ECO 650: Final Exam 2017

December, 13, 2017

1 About Bundling

CMG Sports Club offers 3 basics services (Fitness facilities, Swimming pool, Spa & Massages). CMG Sports Club only offers annual membership. Consumers fall into three types:

Consumers	Fitness facilities	Swimming pool	Spa & Massages
Type A	120	30	50
Type B	80	60	20
Type C	100	20	70

On an annual basis, Fitness facilities and Spa& Massages each cost 20 per member and access to the pool costs 5 per member. The firm cannot discriminate among consumers. We assume there is 1 consumer of each type (A, B and C) and who wants one unit of each product.

Questions:

1. Determine the best pricing strategy for CMG Sports Club if it offers an annual card fee per service?

    - **For Fitness facilities**: $(80-20).3 = 180 > (100-20).2 = 160 > 120 - 20 = 100$.
    - **For Swimming Pool**: $(20 - 5).3 = 45 < (30 - 5).2 = 50 < 60 - 5 = 55$.
    - **For Spa& Massages**: $(20 - 20).3 = 0 < (50 - 20).2 = 60 > 70 - 20 = 50$.
    - Selling access to Fitness facilities at a price of 80, to the Swimming Pool at a price of 60 and to Spa& Massages at a price of 50 makes a total profit of $45 + 55 + 60 = 160$.

2. Determine the optimal price if CMG offers only a Gold card membership (Free access to the 3 services- pure bundling)?

    - For total access Type A has a reservation price of 200, Type B 160 and Type C 190. Total access cost card is $20 + 20 + 5 = 45$.
    - $(160 - 45).3 = 345 > (190 - 45).2 = 290 > (200 - 45) = 155$.
    - Selling a total access card at price 160 to all consumers brings a profit of 345.

3. Determine the optimal fee for any Silver access card (2-services access card-mixed bundling)?

    - There are three types of Silver access cards that we respectively denote (F-S), (F-SM) and (S-SM).

- (F-S) costs 25. Type A is willing to pay 150, Type B 140 and Type C 120. $(120-25).3 = 285 > (140-25).2 = 230 > (150-25).1 = 125$. Combining a card (F-S) with a single access for SM at price 50 brings a total profit of $285 + 60 = 345$.
- (F-SM) costs 40. Type A is willing to pay 170, Type B 100 and Type C 170. $(170-40).2 = 260 > (100-40).3 = 180$. Combining a card (F-SM) with a single access for S at price 60 brings a total profit of $260 + 55 = 315$.
- (S-SM) costs 25. Type A is willing to pay 80, Type B 80 and Type C 90. $(80-25).2 = 110 > (90-25).1 = 65$. Combining a card (S-SM) with a single access for F at price 80 brings a total profit of $110 + 180 = 290$.

4. Comparing profits in each alternative case, what is the best pricing strategy for CMG Sports Club?

   CMG sports Club is indifferent between offering a Gold card at 160 and a Silver card (F-S) at 120 combined with a separate price for SM at 50. The profit is 345 in the two cases.

2 Vertical Relations

Assume there is one upstream firm U that relies on one downstream firm D to sell its product to consumers. The unit cost of the product is c. Consumers' demand is given by $q = a - p$, where $a > 0$ is a parameter, q is the quantity demanded and p is the final price charged to consumers.

Questions:

1. U offers a take-it-or-leave-it linear contract to D, i.e. a unit price w. Determine the solution of the two stage game in which U offers its contract w and then, D chooses its final price p. Determine the profit of each firm.

   D maximises $(p-w)(a-p)$ with respect to p and chooses $p = \frac{a+w}{2}$. U thus chooses w to maximize $(w-c)\frac{a-w}{2}$ and we obtain $w^* = \frac{a+c}{2}$, $p^* = \frac{3a+c}{4}$ and respective profits $\pi_U = \frac{(a-c)^2}{8}$ and $\pi_D = \frac{(a-c)^2}{16}$ with a total industry profit of $\pi_U = \frac{3(a-c)^2}{16}$.

2. Assume now that U and D merge. Determine the optimal price p and profit. Compare the result with question 1 and comment. $U-D$ maximize $(p-c)(a-p)$ with respect to p and therefore $p^M = \frac{a+c}{2}$ and the industry profit is $\pi^M = \frac{(a-c)^2}{4}$. Double-Marginalisation.

3. What happens if U offers a take-it-or-leave-it two-part tariff contract (w, F) to D and then D chooses its price? What happens if U offers a Resale Price Maintenance contract defined by $(w, \bar{p})$ to D in which $\bar{p}$ is the resale price? D maximises $(p-w)(a-p)$ with respect to p and chooses $p = \frac{(a+w)}{2}$. Then U offers a contract that maximizes w to maximize $(w-c)\frac{(a-w)}{2} + F$ under the constraint $\frac{(a-w)^2}{4} - F \geq 0$. Then the optimum is such that $w = c$ and $F = \frac{(a-c)^2}{4}$. Total industry profit is the same as vertical integration. D makes no profit. $\bar{p} = w = p^M = \frac{a+c}{2}$ and U gets the monopoly profit.

4. Assume now that first U and D bargain (with equal power) over the contract w and second D choose its price. What happens? Compare

the result with that of question 1.

D maximises $(p-w)(a-p)$ with respect to p and chooses $p = \frac{(a+w)}{2}$. Then the Nash program is $[\frac{(a-w)^2}{4}][(w-c)\frac{a-w}{2}]$ and it is maximized in w for $w = \frac{1}{4}(a+3c)$. Profits are $\pi_U = \frac{3}{32}(a-c)^2$, $\pi_D = \frac{9}{64}(a-c)^2$ and the industry profit is $\pi_I = \frac{15}{64}(a-c)^2 > \frac{3(a-c)^2}{16}$.

5. Assume now that first U and D bargain (with equal power) over the contract (w, F) and second D choose its price. What happens? Compare the result with that of question 3.

   D maximises $(p-w)(a-p)$ with respect to p and chooses $p = \frac{(a+w)}{2}$. Then, the Nash program is $[\frac{(a-w)^2}{4} - F][(w-c)\frac{a-w}{2} + F]$ and it is maximized in w for $w = c$ and F is such that profits are $\pi_U = \frac{(a-c)^2}{8}$, $\pi_D = \frac{(a-c)^2}{8}$ and the industry profit is $\pi_I = \frac{(a-c)^2}{4}$.

6. Assume that D can also buy the product at cost $\bar{c} \in [c, a[$ from a competitive fringe. Determine the new equilibrium contract (w, F), price and profit depending on whether U makes a take-it-or-leave-it offer or U and D bargain. Comment. $w^* = c$ in the two cases and in case of tiolit offer, $F^* = \frac{(a-c)^2}{4} - \frac{(a-\bar{c})^2}{4}$ and in case of bargaining $F = \frac{(a-c)^2}{8} - \frac{(a-\bar{c})^2}{8}$.