## Firms' Strategies and Markets Course 4: Dynamic Pricing

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## **Dynamic Pricing**

- Repeated interactions among firms may enable collusive strategies (IO class M1)
  - High prices over time.
- Reputation or Signaling strategies can occur (Class / Advertising & Entry )
  - Either a low or a high price can signal a high quality to an uninformed consumer in a first period.
  - Fighting on one market can create the reputation of being tough.
- We focus here on "consumer inertia" which may come from different sources and imply various firm's dynamic pricing strategies.
  - Durable Goods
  - Search costs  $\rightarrow$  generate temporal price dispersion.
  - Switching costs  $\rightarrow$  Consumers are *locked-in* within the same firm

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**Durable goods**: Goods that are not consumed or destroyed in use; Consumers derive the benefit of their purchase for a period of time (several years).

► Cars, Washing Machines, Computers, Smartphones ...

**Insights:** A durable good monopoly who cannot discriminate in a given period among heterogenous consumers can use intertemporal discrimination to extract more surplus from consumers.

- Some consumers buy in the first period;
- Others delay their purchase expecting a lower price.

#### Assumptions

- A durable monopoly with a production cost 0.
- A continuum of heterogenous consumers live two periods t = {1,2}. Consumers buy either 0 or 1 unit and their valuation for the good v is uniformly distributed over [0, 1].
- $\delta$  is the discount factor.
- The monopoly sets  $p_1$  in t = 1 and  $p_2$  in t = 2.

Consider first the simple case in which the monopoly can sell only in t = 1 at price p.

A consumer is willing to purchase the good if  $(1 + \delta)v - p > 0$  in t = 1. The demand is  $D(p) = 1 - \frac{p}{1+\delta}$ .

The price that maximizes the monopoly profit  $p(1 - \frac{p}{1+\delta})$  is:  $p = \frac{1+\delta}{2}$ . The corresponding profit  $\Pi = \frac{1+\delta}{4}$ .

#### Consider now the two period pricing strategy

- For a given couple of prices  $(p_1, p_2)$  we determine the consumer indifferent between purchasing in t = 1 and in t = 2.

$$\underbrace{(1+\delta)\tilde{v}-p_1}_{t=1}=\underbrace{\delta(\tilde{v}-p_2)}_{t=2}\Rightarrow\tilde{v}(p_1,p_2)=p_1-\delta p_2$$

- Suppose that consumers with  $v > \tilde{v}$  have purchased the good in t = 1. The residual demand for the good in t = 2 is

$$D_2(p_1, p_2) = \tilde{v}(p_1, p_2) - p_2.$$

In t = 2, the monopoly chooses  $p_2$  to maximise  $p_2D_2(p_1, p_2)$  and this gives

$$p_2(p1)=\frac{p_1}{2(1+\delta)}$$

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The price in the second period is lower than half of the price in the first period.

- in t = 1 now, the demand is

$$D_1(p_1, p_2) = 1 - \tilde{v}(p_1, p_2)$$

and the monopoly sets  $p_1$  to maximise its intertemporal profit

$$\Pi_{1,2} = p_1 D_1(p_1, p_2) + p_2 D_2(p_1, p_2)$$

under the constraint that  $p_2(p_1) = rac{p_1}{2(1+\delta)}.$  This leads to

$$p_1=rac{(2+\delta)^2}{2(4+\delta)}$$

and the profit is:

$$\Pi_{1,2} = \frac{(2+\delta)^2}{4(4+\delta)} < \Pi$$

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#### The durable good monopolist

-Obtains lower profit in selling over the two periods than only in the first.

-Cannot prevent from competing with itself.

## Remember

- A durable good monopolist may compete with itself throughout time
- Some business practices may limit this phenomenon
  - Renting the good instead of selling it! Here renting at price  $p_1 = p_2 = \frac{1}{2}$  at each period brings  $\Pi$ .
  - Return policies, money back guarantees or repurchase agreements, ...Contracts that are offered by *M* to protect the consumers in *t* = 1 against any future price cut.
  - Reputation
  - Technology (capacity constraints, planned obsolescence, new version of the product...)

