

# Firms' Strategies and Markets

## Course 1: Dynamic Pricing

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# Dynamic Pricing

- ▶ Repeated interactions among firms may enable collusive strategies (IO class M1)
  - ▶ High prices over time.
- ▶ Reputation strategies can occur (Class / Entry & Advertising)
  - ▶ Either a low or a high price can signal a high quality to an uninformed consumer in a first period.
- ▶ We focus here on “consumer inertia” which may come from different sources and imply various firm’s dynamic pricing strategies.
  - ▶ Durable Goods
  - ▶ Search costs → generate temporal price dispersion.
  - ▶ Switching costs → Consumers are *locked-in* within the same firm

**Durable goods:** Goods that are not consumed or destroyed in use; Consumers derive the benefit of their purchase for a period of time (several years).

- ▶ Cars, Washing Machines, Computers, Smartphones ...

**Insights:** A durable good monopoly who cannot discriminate in a given period among heterogenous consumers can use intertemporal discrimination to extract more surplus from consumers.

- ▶ Some consumers buy in the first period;
- ▶ Others delay their purchase expecting a lower price.

## Assumptions

- ▶ A durable monopoly with a production cost 0.
- ▶ A continuum of heterogenous consumers live two periods  $t = \{1, 2\}$ . Consumers buy either 0 or 1 unit and their valuation for the good  $v$  is uniformly distributed over  $[0, 1]$ .
- ▶  $\delta$  is the discount factor.
- ▶ The monopoly sets  $p_1$  in  $t = 1$  and  $p_2$  in  $t = 2$ .

**Consider first the simple case in which the monopoly can sell only in  $t = 1$  at price  $p$ .**

A consumer is willing to purchase the good if  $(1 + \delta)v - p > 0$  in  $t = 1$ . The demand is  $D(p) = 1 - \frac{p}{1+\delta}$  and the price that maximizes the monopoly profit is:  $p = \frac{1+\delta}{2}$  and the corresponding profit  $\Pi = \frac{1+\delta}{4}$ .

## Consider now the two period pricing strategy

- For a given couple of prices  $(p_1, p_2)$  we determine the consumer indifferent between purchasing in  $t = 1$  and in  $t = 2$ .

$$\underbrace{(1 + \delta)\tilde{v} - p_1}_{t=1} = \underbrace{\delta(\tilde{v} - p_2)}_{t=2} \Rightarrow \tilde{v}(p_1, p_2) = p_1 - \delta p_2$$

- Suppose that consumers with  $v > \tilde{v}$  have purchased the good in  $t = 1$ . The residual demand for the good in  $t = 2$  is

$$D_2(p_1, p_2) = \tilde{v}(p_1, p_2) - p_2.$$

In  $t = 2$ , the monopoly chooses  $p_2$  to maximise  $p_2 D_2(p_1, p_2)$  and this gives

$$p_2 = \frac{\tilde{v}(p_1, p_2)}{2} \Rightarrow p_2(p_1) = \frac{p_1}{2(1 + \delta)}$$

The price in the second period is lower than half of the price in the first period.

- in  $t = 1$  now, the demand is

$$D_1(p_1, p_2) = 1 - \tilde{v}(p_1, p_2)$$

and the monopoly sets  $p_1$  to maximise its intertemporal profit

$$\Pi_{1,2} = p_1 D_1(p_1, p_2) + p_2 D_2(p_1, p_2)$$

under the constraint that  $p_2(p_1) = \frac{p_1}{2(1+\delta)}$ .

This leads to

$$p_1 = \frac{(2 + \delta)^2}{2(4 + \delta)}$$

and the profit is:

$$\Pi_{1,2} = \frac{(2 + \delta)^2}{4(4 + \delta)} < \Pi$$

## The durable good monopolist

- Obtains lower profit in selling over the two periods than only in the first.
- Cannot prevent from competing with itself.

## Business practices- Commitment on future prices

- ▶ Renting the good instead of selling it! Here renting at price  $p_1 = p_2 = \frac{1}{2}$  at each period brings  $\Pi$ .
- ▶ Return policies, money back guarantees or repurchase agreements, ...Contracts that are offered by M to protect the consumers in  $t = 1$  against any future price cut.
- ▶ Reputation
- ▶ Technology (capacity constraints, planned obsolescence, new version of the product...)

