# Firms' Strategies and Markets
# Entry

Claire Chambolle

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# Introduction

- Entrant's strategy: "Judo economics"
  - A case study
  - Exercice
- Incumbent's strategies vis-à-vis entry
  - Entry deterred
  - Entry Accomodated

# Entrant's strategy: Judo Economics

**In the art of “judo", a combatant uses the weight and strenght of his opponent to his own advantage.**

- *Rule-based* judo strategy
- *Value-based* judo strategy

- Case study: Four short stories about small firms challenging large incumbent firms!

  1. Softsoap on the liquid soap market
  2. Red Bull on the energy drinks market
  3. UK supermarket chains on the gazoline retail
  4. Freeserve against AOL.

# Judo Economics: Exercice

- Consumers have an inelastic demand of size $D$ if $p \leq p_{max}$.
- An incumbent $I$ has an installed capacity $D$ and no production cost.
- An entrant $E$ has a variable cost $c_E > 0$

1. Determine the price and profit of a monopoly $I$.

A monopolist $I$ sets a price $p_{max}$ and its profit is $p_{max}D$.

2. If $E$ chooses to enter the market, he chooses both its capacity $K_E$ and its price $p_E$. If $E$ enters, $I$ observes $K_E$ and $p_E$ and adapts its price denoted $p_I$.

a. What is the demand for each firm with respect to $p_E$, $p_I$ and $K_E$?

- If $p_I > p_E$ the firm $E$ $D_E = K_E$ and $D_I = D - K_E$
- If $p_I \leq p_E$, the firm I has a demand $D_I = D$ and $D_E = 0$

b. Given $(K_E, p_E)$, determine the best pricing strategy of firm I, denoted $p_I(K_E, p_E)$.

- If $p_I > p_E$ the firm $E$ $D_E = K_E$ and $D_I = D - K_E$.
- If $p_I \leq p_E$, the firm I has a demand $D_I = D$ and $D_E = 0$. Given $(K_E, p_E)$, the firm I can sell at $p_{max}$ and obtain a profit

$$p_{max}(D - K_E)$$

.

The firm can also sell at $p_E$ and obtain $p_E D$. I chooses the price that maximizes its profit i.e.: $p_{max}$ if $p_E \leq \frac{p_{max}(D-K_E)}{D}$ and $p_E$ otherwise.

c. Given the reaction of firm $I$, determine the optimal decisions $(K_E, p_E)$ of the entrant. What is the effect of $c_E$ on these decisions?

- The firm $E$ can sell if and only if I chooses $p_{max}$. Therefore, $E$ must set $p_E = \frac{(D-K_E)p_{max}}{D}$, that is a sufficiently low price and maximises

$$K_E(\frac{D-K_E}{D}p_{max} - c_E)$$

which gives $K_E^* = \frac{D}{2}(1 - \frac{c_E}{p_{max}})$ and $p_E^* = \frac{p_{max}+c_E}{2}$.

- If $c_E = 0$, i.e; the entrant is as efficient as the incumbent, $K_E^* = \frac{D}{2}$, the two firms share the market and the price is $\frac{p_{max}}{2}$.



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d. Determine the profit of the two firms. In equilibrium, profits are:

$$\Pi_I = p_{max}(D - K_E^*) = \frac{D(p_{max} + c_E)}{2}$$

$$\Pi_E = \frac{D}{p_{max}} \frac{(p_{max} - c_E)^2}{4}$$

A less efficient entrant can enter the market and realize a positive profit when facing an incumbent more efficient and with more capacity. The entrant chooses a relatively low size to make it very costly for the incumbent to go into a price war.

e. What is the equilibrium if the incumbent can set a personalized price for each customer?

With personnalized prices, I would sell at $p_E - \epsilon$ at population $K_E$ but at $P_{max}$ to other consumers and entry would be always deterred.