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If discrete classes of consumers can be identified, intertemporal discrimination can become profitable (See Exercice 1!).

#### Exercice 1

#### Assumptions

- ► A durable good monopoly, M, with a production cost *c*.
- ► Two consumers who live two periods t = {1; 2}. Two consumers who buy either 0 or 1 unit and C1 has a valuation 1 and C2 c < v<sub>l</sub> < 1.</p>
- $\delta$  is the discount factor.
- M sets  $p_1$  in t = 1 and  $p_2$  in t = 2.

#### Questions

1. Determine the price equilibrium p and profit  $\Pi$  if M only sells in t = 1.

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## Solution-Exercice 1

- 1. Determine the price equilibrium p and profit  $\Pi$  if M only sells in t = 1.
- ▶ If M sells only to C1,  $p = 1 + \delta$  and its profit is  $\Pi = 1 + \delta c$ .
- If M sells to C1 and C2  $p = v_l(1 + \delta)$  and its profit is  $\Pi = 2(v_l(1 + \delta) c)$ .
- The first option is chosen if  $c < v_l < \frac{1}{2}(1 + \frac{c}{1+\delta})$ .

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- 2. Determine the two period equilibrium  $(p_1, p_2)$  and profit  $\Pi_{1,2}$  of M.
- M wishes to serve C1 in t = 1 and C2 in t = 2. To make sure C1 buys in t = 1:  $1 + \delta - p_1 > \delta(1 - p_2) \Rightarrow$

$$p_1 < 1 + \delta p_2 \tag{1}$$

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- ▶ Now,  $p_2$  depends on the behavior of C1 in t = 1. If C1 has not purchased the good in t = 1,
  - If  $v_l < \frac{1}{2}(1+c)$ , M sets  $p_2 = 1$ . Therefore, given (1) M sets  $p_1 = 1 + \delta$  and sells to C1 and  $p_2 = v_l$  and sells to C2. M obtains  $\Pi_{1,2} = 1 + \delta - c + \delta(v_l - c)$ .
  - If  $v_l > \frac{1}{2}(1+c)$ , M sets  $p_2 = v_l$ . Thus, given (1), M sets  $p_1 = 1 + \delta v_l$  and sell to C1 and  $p_2 = v_l$  and sets to C2. M obtains  $\Pi_{1,2} = 1 + \delta v_l - c + \delta(v_l - c)$ .

## Solution-Exercice 1

- 3. Compare the two profits when  $c < v_l < \frac{1}{2}(1 + \frac{c}{1+\delta})$ . What happens if  $v_l > \frac{1}{2}(1+c)$ ?
- ► If  $c < v_l < \frac{1}{2}(1 + \frac{c}{1+\delta})$ ,  $\Pi = 1 + \delta c < \Pi_{1,2} = 1 + \delta c + \delta(v_l c)$ . Intertemporal discrimination is profitable!
- The reverse is true when  $v_l > \frac{1}{2}(1+c)!$

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## Search Costs & The Diamond Paradox

Search costs: Consumers might be imperfectly informed about prices

- If getting information is costly,  $p_1 = p_2 > c$  can be an equilibrium.
- Diamond Paradox: in a duopoly p<sub>1</sub> = p<sub>2</sub> = p<sup>M</sup> might be an equilibrium
  - All consumers are uninformed about prices

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For any p<sub>1</sub> = p<sub>2</sub> = p < p<sup>M</sup>, a firm has an incentive to deviate towards p + <sup>ε</sup>/<sub>2</sub>!

