ECO 650: Firms' Strategies and Markets

Innovation

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Exercice 1:

Assumptions:

- Demand is linear, $p = a - q$
- Without innovation, the cost is $\overline{c}$.
- If one firm innovates, she has a unit cost $\underline{c}$.
- N firms compete a la Cournot.
- We denote $\phi = \frac{\overline{c} - \underline{c}}{a - \underline{c}}$.

Questions:

1. Determine the asymetric Cournot equilibrium when one firm innovates.
2. Determine the symmetric Cournot gain without innovation and the net gain to innovation.
3. How does this net gain vary with n?
4. What is the optimal market structure in terms of innovation incentive when $\phi = \frac{1}{4}$? when $\phi = \frac{1}{2}$? when $\phi = \frac{2}{3}$?

Exercice 1: Solution

1. Firm 1 maximizes $(a - q_1 - \sum_{j=2}^{n} q_j - \underline{c})q_1$ Firm $j \neq 1$ maximizes $(a - q_1 - \sum_{j=2}^{n} q_j - \overline{c})q_j$. First order conditions are:

$$
\begin{aligned}
a - 2q_1 - \sum_{j=2}^{n} q_j - \underline{c} &= 0 \\
a - q_1 - 2q_j - \sum_{k=2, k\neq j}^{n} q_k - \overline{c} &= 0
\end{aligned}
$$

   In equilibrium, $q_1 = \frac{a - \underline{c} - (n-1)q_j}{2}$ and $q_j = \frac{a - \overline{c} - q_1}{n}$,
   $\pi_1 = \frac{((a-\overline{c}) + n(\overline{c} - \underline{c}))^2}{(n+1)^2} = \frac{(a-\overline{c})^2}{(n+1)^2}(1 + n\phi)^2$ and
   $\pi_j = \frac{((a-\overline{c}) - (\overline{c} - \underline{c}))^2}{(n+1)^2} = \frac{(a-\overline{c})^2}{(n+1)^2}(1 - \phi)^2$.

2. Firm j maximizes $(a - \sum_{j=1}^{n} q_j - \overline{c})q_j$. First order conditions are $a - 2q_j - \sum_{k\neq j}^{n} q_k - \overline{c} = 0$. In equilibrium $q_j = \frac{(a-\overline{c})}{n+1}$ and $\pi_j = \frac{(a-\overline{c})^2}{(n+1)^2}$.

Exercice 1: Solution

3. $\Delta = \frac{(a-\overline{c})^2}{(n+1)^2}((1+n\phi)^2 - 1) = \phi(a-\overline{c})^2 \frac{n(n\phi+2)}{(n+1)^2}$

$$\frac{\partial \Delta}{\partial n} = \partial_n \frac{n(n\phi+2)}{(n+1)^2} = \frac{2}{(n+1)^3}(n\phi + 1 - n)$$

$$\frac{\partial \Delta}{\partial n} = 0 \Rightarrow \hat{n} = \frac{1}{1-\phi}$$

$\frac{\partial^2 \Delta}{\partial n^2}\big|_{n=\hat{n}} = \frac{2(\phi-1)^4}{(\phi-2)^3} < 0$ since $\phi < 1$. There exists a maximum in n which increases with the innovation level.

4. When $\phi = \frac{1}{4}$, $\hat{n} = 1$ when $\phi = \frac{1}{2}$, $\hat{n} = 2$ and when $\phi = \frac{2}{3}$, $\hat{n} = 3$.

   Conclusion: the market structure that gives the largest incentive to innovate is the monopoly when the innovation size is low and an oligopoly of intermediate level when the innovation size increases.