## Exercise 1

### Assumptions

- ▶ A durable monopoly,  $M$ , with a production cost  $c$ .
- ▶ Two consumers who live two periods  $t = \{1; 2\}$ . Two consumers who buy either 0 or 1 unit and  $C1$  has a valuation 1 and  $C2$   $c < v_I < 1$ .
- ▶  $\delta$  is the discount factor.
- ▶  $M$  sets  $p_1$  in  $t = 1$  and  $p_2$  in  $t = 2$ .

### Questions

1. Determine the price equilibrium  $p$  and profit  $\Pi$  if  $M$  only sells in  $t = 1$ .
2. Determine the two period equilibrium  $(p_1, p_2)$  and profit  $\Pi_{1,2}$  of  $M$ .
3. Compare the two profits when  $c < v_I < \frac{1}{2}(1 + \frac{c}{1+\delta})$ . What happens if  $v_I > \frac{1}{2}(1 + c)$ ?



## Search Costs & The Diamond Paradox

**Search costs:** Consumers might be imperfectly informed about prices-

- ▶ If getting information is costly,  $p_1 = p_2 > c$  can be an equilibrium.
- ▶ Diamond Paradox: in a duopoly  $p_1 = p_2 = p^M$  might be an equilibrium
  - ▶ All consumers are uninformed about prices
  - ▶ They have no cost to learn one price and a cost  $\epsilon$  to learn the second price!
  - ▶ For any  $p_1 = p_2 = p < p^M$ , a firm has an incentive to deviate towards  $p + \frac{\epsilon}{2}$ !

# Search Costs and Temporal Price Dispersion

## Assumptions

- ▶ Monopolistic competition among  $n$  symmetric firms with free entry
- ▶  $I$  informed consumers and  $U = \frac{M}{n}$  uninformed consumers per store.
- ▶  $r$  is the reservation price of consumers.
- ▶  $c(q)$  is a firm's cost function with strictly decreasing average cost (ex:  $cq + f$ ).
- ▶ If a firm sets the lowest price, it obtains  $I + U$  consumers. If the firm does not set the lowest price, it obtains  $U$  consumers. If several firms have the identical lowest price, there is a tie and they share equally the  $I$  consumers among them.

## There exists no symmetric pure strategy Nash equilibrium

- ▶ First, the relevant range of prices is  $[p^*, r]$ . If  $p > r$ , there is no demand and if  $p < p^* = \frac{C(I+U)}{I+U}$  the firm obtains a negative profit even in the best case, i.e. when serving all consumers.
- ▶ If all firms set  $p = p^*$ , there is a tie and then profits are negative:  $p^*(U + \frac{I}{n}) - C((U + \frac{I}{n})) < 0$ .
- ▶ If all firms set  $p \in ]p^*, r]$ , a slight price cut by one of the firms enables to capture all informed customers and realize a positive profit.

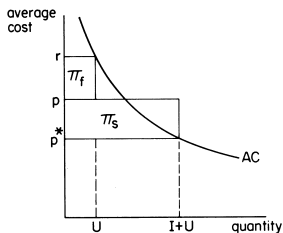
## There is a symmetric equilibrium in mixed strategy.

- ▶ Each firm randomly chooses a price according to the same density of probability  $f(p)$  ( $F(p)$  is the distribution function)  $\Rightarrow$  Temporal price dispersion arises!  
( Original paper by Varian (1980): A model of "sales").

Assume that all firms have the same distribution  $F(p)$ .

**We build the expected profit function for a firm for any price  $p$**

- ▶ With probability  $(1 - F(p))^{n-1}$ ,  $p$  is the lowest price and then the firm earns  $\pi_s = p(U + I) - C(U + I)$  (Success).
- ▶ With probability  $1 - (1 - F(p))^{n-1}$ ,  $p$  is not the lowest price and it obtains  $\pi_f = pU - C(U)$ .



- ▶ The expected profit of the firm therefore is:

$$\int_{p^*}^r [\pi_s(p)(1 - F(p))^{n-1} + \pi_f(1 - (1 - F(p))^{n-1})]f(p)dp$$

- ▶ Maximizing the above profit with respect to  $p$ , the FOC is:

$$\pi_s(p)(1 - F(p))^{n-1} + \pi_f(p)(1 - (1 - F(p))^{n-1}) = 0$$

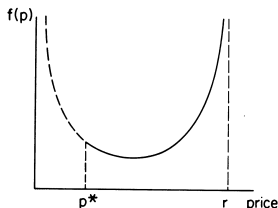
Rearranging, we obtain:

$$F(p) = \begin{cases} 0 & p < p^* \\ 1 - \left(\frac{\pi_f(p)}{\pi_f(p) - \pi_s(p)}\right)^{\frac{1}{n-1}} & p \in [p^*, r] \\ 1 & p > r \end{cases}$$

- ▶ If firms compete in a market with both informed and uninformed consumers, temporal price dispersion may arise in equilibrium. In equilibrium firms alternate relatively high prices and periods of sales.