# Search Costs and Temporal Price Dispersion Varian (1980): A model of "sales".

#### Assumptions

- Monopolistic competition among n symmetric firms with free entry.
- ▶ I informed consumers and  $U = \frac{M}{n}$  uninformed consumers per store.
- r is the reservation price of consumers.
- C(q) is a firm cost function with strictly decreasing average cost (ex: cq + f).
- If a firm sets the lowest price, it obtains I + U consumers.
- ▶ If the firm does not set the lowest price, it obtains U consumers.
- If several firms have the identical lowest price, there is a tie, and they share equally I consumers among them.

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#### There exists no symmetric pure strategy Nash equilibrium

- ► First, the relevant range of prices is [p\*, r]. If p > r, there is no demand and if p < p\* = C(1+U) / (1+U) the firm obtains a negative profit even in the best case, i.e. when serving all consumers.</p>
- ▶ If all firms set  $p = p^*$ , there is a tie and then profits are negative:  $p^*x(U + \frac{1}{n}) - C(U + \frac{1}{n}) < 0.$
- If all firms set p ∈]p\*, r], a slight price cut by one of the firms enables to capture all informed customers and realize a positive profit.

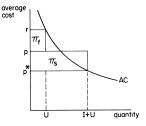
#### There is a symmetric equilibrium in mixed strategy.

► Each firm randomly chooses a price according to the same density of probability f(p) (F(p) is the distribution function) ⇒ Temporal price dispersion arises!

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Assume that all firms have the same distribution F(p). We build the expected profit function for a firm for any price p

- ▶ With probability  $(1 F(p))^{n-1}$ , p is the lowest price and then the firm earns  $\pi_s(p) = p(U + I) C(U + I)$  (Success).
- With probability  $1 (1 F(p))^{n-1}$ , p is not the lowest price and it obtains  $\pi_f(p) = pU C(U)$ .



The expected profit of the firm therefore is:

$$\int_{p^*}^r [\pi_s(p)(1-F(p))^{n-1} + \pi_f(p)(1-(1-F(p))^{n-1})]f(p)dp$$



Maximizing the above profit with respect to p, the FOC is:

$$\pi_s(p)(1-F(p))^{n-1}+\pi_f(p)(1-(1-F(p))^{n-1})=0$$

Rearranging, we obtain:

$$F(p) = \begin{cases} 0 & p < p^* \\ 1 - \left(\frac{\pi_f(p)}{\pi_f(p) - \pi_s(p)}\right)^{\frac{1}{n-1}} & p \in [p^*, r] \\ 1 & p > r \end{cases}$$

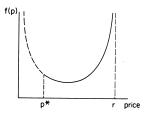
If firms compete in a market with both informed and uninformed consumers, temporal price dispersion may arise in equilibrium. In equilibrium firms alternate (ramdomly) relatively high prices and periods of sales.

## An example with c(q) = f

• 
$$\pi_f(r) = rU - f = 0 \Rightarrow U = \frac{f}{r}$$

$$\pi_s(p^*) = p^*(I+U) - f = 0 \Rightarrow p^* = \frac{f}{I+\frac{f}{r}}$$

▶ The corresponding *f*(*p*) has the following shape:



- Firms tend to charge extreme prices with higher probability.
- Prices are lower as *l* increases and *f* is low (more competitive) but high prices are always charged with positive probability.

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- This model also applies to competition among stores that have a base of *loyal consumers* and other *consumers that tend to switch among stores* when the store cannot distinguish among these consumers (see Narasimhan, 1988).
  - ► There is a tension between an incentive to set the monopoly price to loyal consumers and a competitive price for those who may go to the rival → Mixed strategy equilibrium
- These results on temporal price dispersion are robust if consumers can endogenously decide whether they want to acquire additional information through costly search.
- Empirical evidence for search costs online vs offline.

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## Switching costs

**Definition**: The presence of switching costs give consumers an incentive to purchase repeatedly from the same supplier.