# Strategic Incumbent and entry

1. A taxonomy of incumbent's investments strategies
   - "Top-dog strategy": investment in capacity
   - "Lean and hungry look strategy": an innovation model
2. The chain store paradox : a reputation game
3. Exclusive dealing: a contracting strategy

# A taxonomy of incumbent's investments strategies

- In stage 1, the incumbent chooses the level of some irreversible investment $K_1$.
- In stage 2, after observing $K_1$, $E$ decides to enter or not. Products market decisions are taken, denoted $\sigma_1$ and $\sigma_2$ (price or quantity).
  - If E enters, $\sigma_1$ and $\sigma_2$ are chosen simultaneously, and profits are denoted $\pi_1(K_1, \sigma_1, \sigma_2)$ and $\pi_2(K_1, \sigma_1, \sigma_2)$.

    We assume that $\pi_2(K_1, \sigma_1, \sigma_2)$ includes entry cost if any.

    We assume that there exists a unique Nash equilibrium of this competition stage that results in $(\sigma_1^*(K_1), \sigma_2^*(K_1))$.
  - If E does not enter, the incumbent obtains $\pi_1^m(K_1, \sigma_1^m(K_1))$.
- Two strategies: Entry deterrence and Accomodation.

# Entry deterrence

- $K_1$ is set at a level sufficient to deter entry i.e. such that:

$$\pi_2(K_1, \sigma_1^*(K_1), \sigma_2^*(K_1)) = 0$$

- To see how $K_1$ must be distorted, we totally differentiate $\pi_2$ with respect to $K_1$ :

$$\frac{d\pi_2}{dK_1} = \underbrace{\frac{\partial \pi_2}{\partial K_1}}_{\texttt{Direct Effect}} + \underbrace{\frac{\partial \pi_2}{\partial \sigma_1}\frac{\partial \sigma_1^*(K_1)}{\partial K_1}}_{\texttt{Strategic Effect}} + \underbrace{\frac{\partial \pi_2}{\partial \sigma_2}\frac{\partial \sigma_2^*(K_1)}{\partial K_1}}_{\texttt{0 Envelop theorem}}$$

  - Sign of direct effects :advertising informative ($\frac{\partial \pi_2}{\partial K_1} > 0$) or persuasive ($\frac{\partial \pi_2}{\partial K_1} < 0$), investment in capacity ($\frac{\partial \pi_2}{\partial K_1} = 0$)
  - Strategic effect : given $K_1$ it is a commitment for the incumbent to be tough or weak in its decision of $\sigma_1(K_1)$
  - If $\frac{d\pi_2}{dK_1} < 0$, investment makes the incumbent tough: "top dog"; If $\frac{d\pi_2}{dK_1} > 0$, investment makes the incumbent soft: "lean and hungry look".

# Entry accomodation

- $K_1$ is set at its best accomodating level, i.e. :

$$\max_{K_1} \pi_1(K_1, \sigma_1^*(K_1), \sigma_2^*(K_1))$$

- To see how $K_1$ must be distorted, we totally differentiate $\pi_1$ with respect to $K_1$ :

$$\frac{d\pi_1}{dK_1} = \underbrace{\frac{\partial \pi_1}{\partial K_1}}_{\texttt{Direct Effect}} + \underbrace{\frac{\partial \pi_1}{\partial \sigma_1}\frac{\partial \sigma_1^*(K_1)}{\partial K_1}}_{\texttt{0 Envelop theorem}} + \underbrace{\frac{\partial \pi_1}{\partial \sigma_2}\frac{\partial \sigma_2^*(K_1)}{\partial K_1}}_{\texttt{Strategic Effect}}$$

  - The direct effect is the “profit maximizing effect" with no effect on firm 2.
  - The strategic effect:

$$Sign(\frac{\partial \pi_1}{\partial \sigma_2}\frac{\partial \sigma_2^*(K_1)}{\partial K_1}) = Sign(\frac{\partial \pi_2}{\partial \sigma_1}\frac{\partial \sigma_1^*(K_1)}{\partial K_1}) x Sign(\frac{d\sigma_2^*}{d\sigma_1})$$

Table: TAXONOMY

| | | |
|---|---|---|
| Strategic substitutes | (D) Top Dog | (D) Lean & Hungry |
| $\frac{d\sigma_2^*}{d\sigma_1} < 0$ | (A) Top Dog | (A) Lean & Hungry |
| Strategic complements | (D) Top Dog | (D) Lean & Hungry |
| $\frac{d\sigma_2^*}{d\sigma_1} > 0$ | (A) Puppy Dog | (A) Fat Cat |

- Top Dog: Overinvestment;
- Lean & Hungry: Underinvestment;
- Puppy Dog: Overinvestment for (D) and Underinvestment for (A);
- Fat Cat: Underinvestment for (D) and Overinvestment for (A).

