ECO 650: Firms' Strategies and Markets

Innovation

Claire Chambolle

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Exercice 2: R&D Cooperation

Assumptions:

- Demand is linear, $p = 2 - Q$ and two firms $i \in \{1, 2\}$ compete à la Cournot.
- The cost of firm i is a function $c_i(w_i, x_j) = 1 - x_i - \beta x_j$ with $0 < \beta < 1$ representing a spillover, i.e. benefit a firm obtains from its rival's discovery (public part).
- We denote $\phi(x_i) = \frac{x_i^2}{2}$ the innovation cost.
- The timing we consider is 1) an investment stage which can be either non cooperative or cooperative, and 2) a competition stage.

Questions:

1. Determine the Cournot equilibrium in stage 2
2. Non Cooperative $R\&D$: firms in stage 1 choose x_i and x_j. What is the equilibrium profit and quantity ?
3. Cooperative $R\&D$: firms in stage 1 choose x_i and x_j that maximizes their joint profit. What is the equilibrium profit and quantity ?
4. Compare the outcomes in the two cases.

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Solution 2: R&D Cooperation

1. Each firm i maximizes $(1 - q_i - q_j + x_i + \beta x_j)q_i$. The FOC is $1 - 2q_i - q_j + x_i + \beta x_j = 0$. Crossing best reaction functions, we obtain $q_i(x_i, x_j) = \frac{1}{3}(1 + (2 - \beta)x_i - (1 - 2\beta)x_j)$ and a profit $\pi_i = \frac{1}{9}(1 - x_j + (2 - \beta)x_i + 2\beta x_j)^2$
2. Profit $\pi_i = \frac{1}{9}(1 - x_j + (2 - \beta)x_i + 2\beta x_j)^2 - \frac{x_i^2}{2}$. Maximizing this function with respect to x_i, the FOC is: $\frac{2}{9}(1 - x_j + (2 - \beta)x_i + 2\beta x_j)(2 - \beta) - x_i = 0$. Using symmetry, $x_j = x_i$, we obtain

$$x_i^* = x_j^* = \frac{4 - 2\beta}{2\beta^2 - 2\beta + 5}$$

. The equilibrium profit is

$$\pi_i^* = \frac{-2\beta^2 + 8\beta + 1}{(2\beta^2 - 2\beta + 5)^2} > 0$$

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Solution 2: R&D Cooperation

3. Each i maximizes $\sum_i \pi_i$ with respect to x_i Maximizing this function with respect to x_i, the FOC is: $\frac{2}{9}(1 - x_j + (2-\beta)x_i + 2\beta x_j)(2-\beta) - x_i + \frac{2}{9}(1 - x_i + (2-\beta)x_j + 2\beta x_i)(2\beta - 1) = 0$. Using symmetry, $x_j = x_i$, we obtain

$$x_i^{coop} = x_j^{coop} = \frac{2(\beta+1)}{7 - 2\beta^2 - 4\beta}.$$

The equilibrium profit is

$$\pi_i^{coop} = \frac{1}{7 - 2\beta^2 - 4\beta} > 0$$

4. A comparison shows that $\pi_i^{coop} > \pi_i^*$ for all $\beta \in [0,1]$. Firms could when cooperating still choose the same investment as before, so this is necessarily profitable. Then, $x_i^* > x_i^{coop}$ when $\beta < \frac{1}{2}$ to limit the R&D cost race. But $x_i^* < x_i^{coop}$ when $\beta > \frac{1}{2}$ because firms internalize the strong spillover effect they exert on eachother which encourage them to invest.

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