- Transaction costs: Time and effort to change supplier (e.g. changing bank accounts, insurances, telephone company, etc...)
- Contractual costs : Mobile phone company that offers a contract with a phone at low price for a 24 month lock-in contract.
- Shopping costs : Purchasing several goods from one supplier rather than shopping around for different products.

Search costs

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## Imperfect competition and switching costs Assumptions

- ► Two-period model with imperfect competition.
- ► Consumers are uniformly distributed along a Hotelling line [0, 1] with a linear transportation cost -x for a distance x. Two firms A and B are located at the extremes.

#### Switching costs

- After t = 1, a share λ of consumers leaves the market and is replaced by new consumers.
- The remaining share of consumers  $(1 \lambda)$  who has bought from firm K = A, B in t = 1 incurs a cost z to switch to the other firm in t = 2.
- Old consumers keep their preference from one period to the next.
- Consumers have a reservation price r such that the market is fully covered.
- Consumers are **myopic**.

## Benchmark without switching cost

- Both periods are identical and independent.
- Old and new consumers behave in the same way:
  - A consumer x buys from A in t = 1, 2 if:

$$r-x-p_A^t \geq r-(1-x)-p_B^t \Rightarrow x \geq ilde{x} = rac{1}{2}(1+p_B^t-p_A^t)$$

• In each t = 1, 2 firm A (resp. firm B) maximizes :

$$p_A^t \tilde{x} \Rightarrow p_A^t = p_B^t = 1$$

• Equilibrium profits are  $\Pi_K^t = \frac{1}{2}$  for each firm.

## Competition in t = 2

- Assume that in t = 1, each firm A and B has obtained respectively a share α and 1 − α of the market.
- A fraction  $(1 \lambda)$  of consumers remain
  - ▶ A consumer x who bought from A in t = 1 buys again from A if:

$$r - x - p_A^2 \ge r - (1 - x) - p_B^2 - z \Rightarrow x \le \hat{x}_A = \frac{1}{2}(1 + p_B^2 - p_A^2 + z)$$

- A fraction  $\lambda$  are new consumers
  - A new consumer x buys from A in t = 2 if:

$$r-x-p_A^2 \ge r-(1-x)-p_B^2 \Rightarrow x \le \hat{x} = rac{1}{2}(1+p_B^2-p_A^2)$$

• Assume  $\hat{x}_A > \alpha$  (we check *ex post* this condition), the demand is:

$$q_A^2(p_A^1, p_B^1, p_A^2, p_B^2) = (1 - \lambda)\alpha(p_A^1, p_B^1) + \lambda \frac{1}{2}(1 + p_B^2 - p_A^2)$$

▶ The same reasoning applies for *B*.

#### Competition in t = 2

The FOC writes as:

$$\frac{\partial \pi_A^2}{\partial p_A^2} = q_A^2 + p_A^2 \frac{\partial q_A^2}{\partial p_A^2} = 0$$

We obtain :

$$p_A^2(p_B^2) = rac{1-\lambda}{\lambda}lpha + rac{1}{2}(1+p_B^2)$$

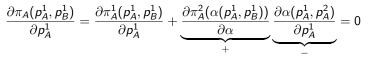
- Firms compete more aggressively to gain new costumers:  $\frac{\partial p_A^2(p_B^2)}{\partial \lambda} < 0$
- Firms compete less aggressively as the share of "captive consumer" increases:  $\frac{\partial p_A^2(p_B^2)}{\partial \alpha} > 0$
- ► In t = 2 equilibrium,  $\pi_A^2(\alpha(p_A^1, p_B^1)) = \frac{1}{2\lambda}(1 + \frac{1}{3}(2\alpha 1)(1 \lambda))^2$ with  $\alpha(p_A^1, p_B^1) = \frac{1}{2}(1 + p_B^1 - p_A^1)$ .