# A top dog example: Investment in capacity

- In stage 1, an incumbent firm 1 sets its capacity $\bar{q_1}$.
- In stage 2, the entrant 2 decides to enter or not. In case of entry the two firms set additional capacity $\Delta\bar{q_1}$ and $\Delta\bar{q_2}$ respectively and produce at most $\bar{q_1} + \Delta\bar{q_1}$ for the incumbent and $\Delta\bar{q_2}$ for the entrant.
- Products are homogeneous and the inverse demand function is $P = 1 - q_1 - q_2$.
- Entry cost : $e$
- $k$ is the marginal cost of capacity.
- $c$ the marginal cost of production.


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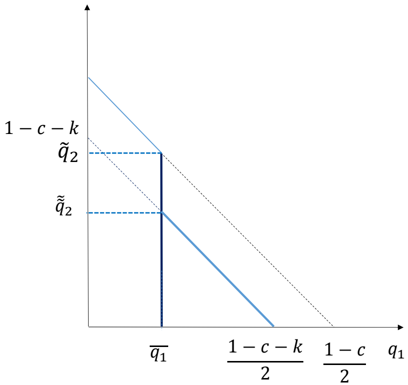
Assume that 2 has entered. The incumbent's profit is:

$$\pi_1 = (1 - q_1 - q_2 - c)q_1 - k\Delta\bar{q_1}$$

Maximizing this function with respect to $q_1$ it follows that the best reaction function is:

$$q_1(q_2) = \begin{cases} \frac{1}{2}(1 - q_2 - c - k) & \text{for } q_1 > \bar{q_1}, \\ \frac{1}{2}(1 - q_2 - c) & \text{for } q_1 \leq \bar{q_1} \end{cases}$$



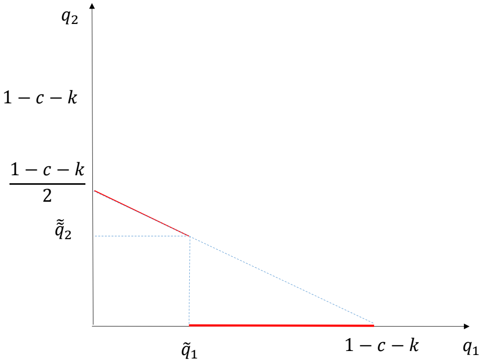
The entrant's profit is:

$$\pi_2 = (1 - q_1 - q_2 - c)q_2 - k\Delta\bar{q}_2 - e$$

Maximizing this function w.r.t. $q_2$, the best reaction function is:

$$q_2(q_1) = \begin{cases} \frac{1}{2}(1 - q_1 - c - k) & \text{for } q_1 < \tilde{q}_1, \\ 0 & \text{for } q_1 \geq \tilde{q}_1 \end{cases}$$

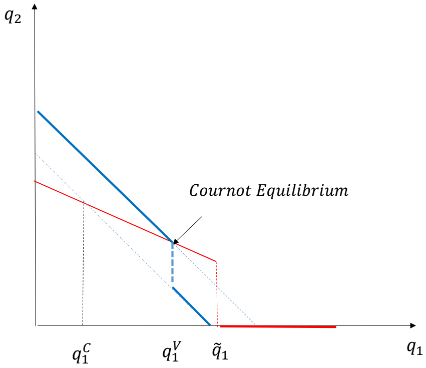
$$\tilde{q}_1 = 1 - c - k - 2\sqrt{e} \Leftrightarrow \pi_2(q_2(q_1), q_1) = \frac{1}{4}(1 - q_1 - c - k)^2 - e = 0$$