## An example with $c(q) = f$

- ▶  $\pi_f(r) = rU - f = 0 \Rightarrow U = \frac{f}{r}$
- ▶  $\pi_s(p^*) = p^*(1 + U) - f = 0 \Rightarrow p^* = \frac{f}{1 + \frac{f}{r}}$
- ▶ The corresponding  $f(p)$  has the following shape:



- ▶ Firms tend to charge extreme prices with higher probability.
- ▶ Prices are lower as  $l$  increases and  $f$  is low (more competitive) but high prices are always charged with positive probability.

- ▶ This model also applies to competition among stores that have a base of *loyal consumers* and other *consumers that tend to switch among stores* when the store cannot distinguish among these consumers (see Narasimhan, 1988).
  - ▶ There is a tension between an incentive to set the monopoly price to loyal consumers and a competitive price for those who may go to the rival → Mixed strategy equilibrium
- ▶ These results on temporal price dispersion are robust if consumers can endogenously decide whether they want to acquire additional information through costly search.
- ▶ Empirical evidence for search costs - online vs offline.

## Switching costs

**Definition:** The presence of switching costs give consumers an incentive to purchase repeatedly from the same supplier.

- ▶ *Transaction costs:* Time and effort to change supplier (e.g. changing bank accounts, insurances, telephone company, etc...)
- ▶ *Contractual costs :* Mobile phone company that offers a contract with a phone at low price for a 24 month lock-in contract.
- ▶ *Shopping costs :* Purchasing several goods from one supplier rather than shopping around for different products.
- ▶ *Search costs*
- ▶ ...



# Imperfect competition and switching costs

## Assumptions

- ▶ Two-period model with imperfect competition.
- ▶ Consumers are uniformly distributed along a Hotelling line  $[0, 1]$  with a linear transportation cost  $-x$  for a distance  $x$ . Two firms  $A$  and  $B$  are located at the extremes.
- ▶ **Switching costs**
  - ▶ After  $t = 1$ , a share  $\lambda$  of consumers leaves the market and is replaced by new consumers.
  - ▶ The remaining share of consumers  $(1 - \lambda)$  who has bought from firm  $K = A, B$  incurs a cost  $z$  to switch to the other firm.
  - ▶ Old consumers keep their preference from one period to the next.
- ▶ Consumers have a reservation price  $r$  such that the market is fully covered.

## Competition in $t = 2$

- ▶ Assume that in  $t = 1$ , each firm  $A$  and  $B$  has obtained respectively a share  $\alpha$  and  $1 - \alpha$  of the market.
- ▶ Remaining consumers ( $1 - \lambda$ )

- ▶ A consumer  $x$  who bought from  $A$  in  $t = 1$  buys again from  $A$  if:

$$r - x - p_A^2 \geq r - (1 - x) - p_B^2 - z \Rightarrow x \leq \hat{x}_A = \frac{1}{2}(1 + p_B^2 - p_A^2 + z)$$

- ▶ New consumers ( $\lambda$ )

- ▶ A new consumer  $x$  buys from  $A$  in  $t = 2$  if:

$$r - x - p_A^2 \geq r - (1 - x) - p_B^2 \Rightarrow x \leq \hat{x} = \frac{1}{2}(1 + p_B^2 - p_A^2)$$

- ▶ Assume  $\hat{x}_A > \alpha$  (We check *ex post* this condition), the demand is:

$$q_A^2(p_A^1, p_B^1, p_A^2, p_B^2) = (1 - \lambda)\alpha(p_A^1, p_B^1) + \lambda \frac{1}{2}(1 + p_B^2 - p_A^2)$$

- ▶ The same reasoning applies for  $B$ .