## Competition in t = 1

In t = 1 firms take into account their intertemporal profit over the two periods.

$$\pi_A(\boldsymbol{p}_A^1,\boldsymbol{p}_B^1) = \pi_A^1(\boldsymbol{p}_A^1,\boldsymbol{p}_B^1) + \pi_A^2(\alpha(\boldsymbol{p}_A^1,\boldsymbol{p}_B^1))$$

The FOC is:



- ▶ For  $\lambda > \frac{2}{5}$ , in equilibrium  $\alpha = \frac{1}{2}$ , and  $p_K^1 = \frac{5\lambda-2}{3}$  and  $p_K^2 = \frac{1}{\lambda}$ . For  $\lambda \le \frac{2}{5}$ , in equilibrium  $\alpha = \frac{1}{2}$ , and  $p_K^1 = 0$  and  $p_K^2 = \frac{1}{\lambda}$ . BOUTON
- ▶ In the benchmark case without switching costs:  $p_K^1 = p_K^2 = 1$ .
- In the first period p<sup>1</sup><sub>K</sub> < 1 is lower to lock in as much consumers as possible ( second period profit effect).</p>
- In the second period though, p<sup>2</sup><sub>K</sub> > 1 the equilibrium price is higher because firms compete only for new consumers. → (=) → (



- ► In terms of profit, each firm loses in t = 1 but earns more in t = 2 than absent switching costs.
- ▶ In equilibrium the intertemporal profit with switching costs is:

$$\pi_{\mathcal{A}} = \begin{cases} \frac{1}{6} \left( \frac{1}{\lambda} + 5 \right) & \text{ for } \lambda > \frac{2}{5}, \\ \frac{1}{2\lambda} & \text{ for } \lambda < \frac{2}{5} \end{cases}$$

- ▶ In equilibrium, the intertemporal profit without switching cost is 1.
- Here firms are always better off when they can lock-in consumers and the effect on consumers is negative.

## Endogenous switching cost: Coupons

- Coupons are discount offered on the price of the product at the next purchase.
- ▶ The oldest "Coupon" by TheCCC



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#### Assumptions

- Consumers redraw their types in t = 2.
- In t = 1 firms can offer coupons c<sub>K</sub> ≥ 0 to their loyal consumers. In t = 2 the consumer will pay p<sub>A</sub><sup>2</sup> − c<sub>A</sub> if he buys again from A.
- Consumers are forward looking.

#### Competition in period 2

- A consumer who purchased from A in t = 1, buys from A again if its new address x is such that  $r - x - (p_A^2 - c_A) > r - (1 - x) - p_B^2 \Rightarrow x < \hat{x}_A = \frac{1}{2}(1 + p_B^2 - p_A^2 + c_A)$
- Similarly, consumers who purchased from B in t = 1 buys from B again if x > x̂<sub>B</sub> = ½(1 + p<sup>2</sup><sub>B</sub> − p<sup>2</sup><sub>A</sub> − c<sub>B</sub>)
- ▶ We assume that  $0 < \hat{x}_B \le \hat{x}_A < 1$  i.e., that there is switching in equilibrium. (We check *ex post* this condition)

- ▶ In t = 2, A sells to consumers who had bought from A in t = 1 ( $\alpha$ ) and do not switch ( $x < \hat{x}_A$ ), and those who bought from B ( $1 \alpha$ ) and switch ( $x < \hat{x}_B$ ).
- The maximization program is:

$$\max_{p_A^2} \alpha \hat{x}_A (p_A^2 - c_A) + (1 - \alpha) \hat{x}_B p_A^2$$

The best reaction function is:

$$p_A^2(p_B^2) = \frac{1}{2}(1+p_B^2+2\alpha c_A-(1-\alpha)c_B)$$

• Conversely, we obtain:  $p_B^2(p_A^2) = \frac{1}{2}(1 + p_A^2 - \alpha c_A + 2(1 - \alpha)c_B)$ • In equilibrium,

$$p_A^2 = 1 + \alpha c_A, p_B^2 = 1 + (1 - \alpha) c_B$$

Prices paid by switching (resp. loyal) consumers are higher (resp. lower).