# 4 cases to consider

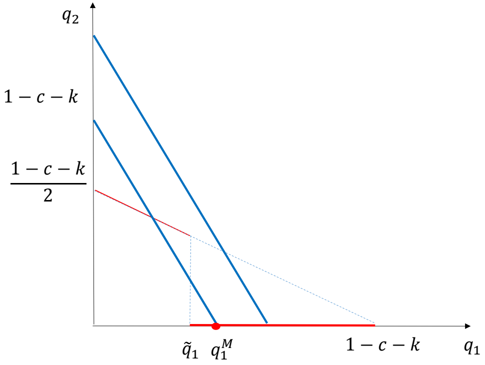
1. Inevitable entry: $\tilde{q}_1 > q_1^V \Rightarrow e < e^- = \frac{1}{9}(1-c-2k)^2$. $q_1^V$ corresponds to a Nash equilibrium between the entrant 2 and an unconstrained firm 1.

- if $\bar{q}_1 = q_1^V \Rightarrow \pi_1 = \frac{1}{9}(1-c+k)(1-c-2k)$
- if $\bar{q}_1 = q_1^C \Rightarrow \pi_1^C = \frac{1}{9}(1-c-k)^2$.



# 4 cases to consider

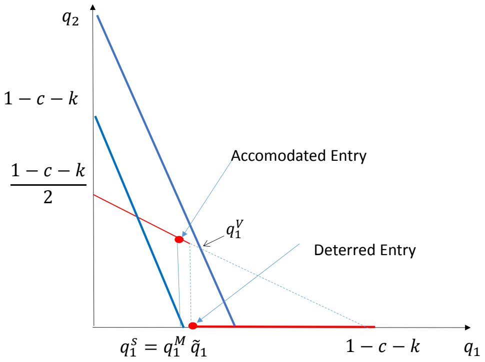
2. Blockaded entry
   $q_1^M = \frac{1}{2}(1-c-k)$ and $q_1^M > \tilde{q}_1 \Rightarrow e > e^+ = \frac{1}{16}(1-c-k)^2$
   - Then $\bar{q}_1 = q_1^M \Rightarrow \pi_1^M = \frac{1}{4}(1-c-k)^2$



# 4 cases to consider

If $q_1^M < \tilde{q}_1 < q_1^V \Leftrightarrow e^- < e < e^+$

3. Deterred entry $\bar{q}_1 = \tilde{q}_1$ — Commitment from 1 to be on its highest reaction function $\Rightarrow$ credible that $q_1 = \tilde{q}_1$ and no entry.
4. Accomodated entry
   - $\bar{q}_1 = q_1^S = \frac{1}{2}(1 - c - k) = q_1^M < \tilde{q}_1$. In the competition stage, 1 is on the high reaction function only if $q_1 \leq q_1^M < q_1^V$.



If $q_1^M < \tilde{q}_1 < q_1^V \Leftrightarrow e^- < e < e^+$

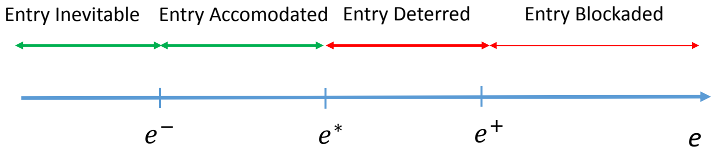
- The profit obtained in case of accomodation is:

$$\max_{q_1^s} \pi_1(q_1^s, q_2(q_1^s)) = \frac{1}{2}(1 - c - k - q_1^S)q_1^S \Rightarrow \pi_1^A = \frac{1}{8}(1 - c - k)^2$$

- To deter entry, the incumbent must install a larger capacity $\tilde{q}_1$ and its profit is:

$$\pi_1^D = (1 - c - k - \tilde{q}_1)\tilde{q}_1 = 2\sqrt{e}(1 - c - k - 2\sqrt{e})$$

It is possible to show that $\pi_1^D > \pi_1^A$ if $e > e^* = \frac{(2-\sqrt{2})^2(1-c-k)^2}{64}$.