## Competition in $t = 2$

The FOC writes as:

$$\frac{\partial \pi_A^2}{\partial p_A^2} = q_A^2 + p_A^2 \frac{\partial q_A^2}{\partial p_A^2} = 0$$

We obtain :

$$p_A^2(p_B^2) = \frac{1 - \lambda}{\lambda} \alpha + \frac{1}{2}(1 + p_B^2)$$

- ▶ Firms compete more aggressively to gain new costumers:

$$\frac{\partial p_A^2(p_B^2)}{\partial \lambda} < 0$$

- ▶ Firms compete less aggressively as the share of "captive consumer" increases:  $\frac{\partial p_A^2(p_B^2)}{\partial \alpha} > 0$

- ▶ In  $t = 2$  equilibrium,  $\pi_A^2(\alpha(p_A^1, p_B^1)) = \frac{1}{2\lambda}(1 + \frac{1}{3}(2\alpha - 1)(1 - \lambda))^2$   
with  $\alpha(p_A^1, p_B^1) = \frac{1}{2}(1 + p_B^1 - p_A^1)$ .

## Competition in $t = 1$

In  $t = 1$  firms take into account their intertemporal profit over the two periods.

$$\pi_A(p_A^1, p_B^1) = \pi_A^1(p_A^1, p_B^1) + \pi_A^2(\alpha(p_A^1, p_B^1))$$

The FOC is:

$$\frac{\partial \pi_A(p_A^1, p_B^1)}{\partial p_A^1} = \frac{\partial \pi_A^1(p_A^1, p_B^1)}{\partial p_A^1} + \underbrace{\frac{\partial \pi_A^2(\alpha(p_A^1, p_B^1))}{\partial \alpha}}_{+} \underbrace{\frac{\partial \alpha(p_A^1, p_B^1)}{\partial p_A^1}}_{-} = 0$$

- ▶ For  $\lambda > \frac{2}{5}$ , in equilibrium  $\alpha = \frac{1}{2}$ , and  $p_K^1 = \frac{5\lambda-2}{3}$  and  $p_K^2 = \frac{1}{\lambda}$ . For  $\lambda \leq \frac{2}{5}$ , in equilibrium  $\alpha = \frac{1}{2}$ , and  $p_K^1 = 0$  and  $p_K^2 = \frac{1}{\lambda}$ . BOUTON
- ▶ In the benchmark case without switching costs:  $p_K^1 = p_K^2 = 1$ .
- ▶ In the first period  $p_K^1 < 1$  is lower to lock in as much consumers as possible ( second period profit effect).
- ▶ In the second period though,  $p_K^2 > 1$  the equilibrium price is higher because firms compete only for new consumers.

- ▶ In terms of profit, each firm loses in  $t = 1$  but earns more in  $t = 2$  than absent switching costs.
- ▶ In equilibrium the intertemporal profit with switching costs is:

$$\pi_A = \begin{cases} \frac{1}{6} \left( \frac{1}{\lambda} + 5 \right) & \text{for } \lambda > \frac{2}{5}, \\ \frac{1}{2\lambda} & \text{for } \lambda < \frac{2}{5} \end{cases}$$

- ▶ In equilibrium, the intertemporal profit without switching cost is 1.
- ▶ Here firms are always better off when they can lock-in consumers and the effect on consumers is negative.

## Endogenous switching cost: Coupons

- ▶ **Coupons** are discount offered on the price of the product at the next purchase.
- ▶ The oldest "Coupon" by TheCCC



## Assumptions

- ▶ Consumers redraw their types in  $t = 2$ .
- ▶ In  $t = 1$  firms can offer coupons  $c_K \geq 0$  to their loyal consumers. In  $t = 2$  the consumer will pay  $p_A^2 - c_A$  if he buys again from  $A$ .

## Competition in period 2

- ▶ A consumer who purchased from  $A$  in  $t = 1$ , buys from  $A$  again if its new address  $x$  is such that
 
$$r - x - (p_A^2 - c_A) > r - (1 - x) - p_B^2 \Rightarrow x < \hat{x}_A = \frac{1}{2}(1 + p_B^2 - p_A^2 + c_A)$$
- ▶ Similarly  $\hat{x}_B = \frac{1}{2}(1 + p_A^2 - p_B^2 - c_B)$
- ▶ We assume that  $0 < \hat{x}_B \leq \hat{x}_A < 1$  i.e., that there is switching in equilibrium. (We check *ex post* this condition)

- ▶ In  $t = 2$ ,  $A$  sells to consumers who had bought to  $A$  in  $t = 1$  ( $\alpha$ ) and do not switch ( $x < \hat{x}_A$ ), and those who bought from  $B$  ( $1 - \alpha$ ) and switch ( $x < \hat{x}_B$ ).
- ▶ The maximization program is:

$$\max_{p_A^2} \alpha \hat{x}_A (p_A^2 - c_A) + (1 - \alpha) \hat{x}_B p_A^2$$

- ▶ The best reaction function is:

$$p_A^2(p_B^2) = \frac{1}{2}(1 + p_B^2 - p_A^2 + 2\alpha c_A - (1 - \alpha)c_B)$$

- ▶ Using symmetry, we obtain:

$$p_A^2 = 1 + \alpha c_A, p_B^2 = 1 + (1 - \alpha)c_B$$

Prices paid by switching consumers (absent coupon) are higher.