• Equilibrium profit in t = 2 is:  $\pi_A^2 = \frac{1}{2} - \frac{1}{2}\alpha(1 - \alpha)c_A(c_A + c_B) < \frac{1}{2}\alpha_{AB}$ 

#### Competition in t = 1

• In t = 1, A maximizes its intertemporal profit:

$$\max_{p_A^1, c_A} p_A^1 \alpha + \pi_A^2(\alpha, c_A)$$

To determine α we need to find the address of the indifferent consumer. Assuming consumers are forward looking, we compute the difference in consumer's surplus in t = 1:

$$\Delta_{s}^{1} = (r - \alpha - p_{A}^{1}) - (r - (1 - \alpha) - p_{B}^{1}) = 1 - 2\alpha + p_{B}^{1} - p_{A}^{1}$$

and the difference in consumer's surplus in t = 2:

$$\Delta_s^2 = \int_0^{\hat{x}_A} (r - (p_A^2 - c_A) - x) dx + \int_{\hat{x}_A}^1 (r - p_B^2 - (1 - x)) dx$$
  
- 
$$\int_0^{\hat{x}_B} (r - p_A^2 - x) dx + \int_{\hat{x}_B}^1 (r - (p_B^2 - c_B) - (1 - x)) dx$$
  
= 
$$\frac{1}{4} ((c_A + c_B)^2 + 2(c_A - c_B)) - \frac{1}{2} (c_A + c_B)^2 \alpha$$

#### Competition in t = 1

• 
$$\Delta_s^1 + \Delta_s^2 = 0$$
 gives:  

$$\alpha = \frac{4(1 + p_B^1 - p_A^1) + (c_A + c_B)^2 + 2(c_A - c_B)}{2(4 + (c_A + c_B)^2)}$$

• Deriving the intertemporal profit  $\max_{p_A^1, c_A} p_A^1 \alpha + \pi_A^2(\alpha, c_A)$  for A and B and focusing on a symetric equilibrium, we find:

$$c_{A} = c_{B} = \frac{2}{3}, p_{A}^{1} = p_{B}^{1} = \frac{13}{9} > 1, p_{A}^{2} = p_{B}^{2} = \frac{4}{3} > 1, \pi_{A} = \pi_{B} = \frac{10}{9} > 1.$$
$$\alpha = \frac{1}{2}, \hat{x}_{A} = \frac{5}{6}, \hat{x}_{B} = \frac{1}{6}$$

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- Without coupons, prices would be equal to 1 in both periods and the intertemporal profit would be 1.
- Prices with coupon are  $p_A^2 c_A = rac{2}{3} < 1$
- Firms are better off with coupons, it enables them to relax competition and all consumers (except loyal costumers in t = 2 who pay <sup>2</sup>/<sub>3</sub>) pay a higher price.

## Exercice 2: Poaching

#### Assumptions

- ► Two firms k ∈ {A, B} are located at the extremes of a Hotelling line and compete during two periods, t ∈ {1,2}. Prices are denoted p<sup>t</sup><sub>k</sub>.
- ► Consumers with a reservation price r uniformly distributed along the line, incur a linear transportation cost -x to travel distance x
- No production cost.

#### Questions

1. Determine the equilibrium of the two period game.

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## Solutions: Exercice 2

- 1. Determine the equilibrium of the two period game.
  - The one shot game is repeated twice: no dynamic effect here!
  - ▶ p<sup>t</sup><sub>A</sub> = p<sup>t</sup><sub>B</sub> = 1 in both periods and each firm gets a market share <sup>1</sup>/<sub>2</sub>, the equilibrium profit is 1.

Firms now observe consumer's identities and can set personalized prices  $p_{kA}^2$  and  $p_{kB}^2$  for consumers who bought from A or B in t = 1.