# Remember

This investment capacity model illustrates the TOP DOG strategy for Deterrence:

- Deterrence $\rightarrow q_1 = \tilde{q}_1$ which corresponds to a capacity expansion above the monopoly level.
- Accomodation $\rightarrow q_1^S = q_1^M$ which corresponds to a capacity expansion above the competition level ($q_1^C = \frac{1-c-k}{3}$).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# Lean and Hungry look: An innovation model

## Assumptions

- **Period 1**: Firm 1 can make an investment $K_1$ to reduce its marginal cost $c(K_1)$ and obtain the corresponding gross profit $\pi^M(c(K_1))$ which strictly increases in $K_1$ in period 1.
- **Period 2** Firm 2 may enter at a fixed cost $F$. When firm 2 enters, 1 and 2 compete in $R\&D$:
  - To innovate with probability $\rho_i$ costs $\rho_i^2/2$.

  Innovation is drastic and leads to a marginal cost $c$.

Table: Gains in period2

| Innovation probabilities | $\rho_2$ | $(1-\rho_2)$ |
|---|---|---|
| $\rho_1$ | $(0,0)$ | $(\pi^M(c),0)$ |
| $(1-\rho_1)$ | $(0,\pi^M(c))$ | $(\pi^M(c(K_1),0)$ |

**Period 2**: Firms 1 and 2 choose their $R\&D$ levels $\rho_1$ and $\rho_2$ to maximize their expected profit:

$$\begin{aligned}\pi_1 &= \rho_1(1-\rho_2)\pi^M(c) + (1-\rho_1)(1-\rho_2)\pi^M(c(K_1)) - \rho_1^2/2,\\ \pi_2 &= \rho_2(1-\rho_1)\pi^M(c) - \rho_2^2/2\end{aligned}$$

FOCS are:

$$\begin{cases}(1-\rho_2^*)(\pi^M(c) - \pi^M(c(K_1)) = \rho_1^*,\\ (1-\rho_1^*)\pi^M(c) = \rho_2^*\end{cases}$$

The equilibrium investments $\rho_1^*$ and $\rho_2^*$ that solve the above system are such that $\frac{\partial \rho_1^*}{\partial K_1} < 0$ and $\frac{\partial \rho_2^*}{\partial K_1} > 0$. FOC

**Deterrence**

$$\frac{d\pi_2(K_1, \rho_1^*, \rho_2^*)}{dK_1} = -\rho_2^*\pi^M(c)\frac{\partial \rho_1^*}{\partial K_1} > 0$$

The deterrence strategy consists in reducing $K_1$.

**Accomodation**

$$\begin{aligned}\frac{d\pi_1(K_1,\rho_1^*,\rho_2^*)}{dK_1} &= \frac{\pi_1(K_1,\rho_1^*,\rho_2^*)}{\partial K_1} - (\rho_1^*\pi^M(c) + (1-\rho_1^*)\pi^M(c(K_1))\frac{\partial\rho_2^*}{\partial K_1} \\ &< \frac{\pi_1(K_1,\rho_1^*,\rho_2^*)}{\partial K_1}\end{aligned}$$

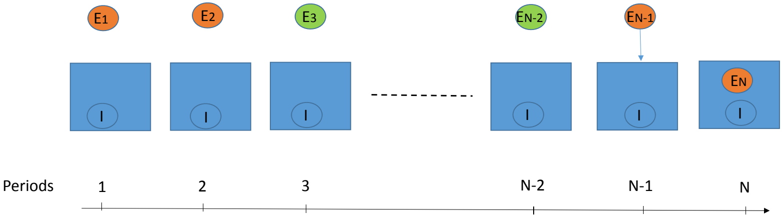
where $\frac{\pi_1(K_1,\rho_1^*,\rho_2^*)}{\partial K_1} = (1-\rho_1^*)(1-\rho_2^*)\frac{\partial\pi^M(c(K_1))}{\partial K_1}$
The accomodation strategy consists in reducing $K_1$.

## Lean and Hungry look

In period 1 firm 1 underinvests in $K_1$ to commit itself to being more aggressive in its $R\&D$ race in period 2. This is the best strategy both to deter entry or accomodate.

**Why?** $R\&D$ investments are strategic substitutes and the larger $K_1$ the higher $\pi^M(c(K_1))$ and therefore the lower the incumbent's incentive to invest in period 2 (Arrow replacement effect).

# The chain store paradox (Selten, 1978)




- An incumbent firm I which owns stores in N markets.
- Entry takes place sequentially
  1. $E_1$ enters or not in period 1 on a first market.
  2. Another $E_2$ enters or not on a second market in period 2.
  3. ...
  4. The last $E_N$ enters or not on market N in period N.