- ▶ Equilibrium profits in  $t = 2$  are:

$$\pi_A^2 = \frac{1}{2} - \frac{1}{2}\alpha(1 - \alpha)c_A(c_A + c_B)$$



Competition in  $t = 1$ 

- ▶ In  $t = 1$ ,  $A$  maximizes its intertemporal profit:

$$\max_{p_A^1, c_A} p_A^1 \alpha + \pi_A^2(\alpha, c_A)$$

- ▶ To determine  $\alpha$  we need to find the address of the indifferent consumer. Assuming consumers are **forward looking**, we compute the difference in consumer's surplus in  $t = 1$ :

$$\Delta_s^1 = (r - \alpha - p_A^1) - (r - (1 - \alpha) - p_B^1) = 1 - 2\alpha + p_B^1 - p_A^1$$

and the difference in consumer's surplus in  $t = 2$ :

$$\begin{aligned} \Delta_s^2 &= \int_0^{\hat{x}_A} (r - (p_A^2 - c_A) - x) dx + \int_{\hat{x}_A}^1 (r - (p_B^2 - (1 - x))) dx \\ &\quad - \int_0^{\hat{x}_B} (r - p_A^2 - x) dx + \int_{\hat{x}_B}^1 (r - (p_B^2 - c_B) - (1 - x)) dx \\ &= \frac{1}{4}((c_A + c_B)^2 + 2(c_A - c_B)) - \frac{1}{2}(c_A + c_B)^2 \alpha \end{aligned}$$

Competition in  $t = 1$ 

- ▶  $\Delta_s^1 + \Delta_s^2 = 0$  gives:

$$\alpha = \frac{4(1 + p_B^1 - p_A^1) + (c_A + c_B)^2 + 2(c_A - c_B)}{2(4 + (c_A + c_B)^2)}$$

- ▶ Deriving the profit and focusing on a symmetric equilibrium, we find:

$$c_A = c_B = \frac{2}{3}, p_A^1 = p_B^1 = \frac{13}{9} > 1, p_A^2 = p_B^2 = \frac{4}{3} > 1, \pi_A = \pi_B = \frac{10}{9} > 1.$$

- ▶ Without coupons, prices would be equal to 1 in both periods and the intertemporal profit would be 1.
- ▶ Firms are better off with coupons, it enables them to relax competition and all consumers (except loyal costumers in  $t = 2$ ) pay a higher price.

## Exercise 2: Poaching

### Assumptions

- ▶ Two firms  $k \in \{A, B\}$  are located at the extremes of a Hotelling line and compete during two periods,  $t \in \{1, 2\}$ . Prices are denoted  $p_k^t$ .
- ▶ Consumers with a reservation price  $r$  uniformly distributed along the line, incur a linear transportation cost  $-x$  to travel distance  $x$
- ▶ No production cost.

### Questions

1. Determine the equilibrium of the two period game.
2. Firms now observe consumer's identities and can set personalized prices  $p_{kA}^2$  and  $p_{kB}^2$  respectively for consumers who bought from  $A$  and from  $B$  in  $t = 1$ . Assuming that  $\alpha$  is the market share of firm  $A$  in  $t = 1$ , determine the second period equilibrium.
3. Consumers are forward looking. Determine the address of the indifferent consumer  $\alpha$  in  $t = 1$ .
4. Determine the first period equilibrium prices.

## Références

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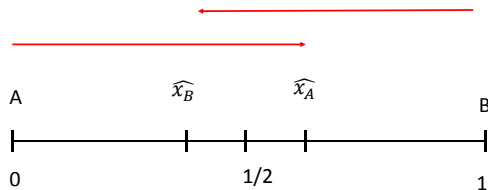
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## Initial Condition

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- ▶ We check here that, in equilibrium, the initial condition is met, i.e. that all old consumers who have bought from A buy again from A in  $t = 2$ .
- ▶ Formally we had assume that  $\hat{x}_A = \frac{1}{2}(1 + z) > \alpha = \frac{1}{2}$ .



Consumers that do not switch.