- 2. If  $\alpha$  is the market share of firm A in t = 1, determine the second period equilibrium.
  - ► The indifferent consumers addresses are  $\hat{x}_A = \frac{1}{2} + \frac{(p_{BA}^2 p_{AA}^2)}{2}$  and  $\hat{x}_B = \frac{1}{2} + \frac{(p_{BB}^2 p_{AB}^2)}{2}$ . Firms *A* and *B*'s maximization problems are:

$$\begin{split} \max_{p_{AA}^2, p_{AB}^2} \quad p_{AA}^2 \hat{x}_A + p_{AB}^2 (\hat{x}_B - \alpha) \\ \max_{p_{BA}^2, p_{BB}^2} \quad p_{BA}^2 (\alpha - \hat{x}_A) + p_{BB}^2 (1 - \hat{x}_B) \end{split}$$

The solution is  $p_{AA}^{2} = \frac{1}{3}(1+2\alpha), p_{AB}^{2} = \frac{1}{3}(3-4\alpha), p_{BA}^{2} = \frac{1}{3}(4\alpha-1), p_{BB}^{2} = \frac{1}{3}(3-2\alpha)$   $\hat{x}_{A} = \frac{1}{6} + \frac{1}{3}\alpha, \hat{x}_{B} = \frac{1}{2} + \frac{1}{3}\alpha$ 

## Solutions: Exercice 2

- 3. Consumers are forward looking. Determine the address of the indifferent consumer  $\alpha$  in t = 1.
  - In t = 1, anticipating a symmetric equilibrium the indifferent consumer anticipates that it will switch in t = 2 and therefore its address is such that:

$$r - \alpha - p_A^1 + (r - (1 - \alpha) - p_{BA}^2) = r - (1 - \alpha) - p_B^1 + (r - \alpha - p_{AB}^2)$$
  
$$\alpha = \frac{4 + 3(p_B^1 - p_A^1)}{8}$$

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BOUTON

- 4. Determine the first period equilibrium prices.
  - Firm A's intertemporal profit is:

$$\max_{\substack{p_A^1\\p_A^2}} p_A^1 \alpha + p_{AA}^2 \hat{x}_A + p_{AB}^2 (\hat{x}_B - \alpha)$$

$$p_{A}^{1} = p_{B}^{1} = \frac{4}{3}, \ \alpha = \frac{1}{2}, \ p_{AA}^{2} = p_{BB}^{2} = \frac{2}{3}, p_{BA}^{2} = p_{AB}^{2} = \frac{1}{3}, \\ \hat{x}_{A} = \frac{1}{3}, \\ \hat{x}_{B} = \frac{2}{3}.$$

- Poaching relaxes competition in period 1 and intensifies it in the second. Total profit is Π<sub>A</sub> = Π<sub>B</sub> = <sup>5</sup>/<sub>6</sub>.
- Firms would be better off if they could refrain from poaching.

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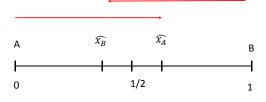
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## Initial Condition

back

- ► We check here that, in equilibrium, the initial condition is met, i.e. that all old consumers who have bought from A buy again from A in t = 2.
- Formally we had assume that  $\hat{x}_A = \frac{1}{2}(1+z) > \alpha = \frac{1}{2}$ .

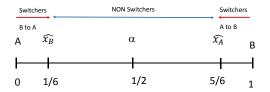


Consumers do not switch.

#### Initial Condition

back

- We check here that, in equilibrium, the initial condition is met, i.e. that all old consumers who have bought from A buy again from A in t = 2.
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Consumers that do not switch.

#### Initial Condition

back

▶ We check here that, in equilibrium, the initial condition is met  $\hat{x}_A < \alpha < \hat{x}_B$ 

FirmA	Poached by firm B	1 00	iched	Firm B	
A	$\widehat{x_A}$	α	$\widehat{x_B}$		В
<b></b>					
•	1/3	1/2	2/3		
0	1/5	1/2	2/5		1

t=2