- Without entry the gain of I in each store is: $a$
- In case of entry, gains of firm I and $E_i$ are:

Table: Payoffs in case of entry

| Choice of I | Fight | Accomodate |
|---|---|---|
| Payoffs (I,$E_i$) | (-1,-1) | (0,b) |

- We solve the game backward.
- In period $N$, if $E_N$ enters, the best choice for player $I$ is to accomodate. Long run consideration do not come in, since after period $N$ the game is over.
- In period $N-1$, a fight in period $N-1$ would not deter player $N$ to enter, therefore in $N-1$ the best strategy for I is to accomodate.
- By induction theory, the unique sequential equilibrium is such that in each period $t$, $E_t$ enters and I accomodates.
- Selten Paradox (1978): Incomplete information framework, i.e. I can be of type tough or weak with a probability => a reputation issue!!

# The chain store game with reputation

- Same framework except that I can be tough (on all markets) with probability (p) and weak with proba (1-p)
- Each $E_i$ can be tough with probability (q) and weak with proba (1-q)
- **Tough I always fights ; Tough $E_i$ always enters.**

Table: Payoffs in case of entry

| Choice of a weak I | Fight | Accomodate |
|---|---|---|
| Payoffs (I,$E_i$) | (-1,-1) | (0,b) |

- We solve the game backward.



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# The case $N = 1$

It is a one period game $\Rightarrow$ **No reputation effect**.

- **A tough I fights**.
- A weak I accomodates.
- $p$ is the probability that the incumbent is tough.
- When the expected gain of a weak $E_1$ is $-p + (1-p)b > 0$, i.e. $p < \underline{p} = \frac{b}{b+1}$, $E_1$ enters. Otherwise, $E_1$ stays out.
- If $p < \underline{p} = \frac{b}{b+1}$, a weak I gains 0. If $p \geq \underline{p} = \frac{b}{b+1}$, I gains $a$.

# The case $N = 2$

It is a two-period game $\Rightarrow$ **A reputation effect may take place**.

- **A tough I fights.**
- What is the strategy for a weak I?
  - If I accomodates in $t = 1$, then, in $t = 2$, $E_2$ knows that I is weak and always enters. The expected gain of a weak I is 0.
  - If I fights in $t = 1$, and if then in $t = 2$ $E_2$ believes that I is tough and stays out, the expected gain of a weak I is $-1 + \delta(1 - q)a$ (with the complementary probability (1-q), $E_2$ **is tough and enters**).

If $-1 + \delta(1 - q)a < 0$, there is **No reputation strategy** for a weak I.

In $t = 1$, a weak $E_1$ enters if $p < \underline{p} = \frac{b}{b+1}$ and stays out otherwise.

- If I accomodates in $t = 1$, a weak $E_2$ enters.
- If I fights in $t = 1$, a weak $E_2$ stays out.

If $-1 + \delta(1-q)a > 0$, **A reputation strategy** for a weak I may arise.

A weak I wants to fight in $t = 1$ with a positive probability $\beta$ to deter entry in $t = 2$. We focus directly on the interesting case in which $E_2$ is a weak entrant.

- If $p > \underline{p}$,
  - If I fights in $t = 1$, the revised probability that I is tough is $p(tough/fight) = \frac{p}{p+\beta(1-p)} > p > \underline{p}$ and a weak $E_2$ stays out. Bayes
  - Because fighting in $t = 1$ always deters entry in $t = 2$, a weak I always fights ($\beta = 1$) in $t = 1$ and earns the expected profit : $-1 + \delta(1-q)a > 0$
  - Not fighting in $t = 1$ brings 0 to I.

If $-1+\delta(1-q)a>0$, a weak I wants to fight in $t=1$ with a positive probability $\beta$ to deter entry in $t=2$.

