi_i = \phi_j = \phi$, the elasticity $\epsilon = \frac{-p_i\partial D_i/\partial p_i}{D_i} = \frac{p\phi}{t(2-\phi)}$ which increases in ϕ.
4. The profit of firm i is:

$$\Pi_i = (p_i-c)D_i - A(\phi_i)$$

   The first order conditions are :

$$2p_i = c+t+p_j+\frac{2(1-\phi_j)t}{\phi_j}$$

$$\phi_i = (p_i-c)\frac{(1-\phi_j+\phi_j\tilde{x})}{a}$$

5. At the symmetric equilibrium $p_i = p_j = p^* = c+\sqrt{2at}$ and $\tilde{x} = \frac{1}{2}$ and $\phi_i = \phi_j = \phi^* = \frac{2}{(1+\sqrt{2a/t})}$.