- If $p<\underline{p}$,
  - If $I$ fights in $t=1$, $E_2$ then revises its beliefs accordingly and now believes that I is tough with a probability: $p(tough/fight)=\frac{p}{p+\beta(1-p)}>p$.
  - In $t=2$, still $E_2$ knows that a weak I accomodates and a tough I fights (last period) but he takes into account the revised probability that $I$ is tough $p(tough/fight)$. A weak $E_2$ is indifferent between entering or not if: $-\frac{p}{p+\beta(1-p)}+(1-\frac{p}{p+\beta(1-p)})b=0$, i.e. if $\beta^*=\frac{p}{(1-p)b}$.
  - Going backward to $t=1$, $E_1$ knows that I plays this reputation effect to deter entry in $t=2$ and therefore anticipates that I fights with a probability $p+(1-p)\beta^*=p\frac{(1+b)}{b}$.
  - A weak $E_1$ prefers to stay out if $-p\frac{(1+b)}{b}+(1-p\frac{(1+b)}{b})b<0$, i.e. if $p>(\frac{b}{1+b})^2$.
  - In $t=1$, the expected profit of a weak I is $-\beta^*+\delta(1-q)a>0$. A higher $\beta$ would be too costly. A lower $\beta$ would let a weak $E2$ enter. Again, not fighting would bring 0 to I.

## Conclusion

Because there are at least two-periods, $E_1$ anticipates that I has an incentive to create a reputation of being tough in $t = 1$ to deter entry in $t = 2$, and therefore $E_1$ is less likely to enter also in $t = 1$.

### The generalization to any N is possible

- Assuming that $N = 3$, we now find that $E_1$ enters if and only if $p < (\frac{b}{1+b})^3$ and so on for $N = T$ for $p < (\frac{b}{1+b})^T$.

# Contracts to deter entry

Vertical contracts between manufacturers and retailers might be used to deter entry.

- For instance bundling or full line forcing practices (Coca-Cola case in Multiproduct pricing class)
- Exclusive dealing contracts: Mars *vs* HB case.
  - The case starts in ireland in 1989. Ice-cream bars are mostly sold in gas stations.
  - HB (Unilever) has 79% of the ice-cream bar market and, in 1989, Mars enters.
  - HB freely supplies small retailers with freezers. Mars market share rises up to 42%.
  - HB requires exclusivity: only HB ice cream bars are stock in my freezers. Mars's market share decreases to 20%. Mars cannot fight back by offering its own freezers because shops are too small.
  - The European Court of Justice confirms the EC's prohibition of free freezers.

# Exercice 2: Aghion and Bolton (1987)

$M$ sells a good to $A$ who is willing to pay at most $p = 1$ for one unit. The unit cost of $M$ is $c_M = \frac{1}{2}$. An entrant, $E$ can produce the same good at an unknown unit cost $c_E$ uniformly distributed over $[0, 1]$.

- In $t = 0$, $A$ and $M$ sign a contract or not;
- In $t = 1$, $E$ observes the contract, learns its unit cost $c_E$ and chooses to enter or not.
- In $t = 2$, firms set their prices.
- In $t = 3$, $A$ decides where to buy.

1 Without contract, the competition is a la Bertrand.

a. Determine the equilibrium and the probability $\phi$ of entry. Bertrand $\Rightarrow p^* = max\{c_E, c_M\}$. E enters only if $c_E < c_M$.

The probability of entry is $\phi = P(c < c_M) = c_M = \frac{1}{2}$.

The situation is efficient, the firm who produces is the firm with the lowest unit cost.

b. What are the expected profits? The expected profits are:

$$\Pi_M = \phi 0 + (1-\phi)(1-c_M) = \frac{1}{4},$$

$$\Pi_E = \int_0^{c_M} (c_M - c)dc + 0 = \frac{c_M^2}{2} = \frac{1}{8},$$

$$\Pi_A = \phi(1-c_M) + (1-\phi)0 = c_M(1-c_M) = \frac{1}{4}.$$

$$W = \Pi_M + \Pi_E + \Pi_A = \frac{5}{8}$$

2 M offers a take-it-or-leave-it contract $(P, P_0)$ where $P$ is the price that $A$ must pay if he chooses to buy the good from $M$ and $P_0$ is the penalty $A$ must pay to $M$ if he buys from $E$.

a. Given $(P, P_0)$, under which conditions does $E$ enter?

$\Pi_A = 1 - P_0 - P_E$ if he buys from $E$.

$\Pi_A = 1 - P$ if he buys from $M$.

Therefore $A$ buys from $E$ if $c_E \leq P_E \leq P - P_0$ i.e. $P - P_0 \geq c_E$ and in that case $P_E = P - P_0$.

b. What is the profit of $A$ if he accepts a contract $(P, P_0)$ ?

$\Pi_A = \frac{1}{4}$ without contract.

With the contract,
$\Pi_A(P, P_0) = (P - P_0)(1 - P_E - P_0) + (1 - P + P_0)(1 - P) = 1 - P$ (as $P_E = P - P_0$).

$A$ accepts the contract only if $1 - P \geq \frac{1}{4} \Rightarrow P \leq \frac{3}{4}$.

# Solution

c. Determine the optimal contract $(P, P_0)$ for $M$.

$$\Pi_M(P, P_0) = (P - P_0)P_0 + (1 - P + P_0)(P - C_M)$$

$$\frac{\partial \Pi(P, P_0)}{\partial P_0} = -2P_0 + P + P - c_M = 0$$

Replacing $c_M = \frac{1}{2}$, we obtain:

$$\Rightarrow P_0 = P - \frac{1}{4}.$$

For $P_0 = P - \frac{1}{4}$, the profit of $M$ is $\frac{1}{4}(P - \frac{1}{4}) + \frac{3}{4}(P - \frac{1}{2}) = P - \frac{7}{16}$.

However we know that $P \geq \frac{3}{4}$ to be accepted by $A$.

The optimal contract is thus $P = \frac{3}{4}$, $P_0 = \frac{1}{2}$.

With the exclusive dealing contract, the probability of entry is reduced to $\frac{1}{4}$.

# Solution

d. What are the expected profits under this contract? Comment!
Expected profits are:

$$\Pi_M = (1 - \frac{1}{4})(\frac{3}{4} - c_M) + \frac{1}{4}\frac{1}{2} = \frac{5}{16} > \frac{1}{4},$$

$$\Pi_E = (1 - \frac{1}{4})0 + \int_0^{\frac{1}{4}} (\frac{1}{4} - c)dc = \frac{1}{32} < \frac{1}{8},$$

$$\Pi_A = (1 - \frac{1}{4})(1 - \frac{3}{4}) + \frac{1}{4}(1 - \frac{3}{4}) = \frac{1}{4}.$$

$$W = \frac{19}{32} < \frac{5}{8}$$

The welfare decreases because efficient entries are blockaded.

# References


- Fudenberg, D. and J. Tirole (1991), “Game Theory", MIT Press, Chapter 9.
- Gelman, J. and S. Salop (1983), “Judo Economics: Capacity Limitation and Coupon Competition", *The Bell Journal of Economics*, 14, 2, p315-325.
- Selten, R. (1978),“The Chain Store Paradox", *Theory and Decision*, 9, p127-159.




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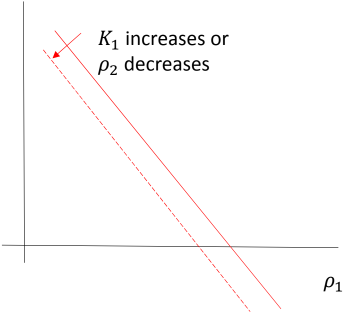
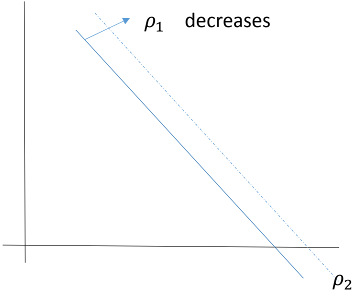
FOC $f(\rho_1, \rho_2, K_1) = 0$

$f_{\rho_1} < 0$




FOC $g(\rho_1, \rho_2, K_1) = 0$

$f_{\rho_2} < 0$



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Two events $A$ and $B$ respectively occur with probability $p(A)$ and $p(B)$. Bayes's rule is as follows:

$$p(A/B) = \frac{p(B/A)P(A)}{p(B)}$$

where conditional probabilities:

- $p(A/B)$ is the likelihood of event A occurring given that B is true;
- $p(B/A)$ is the likelihood of event B occurring given that A is true.